

# An Analysis Of Students' Mathematical Reasoning Ability On Abstract Algebra Course

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**Abstract:** This study aims at revealing the students' mathematical reasoning abilities on abstract algebra courses. The method used a qualitative descriptive method. It involved 31 sixth semester students of the Mathematics Education Study Program, Faculty of Teacher Training and Education, Riau Kepulauan University, Batam, Indonesia in the academic year of 2018/2019. The data collection techniques employed test and interview. The results showed that the four indicators of mathematical reasoning ability obtained the overall mean scores of 42.12% (low category). The lowest indicator was "do mathematical manipulation" with 21.75% (low category), followed by "conclusions drawing, compiling evidence, giving reasons or evidence for validity of the solution" with 40.25% (low category), the indicator of "making a conjecture" was 43.50% (low category), and the indicator of "drawing conclusions from a statement" was 63% (medium category). Based on the findings, the suggestions can be proposed for lecturers to have more exercises related to proofing in order to enhance the students' mathematical reasoning abilities on abstract algebra lectures.

**Index Terms:** Abstract algebra, mathematical reasoning ability, proof, mathematical manipulation, conjecturing, conclusion drawing, generalization

## 1. INTRODUCTION

Mathematics involve the materials that can develop thinking skills, especially reasoning ability. As mentioned by [1] mathematics has axiomatic deductive characteristics that require the reasoning and thinking ability in order to comprehend it. Furthermore, [2] states that there are some different characteristics of mathematics learning for secondary and high education levels. In college, especially mathematics education study program, the materials focus on its fundamental aspects and in details. Meanwhile, mathematics for the secondary schools is on the implementation of theories or mathematical laws that taught in college level. It requires the students to have a high concentration level during the mathematics learning process. The development of mathematics is inseparable from reasoning and proofing. As stated [3], [4], [5], [6] that between mathematics and reasoning cannot be separated one another because understanding mathematics requires reasoning which can be trained through mathematical material. It indicates that mathematical reasoning is very crucial during mathematics learning. In line with this, [7], [8], [9] explain that reasoning is one of the crucial competencies in mathematics as a supporting feature in mathematics learning. [1] adds that reasoning plays a role in solving mathematical problems and the implications of reasoning are usually found in real life. Moreover, [10] points out that reasoning and proofing are the most basic aspects of mathematics learning. One of the courses taught in mathematics education study programs is abstract algebra that aims at developing students' proofing ability. Traditionally, these courses have covered the theoretical aspects of groups, rings, and fields. [11] mentions that one of the major problems

in teaching abstract algebra course is that many students considering it as relatively new experience dealing with an environment that requires them to do rigorous proofing. Similarly, [11], [6] asserted that several abilities must be mastered by students to achieve the goal of abstract algebra learning, one of which is the ability of mathematical reasoning. The reasoning is the thinking process that connects facts or concepts to draw a conclusion [12], [13], [14]. In other words, reasoning can be interpreted as a thinking process to draw conclusions or make a correct statement that has been known to be true. Similarly, [15], [16], emphasize that mathematical reasoning is active thinking that has certain characteristics in finding the truth. People tend to think with various way, structures, or orders in the real world and symbolic situations of objects. Therefore, mathematical proofing is a formal way of expressing certain types of reasoning. Another definition proposed by [17], [18], [19] that reasoning is five interrelated processes in mathematical thinking activities that are categorized as sense-making, conjecturing, convincing, reflecting, and generalizing. Sense-making is closely related to the ability of problem schemes development and represent the knowledge they have. Conjecturing refers to the activity in drawing a conclusion and a theory based on incomplete facts or in other words it is a strategy of completion, argumentation, and communication. Convincing describes the implementation of completion strategy based on the two previous processes. Reflecting contains an activity of re-evaluating those three processes that have been carried out. Based on the above opinion, it can be concluded that reasoning is a thinking activity to perceive conclusions or make a new statement that is correct based on relevant theories. The study from [20] divide indicators of mathematical reasoning ability into four, namely 1) drawing logical conclusions; 2) conjecturing and proofing; 3) giving explanations to the model, making patterns and making connections between facts or concepts; and 4) using the patterns relationships to make analysis, analogy or general conclusions. According to [21], [22] reasoning indicators of mathematics consist of 1) make calculations based on applicable mathematical formulas or rules; 2) draw general conclusions based on visible mathematical processes/concepts; 3) make estimation; and 4) draw conclusions based on the similarity of visible processes/mathematical concepts. Based on the above research, the indicators of mathematical reasoning abilities in this study are presented in Table 1 below.

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**TABLE 1**  
**THE INDICATORS OF MATHEMATICAL REASONING ABILITY**

Reasoning indicators	Descriptions
Making a conjecture	The students ability to make possible solutions according to their knowledge
Performing mathematical manipulation	The students ability to work or solve a problem using certain method so that the goal for problem solving can be achieved
Providing reasons or evidence for the validity of the solution	The students are able to compile evidence or reasons to gain the validity of the solution if they are able to demonstrate it through investigation.
Drawing conclusion	The thinking process that empowers knowledge to propose an idea.

The reasoning ability can help students to grasp mathematics as logical and reasonable course so their beliefs towards mathematics can be nurtured as something that can be understood, learned, and evaluated [23]. It makes mathematical reasoning ability is fundamental in abstract algebra courses where the students must be able to prove the truth of a new theorem based on axioms or theorems that have been verified. Various studies reveal that the ability of mathematical reasoning and proofing is considered difficult among students in various countries [6], [18], [23]. [6] declares that mathematical reasoning abilities can reinforce students' success in understanding abstract algebra material. So, it is very important to analyze mathematical reasoning abilities in abstract algebra courses to provide information for lecturers, especially problems faced by students in the learning process. By having these data, the lecturers can design an effective instructional approach to overcome students' learning difficulties, especially related to reasoning and proofing. Based on the description above, the purpose of this study is to reveal the students' mathematical reasoning abilities in abstract algebra courses.

## 2 RESEARCH METHOD

This research can be categorized as qualitative descriptive research. The research subjects were 31 students of the sixth semester in the Mathematics Education Program, Kepulauan Riau University, Batam, Indonesia in the academic year of 2018/2019. The data collection techniques employed mathematical reasoning test and interview guidelines. The reasoning test instrument consisted of four items that had been tested its content validity through expert judgment. The four test questions were used to measure four predetermined mathematical reasoning indicators, namely 1) making conjectures; 2) performing mathematical manipulation; 3) drawing conclusions, compiling evidence, giving reasons or evidence to guarantee the validity of the solution; and 4) drawing conclusions. To analyze the test results, the researcher applied the holistic rubric scoring to assess students' works on mathematical reasoning ability [24] as

presented in the following Table 2.

**TABLE 2.**  
**THE HOLISTIC RUBRIC FOR SCORING STUDENTS' MATHEMATICAL REASONING ABILITY**

Criteria	Score
Solution is correct and complete. Reasoning in solving problem and its communication are complete. Adequate explanation on the solution, but contain a little defect.	4
Solution is correct. Good reasoning in solving problems and its communication. Explanation on the solution exists though contain some defects.	3
Incomplete solution and adequate reasoning in solving problems and its communication. Defect reasoning emerge clearly. Inaccurate conclusion. Limited understanding on mathematical concept.	2
Problem emerges while imitating mathematical idea and unable to make some development. Lack of reasoning and its communication. A lot of incorrect calculation emerge.	1
No solution exist. No reasoning in solving problem. Neither mathematical understanding nor response on possibilities emerges. Just guessing.	0

categories with the following criteria [25].

**TABLE 3.**  
**CATEGORY OF STUDENTS' MATHEMATICAL REASONING ABILITY**

Achievement percentage of mathematical reasoning abilities	Category
$x_i > 70\%$	High
$55\% < x_i \leq 70\%$	Moderate
$x_i \leq 55\%$	Low

interview session was done with the semi-structured method. It was completed with the interview guideline as the outline of the data that should be obtained in order to clarify the results of the test of the students' reasoning abilities. Of the Six students, 2 were listed as highly capable, 2 were moderate, and the other 2 were low students. They were randomly selected in this interview session. The qualitative data analysis techniques in this study referred to Mile's and Huberman steps which included data reduction, data presentation, and data verification/conclusion drawing. To check the validity of the research findings, the triangulation of the data source was conducted by comparing the tests results of mathematical reasoning abilities and interviews.

## 3 RESEARCH FINDINGS AND DISCUSSION

To obtain the data on students' mathematical reasoning abilities, this study used the test instrument which included four items. The question number 1 was to measure the first indicator, 2 was for the second indicator, number 3 was for the third indicator, and 4 was for the fourth indicator. After the test was given to 31 research respondents, the response were converted into scores according to the rubric in Table 1. The

results of the descriptive analysis in the reasoning ability tests are presented in Table 4 below.

**TABEL 4.**  
**THE SCORES OF STUDENTS' REASONING ABILITY**

Explanation	Scores
Mean	6.74
Standard deviation	2.06
Highest score	10
Lowest score	2
Ideal minimum score	0
Ideal maximum score	16
N	31

Based on Table 4, there were students with the lowest score of 2 and the highest score was only 10 from the maximum score of 16. The overall score was 6.74 and the percentage was 42.12% so that the overall students reasoning abilities included in the low category. Based on the percentage score for each respondent, 4 respondents were included in the medium category and 27 respondents were in the low category. It showed that students' mathematical reasoning ability was still far below the expectation. The following is presented the students' scores distribution for each measured indicator.

**TABLE 5.**  
**THE SCORES OF STUDENTS' REASONING ABILITIES IN EACH INDICATOR**

Test item/Indicator	Frequency of students score					Total	Mean	Standard deviation	Percentage
	4	3	2	1	0				
Making conjecture	1	2	17	10	1	31	1.74	0.77	43.50%
Performing mathematical manipulation	0	0	3	21	7	31	0.87	0.56	21.75%
Drawing conclusions, compiling evidence, giving reasons or evidence for validity of the solution	2	6	9	6	8	31	1.61	1.26	40.25%
Drawing conclusions from a statement	9	7	7	7	0	31	2.52	1.23	63%

Based on the results of the frequency distribution, the three indicators of mathematical reasoning ability were in the low category and one indicator categorized as medium. The lowest percentage was the second indicator, followed by the third indicator, the first indicator, and the fourth indicator. In the indicator 2 on mathematical manipulation, none respondents obtained the score of 4 and 3. The third indicator was quite good where there were 17 respondents obtained the score of 2, 3 and 4. However, the high score on the standard deviation caused the achievement becoming small. It can be seen from the existence of 6 respondents who got the score of 1 and 8 respondents obtained the score of 0 and the mean of 1.61 was obtained on the third indicator. Meanwhile, for the first indicator only one respondent had the score of 4. For the fourth indicator as the indicator with the highest achievement percentage, there were still respondents who scored 0. The following is the sample of respondents' answers to questions number 1 to 4 that includes all four indicators.

2) Dik: a dan b anggota ring komutatif R.  
 ab merupakan zero divisor.  
 Dit: Tunjukkan bahwa a maupun b merupakan zero divisor.  
 Jwb: misalkan:  $\mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6, 7\}$  merupakan satu ring komutatif.  
 $\Rightarrow ab$  merupakan zero divisor.  
 $ab=0$   
 $\Rightarrow a \cdot 2 = b \cdot 4$   
 $ab=0 \mid b \cdot a=0$   
 $2 \cdot a=0 \mid a=0$   
 $\Rightarrow a=4 \quad b=6$   
 $a \cdot 6=0 \mid b \cdot a=0$   
 $A \cdot 6=0 \mid b \cdot a=0$   
 $\therefore a$  maupun  $b$  terbukti merupakan zero divisor dan sb.  
 (Lainnya tidak setiap integral)

**Fig 2.** One of the answers for indicator 2

**Penyelesaian**  
 1) Setiap daerah integral merupakan field  
 Bukti :  
 Daerah integral adalah ring komutatif dengan elemen kesatuan dan tidak memuat pembagi nol. Sehingga, membuktikan bahwa suatu daerah integral berhingga adalah suatu field.  
 Cukup ditunjukkan bahwa setiap elemen tak nolnya memiliki invers terhadap perkalian.

**Fig 1.** One of the answers for indicator 1

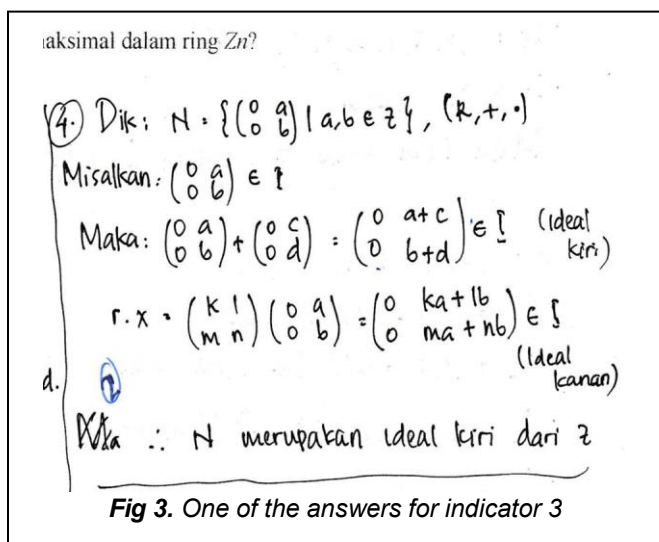


Fig 3. One of the answers for indicator 3

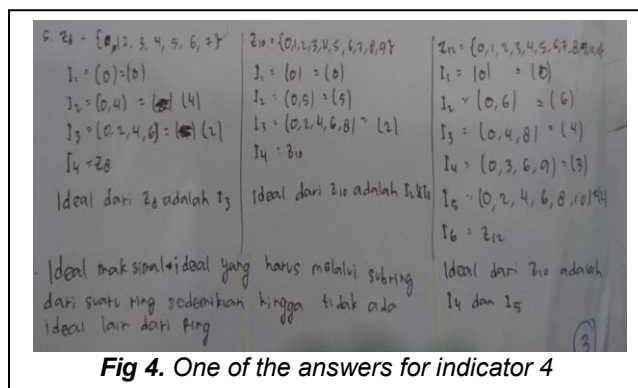


Fig 4. One of the answers for indicator 4

further advancement in mathematics. Moreover, for the question number 3 and 4 that measured the third and fourth indicators, it included the types of routine questions that involved more calculations activities. The item number 3 states "R is the ring of square matrices of order 2 with the matrices addition and multiplication. Is that true that the left ideal of ring R and not the right ideal from R? Please explain your answer!" To solve this problem, respondents simply used one theorem to investigate the ideal of the left or right. Nevertheless, in Fig. 3, there were many procedures missed by the respondent, such as did not write down the terms  $N \neq \emptyset$  and  $N \subset R$ . They also performed the addition operation in the first property that should be subtracted operation, and they were not properly investigating the R and N multiplication properties to prove the ideal left or right. This shows that respondents had not been able to present proof and reasons for the desired solution. The question number 4 contains, "Find for all the maximum ideal of ring  $Z_8, Z_{10}$ , and  $Z_{12}$ . From these results, what you can conclude about the maximum ideal in ring  $Z_n$ ?". In Fig 4, it can be seen that the respondent was able to determine the true ideal and the maximum ideal of  $Z_8, Z_{10}$ , and  $Z_{12}$  but the respondents were not able to conclude or generalize the maximum ideal of  $Z_n$  where it should be known from the existing pattern when completing the maximum ideal for  $Z_8, Z_{10}$ , dan  $Z_{12}$ . To obtain deeper information, the researchers interviewed 6 respondents who had been previously determined. During the interviews session, the six respondents stated that they were lack time to complete the test. The answers presented were in conformity with their abilities. When they were asked about their difficulty in working the test, one of the respondents answered "I am confused where I should start from" Meanwhile, other respondents admitted that they did not really understand the concepts due to many new terms in this course. Many properties and theorems also caused confusion in which theorems must be used in solving reasoning problems, especially those related to proofing. The results of this study are in line with [28], [29], [30] statement that reasoning and proofing abilities are very important for building students' understanding of mathematical concepts. Leithner mentions that reasoning is the foundation of mathematics, unfortunately, students at all grade levels find it difficult in mathematical reasoning problem including undergraduate students of mathematical majors [31], [32], [33]. Therefore, mathematical reasoning skills must be given attention during the lecturing process of abstract algebra courses and should be regularly improved with proving mathematical theorems.

### 4 CONCLUSION

Based on the four indicators of mathematical reasoning ability measured in this study, it can be concluded that the indicator with the lowest percentage was "performing mathematical manipulation" with 21.75% (low category), followed by indicators of "drawing conclusions, compiling evidence, giving reasons or evidence for the validity of the solution" with 40.25% (low category), indicator of "making a conjecture" with 43.50% (low category), and 63% for the indicator of Drawing conclusions from the statement (medium category) respectively. The average percentage of the students' reasoning abilities was 42.12% that can be categorized as low. Most students still experience the main difficulties in proofing and it shows that students' mathematical reasoning

For the question number 1 which measures the indicator of making a conjecture, the student was given that question number 1 stating that "Which of the following two statements is true? Every field is an integral domain or Every integral domain is a field"? Explain and give a conclusion about your answer". Based on the students' answer in Fig. 1, it can be seen that at the beginning the student make the wrong conjecture. This indicates that students did not yet understand the relationship between the integral concepts of the domains and the fields. Although students provide the correct definition for integral domains, the written answers did not synchronize with the test intention and it makes the ability to form the appropriate problem solving failed to be done. For the question number 2 which measured the indicator of mathematical manipulation ability, the respondent was given the question "Suppose that a and b are the members of the commutative ring R and ab are zero-divisors. Please, show that both a and b are zero-divisors!". The results of the respondent's answer in Fig. 2 showed that the respondent performed inappropriate mathematical manipulations by giving an example without doing general proofing as requested in the test item. It indicates that students have not been able to do abstract and manipulative thinking processes in solving proving questions. As expressed by [26] that students hold many misconceptions as they are in transition process from arithmetic to algebraic thinking and these misconceptions can hinder their performance and learning in the subject. On the other hand, [27] propose that by making a transition from concrete arithmetic to the symbolic language of algebra, students develop abstract mathematical cognition for their

ability for abstract algebra courses is still low and needs to be improved. Based on the findings, the suggestions can be proposed for lecturers to have more exercises related to proofing in order to enhance the students' mathematical reasoning abilities on abstract algebra lectures.

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