PERSAMAAN DIFERENSIAL

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Silabus Materi

- Definisi PD dan Peny. PD
- Penggolongan PD
- PD linear order satu
    - PD Terpisah.
    - Fungsi Homogen
    - PD Homogen.
    - PD Eksak
    - Faktor Integral fungsi x saja.
    - Faktor integral fungsi y saja.

Lanjutan silabus

Faktor integral fungsi x dan y.
PD Non Eksak

- Bentuk Umum PD Linear Tingkat Satu
- PD Bernoulli
- Aplikasi PD order satu
- UTS
- PD Linear Order tinggi
- Bentuk Umum
- Penyelesaian umum Persamaan Cauchy-Euler
- Aplikasi PD linear order tinggi

Evaluasi

• Tugas-tugas	25%
• Kuis/Kehadiran	10%
• UTS	30%
• UAS	35%
Jumlah	100%

Referensi

- **A. Wajib :**

[A] Boyce, E.W. & Richard C. DiPrima. 2004. *Elementary Differential Equation and Boundary Value Problems, Eight Edition.* New York: John Wiley&Sons,Inc.

[B] Ross, S.L. 1984**.** *Differential Equations, Third Edition*. New York: John Wiley&Sons,Inc.

- **B. Anjuran :**

[C] Tenenbaum, M. & Harry Pollard. 1963. *Ordinary Differential Equations*. New York: Dover Publication, Inc.

[D] Ayres, F. 1999. *Differential Equations. Schaum's Outline series.* Mc Graw-Hill Company.

[E] Kreyszig, E.2006. *Advanced Engineering Mathematics, 9th ed.* New York: John Wiley & Sons, Inc.

Pendahuluan (Pretes)

- Apa yang dimaksud dengan PD? Berikan contohnya.

- Apa yang Anda ketahui tentang order atau tingkat pada PD?

Lanjutan:

- Sebutkan aplikasi PD yang Anda ketahui.
- Apa yang anda harapkan dari perkuliahan PD semester ini?
- Tulis: Nama, NIM, HP

Pengertian PD

- *suatu bentuk persamaan yang memuat derivatif (turunan) satu atau lebih variabel tak bebas terhadap satu atau lebih variabel bebas suatu fungsi.*

- *Notasi PD:*

  $y' = dy/dx;$

  $x' = dx/dt$

Contoh PD:

1. $$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$$

2. $$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t$$

3. $$\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Klasifikasi PD:

1. PD Biasa :

   sebuah bentuk persamaan yang memuat turunan satu atau lebih variabel tak bebas terhadap satu variabel bebas suatu fungsi.

   Berdasarkan turunan tertinggi;

   - PDB Orde 1 : turunan tertingginya adalah turunan pertama
   - PDB Orde 2 : turunan kedua merupakan turunan tertinggi
   - PDB Orde 3 : turunan ketiga merupakan turunan tertingginya.
   - Dan seterusnya

2. PD Parsial

   Persamaan Differensial yang memiliki lebih dari satu variabel bebas.

PD biasa

$$y' = \sin x + \cos x$$

$$y'' + 7y = 0$$

$$y'' + 3y' - 4y = 0$$

$$y''' - e^x y'' - yy' = (x^2 + 1)y^2$$

PD Parsial

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y} + 2v = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = k$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = e$$

PDB Linear

Dimana $a_0 \neq 0$

Persamaan diferensial biasa linear order *n* dapat dituliskan sebagai:

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_n(x)y = b(x)$$

PD LINEAR ORDER TINGGI

DEFINITION

A linear ordinary differential equation of order *n in the dependent variable y and the independent variable x is an equation that is in, or can be expressed in, the form*

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = F(x), \qquad (4.1)$$

where a_0 is not identically zero. We shall assume that $a_0, a_1, \ldots, a_n$ and F are continuous real functions on a real interval $a \leq x \leq b$ and that $a_0(x) \neq 0$ for any x on $a \leq x \leq b$. The right-hand member F(x) is called the nonhomogeneous term. *If F is identically zero, Equation (4.1) reduces to*

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = 0 \qquad (4.2)$$

and is then called homogeneous.

PD LINEAR ORDER DUA

For $n = 2$, Equation (4.1) reduces to the *second*-order nonhomogeneous linear differential equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = F(x) \tag{4.3}$$

and (4.2) reduces to the corresponding second-order *homogeneous* equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0. \tag{4.4}$$

Here we assume that a_0, a_1, a_2, and F are continuous real functions on a real interval $a \leq x \leq b$ and that $a_0(x) \neq 0$ for any x on $a \leq x \leq b$.

Contoh

- **Example 4.1**

The equation

$$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + x^3y = e^x$$

is a linear ordinary differential equation of the second order.

- **Example 4.2**

The equation

$$\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} - 5y = \sin x$$

is a linear ordinary differential equation of the third order.

Prinsip Superposisi

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = 0. \tag{4.2}$$

THEOREM 4.2 BASIC THEOREM ON LINEAR HOMOGENEOUS DIFFERENTIAL EQUATIONS

Hypothesis. *Let $f_1, f_2, \ldots, f_m$ be any m solutions of the homogeneous linear differential equation (4.2).*

Conclusion. *Then $c_1 f_1 + c_2 f_2 + \cdots + c_m f_m$ is also a solution of (4.2), where $c_1, c_2, \ldots, c_m$ are m arbitrary constants.*

Contoh

Example 4.6

The student will readily verify that sin x and cos x are solutions of

$$\frac{d^2y}{dx^2} + y = 0.$$

Theorem 4.2 states that the linear combination $c_1 \sin x + c_2 \cos x$ is also a solution for any constants c_1 and c_2. For example, the particular linear combination

$$5 \sin x + 6 \cos x$$

is a solution.

Contoh: lanjutan

▶ **Example 4.7**

The student may verify that e^x, e^{-x}, and e^{2x} are solutions of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0.$$

Theorem 4.2 states that the linear combination $c_1 e^x + c_2 e^{-x} + c_3 e^{2x}$ is also a solution for any constants c_1, c_2, and c_3. For example, the particular linear combination

$$2e^x - 3e^{-x} + \tfrac{2}{3}e^{2x}$$

is a solution.

Latihan

4. Consider the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0. \tag{A}$$

(a) Show that each of the functions e^x and e^{3x} is a solution of differential equation (A) on the interval $a \leq x \leq b$, where a and b are arbitrary real numbers such that $a < b$.

(b) What theorem enables us to conclude at once that each of the functions

$$5e^x + 2e^{3x}, \qquad 6e^x - 4e^{3x}, \quad \text{and} \quad -7e^x + 5e^{3x}$$

is also a solution of differential equation (A) on $a \leq x \leq b$?

(c) Each of the functions

$$3e^x, \qquad -4e^x, \qquad 5e^x, \quad \text{and} \quad 6e^x$$

13. Given that x, x^2, and x^4 are all solutions of

$$x^3 \frac{d^3 y}{dx^3} - 4x^2 \frac{d^2 y}{dx^2} + 8x \frac{dy}{dx} - 8y = 0,$$

show that they are linearly independent on the interval $0 < x < \infty$ and write the general solution.

Metode Reduksi Order

THEOREM 4.6

Hypothesis. *Let f be a nontrivial solution of the nth-order homogeneous linear differential equation*

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = 0. \qquad (4.2)$$

Conclusion. *The transformation $y = f(x)v$ reduces Equation (4.2) to an $(n-1)$st-order homogeneous linear differential equation in the dependent variable $w = dv/dx$.*