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WRITING TO ENHANCE MATHEMATICAL UNDERSTANDING

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Abstract

One focus of mathematics teaching and learning is to facilitate students to communicate mathematical ideas or mathematical thinking processes, orally or in writing. Writing is an integral part of mathematics teaching and learning. There are various forms of writing activities in mathematics, such as free writing, biography and autobiography, learning logs and journals, summaries, word problems and mathematical proof. Writing activities that are done consistently not only have an impact on increasing students' writing skills, but also will enhance students' mathematical understanding. Writing enable students to organize and consolidate ideas and their mathematical thinking coherently and systematically. This process allows students to internalize mathematical ideas or concepts strongly and systematically within their cognitive structure. For the teacher, writing is an ideal vehicle for formative assessment, providing teachers with the information they need to adjust their teaching and learning process.

Keywords: writing, enhance, understanding

Introduction

Writing is a very important activity. Through writing, the sciences can be arranged systematically so they can be read and utilized by people across the generations as well as including the underlying technology developments. The ability to write, which is one form of communication, also be comes one of the requirements for individual success in the 21st century. Based upon studies involving a number of countries, Griffin (2012) formulated 21st century skills that he claimed must be learned and mastered by people. The skills are:(1) ways of thinking, which include creativity and innovation, critical thinking, problem solving, decision making, learning to learn and metacognition, (2) ways of working, including communication and collaboration, (3) ways of using working tools, which include information literacy and ICT literacy and (4) ways of living in the world, which includes citizenship (local and global) and life and career, personal and social responsibility; cultural awareness and competencies.

Educational institutions have an important role to develop strategic capabilities, one of which is the writing skill. Is this role performed optimally?

Studies conducted by McGregor (2007) showed that about two-thirds of people in the United States aged 16 to 25 years old stated thate ducational institutions do not equip them with important skills needed to face the challenges of life, including the communication skills. The author's experience in implementing mathematics teaching and learning also reveals that the students' communication skills are not well developed. In solving mathematical problems, many students find it difficult to perform in a coherent mathematical manner or communicate the solutions to the problem appropriately.

Why students' writing skills are not well developed can be caused by several things. One is the limited perception of writing in mathematics. Mathematics teaching and learning is more focused on the application of various mathematical procedures that are not considered directly related to the activity of writing. Today, teachers realize that writing during a mathematics teaching and learning is more than just a way to document information; that it is a way to deepen student learning and a tool for helping students gains new perspectives.

Now, the development of communication skills, including writing skills, is one of the goals of mathematics teaching and learning. Through themathematics teaching and learning, students are expected to communicate mathematical ideas clearly and effectively. (Ministry of Education, 2013). Writing is not a sperate entity from the mathematics curriculum; it is part of it. Among the learning goals that National Council of Teahers of Mathematics (NCTM) has set for all students is to communicate their mathematical thinking (NCTM, 2000)

As suggested by NCTM (2000) the ability to communicate mathematics involves being able to:

1. Express mathematical ideas by speaking, writing, demonstrating and depicting them visually;
2. Understand, interpret and evaluate mathematical ideas that are presented in written, oral or visual forms;
3. Use mathematical vocabulary, notation and structure to present ideas, describe relationships and model situations.

In mathematics, writing activities are not only intended to develop students' writing skills. They are meant to enrich the mathematics teaching and learning and improve students' mathematical understanding. This theoretical paper is intended to explore and describe the role of writing activities todevelop students' mathematical understanding.

Mathematical Understanding

What is the mathematical understanding? Skemp (1976) identified two types of understanding as instrumental understanding and relational understanding. Instrumental understanding refers to the ability to perform a mathematical procedure or apply an algorithm to solve a problem that is not accompanied by a good understanding of why the procedure was applied or why it gives the right solution to the problem. Instrumental understanding was simply described as "rules without reasons". Relational understanding, on other hand, refers to "knowing both what to do and why". Relational understanding also refers to the ability to link a concept with another concept or to define a concept with other concepts and the ability to explain the appropriateness of a strategy or solution to the problem.

Barmby et al. (2007), give some characteristics of mathematics understanding, i.e. being able to see deeper characteristics of a concept, looking for specific information in a situation more quickly, being able to represent situations and envisioning a situation using mental models. Understanding in everyday life is enhanced by the ability to build bridges between one conceptual domain and another. Nickerson (cited in Barmby et al. (2007), also highlighted the importance of knowledge and of relating instruction to student prior knowledge. "The more one knows about a subject, the better one understands it. The richer the conceptual context in which one can embed a new fact, the more one can be said to understand the fact" (p. 2-41).

Hiebert and Carpenter (cited in Barmby et al, 2007) specifically defined degree of mathematical understanding is determined by the number of strength of its connections. A mathematical idea, procedure or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections. Thus, an indication of the understanding of the concept is the ability to link between concepts of mathematics as well as with real life situations. For example, in geometry, understanding of a square concept is shown by the ability to describe its properties and link it to other quadrilaterals. A square can be defined as a rhombus that has exactly one right angle.

How to identify students' mathematical understanding? Errors in solving mathematical problem solving may be an indication of the lack of mathematical understanding, even though it was only instrumental understanding. However, in contrast, the ability to solve a mathematical problem correctly is not necessarily an indication of a good mathematical understanding. Possibly, students may apply certain strategies and perform mathematical procedure well and produce the correct solution, but they may not know why the strategy or the solution is appropriate. The students may have an instrumental understanding, and not a relational understanding. Hiebert and Carpenter (cited in Bramby, et al., 2007) stated that "understanding usually can not be inferred from single response on a single task; any individual task can be performed with out correctly understanding" (p.2-45).

Simple calculations that we often use in the classroom are limited in their ability to indicate understanding. A variety of tasks, then, are needed to generate a profile of behavioural evidence. One way to identify students' mathematical understanding is through writing in a mathematical task. Bramby et al. (2007), say that "if we ask students to explain what they are doing in mathematical tasks, then we can try and infer the links that they have made between different mental representations" (p. 2-45). The role of writing in mathematics to develop a mathematical understanding will be described in the following sections.

Writing in Mathematics

Humans beings are social creatures that require communication. Writing is a form of highly effective verbal communication to express one's thoughts. Writing plays an important role in the development of science. Mathematicians who have found or proven a theorem find it necessary to communicate through writing in order to be understood by other mathematicians.

Writing is a very important activity in the mathematics teaching and learning process. Through writing, a teacher can communicate mathematical concepts systematically. By writing, students can express their understanding. Writing skills need to be developed in the learning activities, including in mathematics teaching and learning activities. Writing has become one of the goals of learning mathematics.

According to Countrymann (cited in McREL, 20007), there are some type of writing in mathematics such as: free writing, biography, autobiography, learning logs, blogs and journals. Free writing involves writing nonstop for a fixed amount of time, usually just a few minutes. Free writing doesn't allow time for students to organize their grammar or spelling; rather, it encourages students to think freely and raise questions about a topic or ideas. Teacher can introduce "The journalists' questions" (Who? What? When? Where? Why? How?). This traditional reporters' style of writing is a way of identifying essential information.

Biography and autobiography are types of narrative non-fiction writing that encourages students to write descriptively, and to identify significant events, personality traits, turning points, and impacts on a person's life. The life stories of important mathematicians will intrigue some of students, and there are many web sites with biographies, (such as Omnibiography.com, allmath.com/biography.php; or Simpsonsmath.com), where students can read the mathematics biographies. At the beginning of a year, a teacher could ask students to write about their experiences learning mathematics, describing the strategies, their instructions used to help them learn.

Learning logs focus on content, whereas journals might focus on students' ideas and questions about a broad range of general topics. While journaling, they might reflect on anything they consider relevant, carry on conversations with their teachers, or both. Learning logs, on the other hand, are less about "feeling" and more about understanding content. Similarly, blogs allows students to share ideas and solutions in real time with each other, other classes and the teacher. A learning log usually consist of some information such as date, name of topic, pages read, length of discussion/explanation, main points and summary of main points. When students write a log, they internalize the processes and the content. For example, on unit of the area of polygons, a students spends 10 minutes writing everything they learned, including formulas and descriptions. They then read over the writing and select one idea to explore further, such as why the formula to determine the area of a triangle works for all triangles, regardless of type. Then 10-minute writing allows them to jutify their thinking or reflect on their understanding.

The teacher can use a double-entry journal especially in the problem solving process. The first column is used to present proof steps and the second column to present reasons, explanations or any rational measures presented in the left column. Here's an example of proving theorems in the form of two columns.

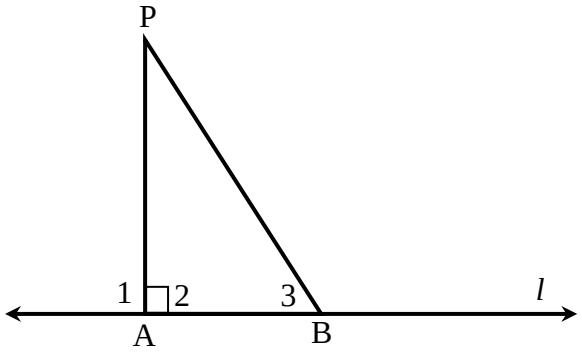
Example 1

Theorem: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Given: $\overline{PA} \perp l$

$\overline{PA}$ is any nonperpendicular segment from A to l

Prove: PB > PA
Proof



Statements	Reasons
1. $\overline{PA} \perp l$	1. Given
2. $\angle 1$ and $\angle 2$ are right angles	2. $\perp$ lines angle are congruent
3. $\angle 1 \cong \angle 2$	3. All right angles are congruent
4. $m\angle 1 = m\angle 2$	4. Definition of congruent angles
5. $m\angle 1 > m\angle 3$	5. Exterior Angle Inequality Theorem
6. $m\angle 2 > m\angle 3$	6. Substitution Property
7. PB > PA	7. If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the the lesser angle

Example 2
Theorem
Opposite angles of a parallelogram are congruent.
Given: parallelogram ABCD
Prove: $\angle A \cong \angle C$, $\angle D \cong \angle B$
Proof

Statements	Reasons
1. Parallelogram ABCD	1. Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	2. Definition of parallelogram
3. $\angle A$ and $\angle D$ are supplementary $\angle D$ and $\angle C$ are supplementary $\angle C$ and $\angle B$ are supplementary	3. If parallel lines are cut by a transversal, consecutive interior angles are supplementary
4. $\angle A \cong \angle C$ $\angle D \cong \angle B$	4. Supplements of the same angles are congruent

Other forms of writing assignments according to Harrell (2003) is summing up the daily instruction and creating mathematical word problem. In summing up the activities of daily instruction, immediately after the completion of learning activities,

students are asked to write a brief description of amathematical concept. One key to practice summing up daily instruction is to provide students with insightful questions that allow students to summarize the lesson. Examples of question to sum up a daily lesson:

1. What strategy would you use to determine two triangles are similar.
2. Describe the characteristics of a trapezoid. List the minimum requirements to show that a quadrilateral is a trapezoid
3. What kinds of angles are formed when streets intersect? Include the types of angles that might be formed by two intersecting lines, and a sketch of intersecting streets with angle measures and angle pairs identified.
4. Explain how parallel lines and planes are used in architecture. Include a description of where you might find examples of parallel lines and parallel planes, and skew lines and nonparallel planes.
5. Explain how slope is used in transportation. Include an explanation of why it is sometimes important to display the grade of a road and an example of slope used in transportation other than roads.

Creating mathematical word problems is another powerful writing tool to support mathematical learning. Writing mathematical problems allow students to relate mathematical operations to real world situastions, communicate mathematical concepts in writing and share their ideas with the peers. For the teacher, student-created word problems provide insight into the depth of students' mathematical understanding processes and helps teachers detect what concepts need reinforcement. Example of a create a problem question:

Create a problem that can be solved using this algorithm. (18 + 4) : 2.

Problems than be created by students:

There were eighteen children playing tag in the park. Four children came and joined them. Now there where twenty two children playing tag in the park. Everyone then decided to play football instead and the eighteen children broke up into two teams. Then there were six children on each team.

Another form of assignment or writing activity is find the error. In this task, the students are presented with two statements about a concept. Students are asked to identify and write down a description of the error on one of these statements. Here's an example of the task.

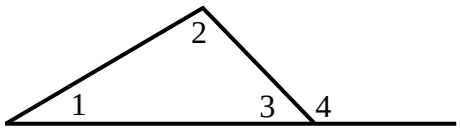
Example 3

Edy and Tono are describing ways to show that a quadrilateral is a parallelogram. Who is correct? Explain your reasoning.

Edy	Tono
A quadrilateral is a parallelogram if one pair of sides is congruent and one pair of opposite sides is parallel	A quadrilateral is a parallelogram if one pair of opposite sides is congruent and parallel

Example 4

Hasan and Atika are discussing the exterior angle theorem. Who is corect?



Hasan	Atika
m∠1 + m∠2 = m∠4	m∠1 + m∠2 + m∠4 = 180

Example 5

Jono and Jony are determining whether 5 – 12 – 13 is a Pythagorean triple. Who is correct? Explain your reasoning.

Jony	Jono
132 + 52 = 122 (?)	$5^2 + 12^2 = 13^2$
169 + 25 = 144 (?)	25 + 144 = 169
193 ≠ 144	169 = 169
NO	YES

Writing and Mathematical Understanding

There are many benefits of implementing writing in mathematics. For teachers, writing can help them understand the students' thinking. Writing assignments such as summary or find an error has the potential to identify student misconceptions. Writing can be seen as one of the effective forms to evaluate students' mathematical understanding. When students write explanations of their work and give examples, teachers can better assess student understanding and progress throughout time. Writing is an ideal vehicle for formative assessment, providing teachers with the information they need to adjust their instruction. Writing can help teachers understand student thinking as well as broaden students understanding of mathematical concepts. Through mathematical writing students are able to clarify their thinking, communicate ideas or questions they were not able to address in class, summarize and connect ideas.

For students, writing activities have very positive impact on the performance of students in understanding mathematical ideas and to solve problems. Countrymann (cited in McREL, 2007) explores the relationship between mathematics and writing and describes the benefits of writing in mathematics. While they are writing, students can restate new material in their own words, identify computations that are esay or difficult, or reflect on aspects that confuse. Pugalee (cited in McREL, 2009) conducted a study with 9th-grade algebra students to determine if writing can be an effective instructional tool in mathematics education and found that it may have a positive effect on problem solving because the writer must organize and describe internal thoughts.

Like reading, improving students' writing skills improves their capacity to learn (National Institute for Literacy, 2007). Writing fosters community in a classroom and, because writing is a social act, it is a vehicle for students to learn more about themselves and others. Alvermann (2002) encourages students to read and write in many different ways, because writing challenges students to problem solve and think critically.

Recent research (Harrel, 2003) showed that writing can help students more fully understand mathematical concepts and in turn improve their progress and

increase their test scores.This can be understood because an indication of mathematical understanding is knowing both what todo and why (Skemp, 1976) and can make the connection between concepts and describe the characteristics or attributes of the concept (Barmby et al, 2007). Writing done consistently has great potential to improve students' mathematical understanding. When students are proving a theorem in the form of two columns, for example, students not only develop an instrumental understanding (Skemp, 1976) which is characterized by the ability to perform mathematical procedures on problem solving activities, but also develop a relational understanding. Relational understanding is characterized by the ability of the student togive a reason, an explanation or rationale to any problem solving steps.

Activities such as find the error are also very powerful instimulating students' mathematical understanding. When students identify errors in a statement, the students have used some of the concepts and their relationships to give an explanation or rationale for these statements. In such a context, the students have met the criteria for relational understanding by Skemp (1976) and according to the classification of mathematical understanding by Barmby et al.(2007).

Improving Students' Mathematical Writing

Teachers should facilitate students to develop writing skills. In addition to consistently practice writing activity, the teacher must also consistently provide feedback on writing assignments produced by students. Students need feedback on writing assignments they are working on that will inform about the correct use of mathematical concepts and procedures.

There are a few guidelines to provide feedback onstudents' writing assignments. Feedback can be associated with aspects of students' mathematical understanding and language aspects. Teachers can ask questions such as "is the mathematical content accurate based on the information given?" or "is mathematical vocabulary accurately and appropriately used to explain the mathematics?" Related to aspects of language, the question can be asked is "do the words, phrases and sentences tie the ideas together?", "is the writing clearly the students' own creation?", "how does the writing sound together?" or "is the grammar and spelling correct?"

Conclusion

Writing is an effective tool for students and teachers, that helps students to explain and reflect on their own mathematical thinking while giving teachers insight into students' mathematical thinking. Writing benefits all students by allowing them to reflect back upon what they have learned and gain a deeper understanding of the mathematics. In practice, it has been shown that students who write in mathematics extend their mathematics skills beyond where they would reach by practice alone.

Writing is an activity that can naturally be added into a mathematics teaching and learning process and will support and enhance student understanding.We will come to the same conlusion as mathematics educator Burns (2008) who said that, "I can no longer imagine teaching mathematics without making writing an integral aspect of students' learning" (p. 1).

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