

HAND OUT MATEMATIKA DISKRET

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PERTEMUAN 1. ATURAN PENCAHAHAN

1. ATURAN PENJUMLAHAN

Aturan penjumlahan digunakan untuk mengetahui banyaknya cara yang dapat digunakan jika terdapat banyak n_1 cara pada kejadian pertama dan n_2 cara pada kejadian kedua. Banyaknya cara keseluruhan adalah $n_1 + n_2$.

Contoh Penerapan Aturan Perkalian adalah:

Berapa banyak cara keseluruhan jika akan menggunakan baju jika mempunyai 8 buah baju jenis lengan pendek dan 17 baju lengan panjang.

Banyak cara menggunakan baju adalah $8 + 17 = 25$ cara.

2. ATURAN PERKALIAN

Aturan perkalian digunakan untuk mengetahui banyaknya cara yang dapat digunakan jika terdapat banyak n_1 cara pada kejadian pertama dan masing –masing kejadian pada kejadian pertama memiliki sebanyak n_2 cara. Banyaknya cara keseluruhan adalah $n_1 \times n_2$.

Contoh penerapan aturan perkalian adalah

There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

Solution: The procedure of choosing a port consists of two tasks, first picking a microcomputer and then picking a port on this microcomputer. Because there are 32 ways to choose the microcomputer and 24 ways to choose the port no matter which microcomputer has been selected, the product rule shows that there are $32 \cdot 24 = 768$ ports. ◀

3. ATURAN PENJUMLAHAN DAN PERKALIAN

Contoh aturan penjumlahan dan perkalian digunakan apabila menggunakan aturan penjumlahan dan aturan perkalian. Di bawah ini contoh penggunaan aturan ini.

In a version of the computer language BASIC, the name of a variable is a string of one or two alphanumeric characters, where uppercase and lowercase letters are not distinguished. (An *alphanumeric* character is either one of the 26 English letters or one of the 10 digits.) Moreover, a variable name must begin with a letter and must be different from the five strings of two characters that are reserved for programming use. How many different variable names are there in this version of BASIC?

Solution: Let V equal the number of different variable names in this version of BASIC. Let V_1 be the number of these that are one character long and V_2 be the number of these that are two characters long. Then by the sum rule, $V = V_1 + V_2$. Note that $V_1 = 26$, because a one-character variable name must be a letter. Furthermore, by the product rule there are $26 \cdot 36$ strings of length two that begin with a letter and end with an alphanumeric character. However, five of these are excluded, so $V_2 = 26 \cdot 36 - 5 = 931$. Hence, there are $V = V_1 + V_2 = 26 + 931 = 957$ different names for variables in this version of BASIC. ◀

4. Permutasi

Permutasi digunakan untuk menentukan banyaknya cara memilih k objek dari sekumpulan n objek dengan memperhatikan urutan. Banyak cara tersebut adalah P_k^n . $P_k^n = \frac{n!}{(n-k)!}$

Contoh penerapan dari aturan permutasi ini adalah

In how many ways can we select three students from a group of five students to stand in line for a picture? In how many ways can we arrange all five of these students in a line for a picture?

Solution: First, note that the order in which we select the students matters. There are five ways to select the first student to stand at the start of the line. Once this student has been selected, there are four ways to select the second student in the line. After the first and second students have been selected, there are three ways to select the third student in the line. By the product rule, there are $5 \cdot 4 \cdot 3 = 60$ ways to select three students from a group of five students to stand in line for a picture.

To arrange all five students in a line for a picture, we select the first student in five ways, the second in four ways, the third in three ways, the fourth in two ways, and the fifth in one way. Consequently, there are $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways to arrange all five students in a line for a picture. ◀

5. Kombinasi

Kombinasi digunakan untuk menentukan banyaknya cara memilih k objek dari n objek tanpa memperhatikan urutan. Banyak cara tersebut adalah $C_k^n = \frac{n!}{(n-k)!k!}$

Contoh:

How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

Solution: The answer is given by the number of 5-combinations of a set with 10 elements. By Theorem 2, the number of such combinations is

$$C(10, 5) = \frac{10!}{5!5!} = 252. \quad \blacktriangleleft$$

A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?

Solution: The number of ways to select a crew of six from the pool of 30 people is the number of 6-combinations of a set with 30 elements, because the order in which these people are chosen does not matter. By Theorem 2, the number of such combinations is

$$C(30, 6) = \frac{30!}{6!24!} = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 593,775. \quad \blacktriangleleft$$

Latihan Soal:

1. There are 18 mathematics majors and 325 computer science majors at a college.
 - a) How many ways are there to pick two representatives so that one is a mathematics major and the other is a computer science major?
 - b) How many ways are there to pick one representative who is either a mathematics major or a computer science major?
2. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?
3. A multiple-choice test contains 10 questions. There are four possible answers for each question.
 - a) How many ways can a student answer the questions on the test if the student answers every question?
 - b) How many ways can a student answer the questions on the test if the student can leave answers blank?
4. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
5. Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco? How many of these pairs involve more than one airline?
6. There are four major autoroutes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?
 7. How many different three-letter initials can people have?
 8. How many different three-letter initials with none of the letters repeated can people have?
 9. How many different three-letter initials are there that begin with an A?
 10. How many bit strings are there of length eight?
 11. How many bit strings of length ten both begin and end with a 1?
 12. How many bit strings are there of length six or less?
 13. How many bit strings with length not exceeding n , where n is a positive integer, consist entirely of 1s?
 14. How many bit strings of length n , where n is a positive integer, start and end with 1s?
 15. How many strings are there of lowercase letters of length four or less?
 16. How many strings are there of four lowercase letters that have the letter x in them?
 17. How many strings of five ASCII characters contain the character @ ("at" sign) at least once? (*Note:* There are 128 different ASCII characters.)
 18. How many positive integers between 5 and 31
 - a) are divisible by 3? Which integers are these?
 - b) are divisible by 4? Which integers are these?
 - c) are divisible by 3 and by 4? Which integers are these?

