# LINEAR MATRIX INEQUALITY BASED ROBUST CONTROL OF A TWO-LINK FLEXIBLE MANIPULATOR

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I declare that this thesis entitled "*Linear Matrix Inequality Based Robust Control of a Two-Link Flexible Manipulator*" is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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## DEDICATION

To my dearest mother, Wari Hasim for her love and blessing.

To my dearly beloved wife, Susiwi for her support and encouragement.

To my children, Fattah Ghiyas Khairi and Noura Lathifa Khairi

for making my life beautiful.

To my great teachers for strongly inspirations.

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# ABSTRACT

This thesis presents modelling and development of a robust control algorithm of a two-link flexible manipulator. A planar two-link flexible robot manipulator incorporating payload that moves in a horizontal plane is considered. A dynamic model of the system is developed using a combined Euler-Lagrange and Assumed Mode Methods (AMM) and simulated using Matlab. Experiments are performed on a lab-scaled two-link flexible manipulator for validation of the developed dynamic model. Two system responses namely hub angular position and deflection at both links are obtained and analysed in time and frequency domains. The effects of payload on the dynamic characteristics of the flexible manipulator are also studied and discussed. Comparisons of the resonance frequencies of the system with a payload of 50 g, relative errors for link-1 are 26 % and 6.4 % for mode 1 and mode 2 respectively, and for link-2 as 12.4 % and 7.4 %. The results show that a close agreement between simulation and experiments is achieved demonstrating an acceptable accuracy of the developed model. For control of the system, a Linear Matrix Inequality (LMI)-based robust Proportional Derivative (PD) controller is developed and analysed. For performance comparison, PD controller based on a closed-loop Ziegler-Nichols (ZN) technique is also designed for control of the flexible manipulator. The performances of the controller are examined in terms of input tracking capability, level of deflection of both links and robustness to various payload conditions. Simulation results show that the LMI-PD controller is robust and able to produces optimal PD gains for various system conditions. Despite using similar PD gains for all conditions, the LMI-PD controller performs better than ZN-PD controller. The proposed controller is also implemented on a lab-scaled two-link flexible manipulator to validate the simulation results and to study real-time capability of the controller. Experimental of the system without payload with the LMI-PD, the settling times are 14.3 % and 43.2 % faster than the results with ZN-PD for link-1 and link-2 respectively. Experimental results show similar result as the simulation results demonstrating good performance of the controller for control of the system with various loading conditions.  For the next investigation other feedback control and control technique can be apply for position control of the system.

# CHAPTER 6

# CONCLUSION AND FUTURE WORK

## 6.1 Conclusion

This thesis has presented investigations into dynamic modelling and control of a two-link flexible manipulator system. Modelling and control approaches have initially been discussed and a research direction has accordingly been identified. Simulation and experimental exercises with a two-link flexible manipulator without and with payload have been performed. The system responses namely hub angular position and deflection at both links have been obtained and presented in the time and frequency domains.

Dynamic modelling of the two-link flexible manipulator has been investigated using the AMM method. A dynamic model of the two-link flexible manipulator incorporating structural damping and payload has been developed. Experiments have been performed on an experimental two-link flexible manipulator and used for verification and assessment of the simulation results. The performance and accuracy of the simulation algorithm has been studied. Moreover, the effects of payload on the dynamic behaviour of the manipulator have been addressed. A close agreement between simulation and experimental results has been achieved in both domains. With increasing payload, the simulation algorithm also shows similar characteristics as the actual systems in terms of time domain specifications and resonance frequencies. It can be concluded that the developed model provides confidence in the accuracy of the model and can be utilised as a test and evaluation platform for development of control strategies for two-link flexible manipulator systems.

The development of LMI-PD robust controller for a two-link flexible manipulator with various payloads has been presented. A set of linear models has been identified to cast the control problem into the LMI framework. Practical steps in designing the robust controller have also been proposed. Besides, ZN-PD controller has also been developed for performance assessment. The performance of the controllers has been evaluated in terms of input tracking capability of hub angular response, level of deflections of both links and robustness to payload variations. For these purposes, a two-link flexible manipulator without payload and payloads of 50 g, 70 g and 100 g is considered.

Simulation and experimental results have shown that LMI-PD controller provide better performance as compared to ZN-PD control. In this case, LMI-PD control gives faster settling time, lower overshoot and low deflection for all loading conditions. Moreover, these are achieved with a single set of PD gains whereas ZN-PD control has to be re-tuned for each loading case. The LMI-PD control technique is robust to payload change in the system where satisfactory responses are achieved under various loading conditions.

### 6.2 Future Work

A number of investigations from this research can be undertaken further. Some of those are identified below: