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A NEW METHOD FOR GENERATING FUZZY RULES FROM TRAINING DATA AND ITS APPLICATION TO FORECASTING INFLATION RATE AND INTEREST RATE OF BANK INDONESIA CERTIFICATE

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Abstract. Table lookup scheme is a simple method to construct fuzzy rules of fuzzy model. That can be used to overcome the conflicting rule by determining each rule degree. The weakness of fuzzy model based on table lookup scheme is that the fuzzy rules may not be complete so the fuzzy rules can not cover all values in the domain. In this paper a new method to generate fuzzy rules from training data will be proposed. In this method, all complete fuzzy rules are identified by firing strength of each possible fuzzy rule. Then, the resulted fuzzy rules are used to design fuzzy model. Applications of the proposed method to predict the Indonesian inflation rate and interest rate of Bank Indonesia Certificate (BIC) will be discussed. The predictions of the Indonesian inflation rate and interest rate of BIC using the proposed method have a higher accuracy than those using the table lookup scheme.

Keywords: fuzzy rule, fuzzy model, firing strength of rule, inflation rate, interest rate of BIC.

1. Introduction

Designing fuzzy rule base is one of some steps in fuzzy modeling. Fuzzy rule base is the heart of fuzzy model. Recently, fuzzy model was developed by some researchers. Wang created fuzzy model based on table lookup scheme, gradient descent training, recursive least squares and clustering methods [8], [9]. The weakness of the fuzzy model based on table lookup scheme is that the fuzzy rule base may not be complete so the fuzzy rule can not cover all values in the domain. Wu and Chen designed membership functions and fuzzy rules from training data using α-cut of fuzzy sets [10]. In Wu and Chen's method, determining membership functions and fuzzy rules needs large computations. To reduce the complexity of computation, Yam *et al*. built a method to decrease fuzzy rules using singular value decomposition [11].

Yen *et al.* [12] developed fuzzy model by combining global and local learning to improve the interpretability. New form of fuzzy inference systems that was highly interpretable based on a judicious choice of membership function was presented by Bikdash [5]. Herrera *et al.* constructed Takagi-Sugeno-Kang models having high interpretability using Taylor series approximation [6]. Then, Pomares *et al*. [7] identified structure of fuzzy systems based on complete rules that can decide which input variables must be taken into account in the fuzzy system and how many membership functions are needed in every selected input variable. Abadi *et al*. designed complete fuzzy rules using singular value decomposition [3]. In this method, the prediction error of training data depended only on taking the number of singular values.

Fuzzy models have been applied to many fields such as in communications, economics, engineering, medicine, etc. Specially in economics, forecasting the Indonesian inflation rate by fuzzy model resulted more accuracy than that by regression method [1]. Then, Abadi *et al.* [2] constructed fuzzy time series model using table lookup scheme to forecast the interest rate of Bank Indonesia certificate (BIC) and its result gave high accuracy. Then, Abadi *et al*. [4] showed that forecasting Indonesian inflation rate based on fuzzy time series data using combination of table lookup scheme and singular value decomposition methods had a higher accuracy than that using Wang's method. Based on the previous research, there are interesting topics in fuzzy model especially in determining fuzzy rules that give good prediction accuracy. To overcome the weakness of table lookup scheme (Wang's method), in this paper, we design complete fuzzy rules of fuzzy model using firing strength of rule and then its result is used to predict the Indonesian inflation rate and interest rate of BIC. The proposed method has higher prediction accuracy than Wang's method in its applications to predicting the Indonesian inflation rate and interest rate of BIC.

The rest of this paper is organized as follows. In section 2, we briefly review the Wang's method to construct fuzzy model. In section 3, we present a new method to construct complete fuzzy rules using firing strength of rule based on training data. In section 4, we apply the proposed method to forecasting the inflation rate and interest rate of BIC. We also compare the proposed method with the Wang's method in the forecasting inflation rate and interest rate of BIC. Finally, some conclusions are discussed in section 5.

2. Wang's method for designing fuzzy rules

In this section, we will introduce the Wang's method to construct fuzzy rules referred from [9]. Suppose that we are given the following N input-output data: $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, $p = 1, 2, 3, ..., N$ where $x_{ip} \in [\alpha_i, \beta_i] \subset R$ and $y_p \in [\alpha_y, \beta_y] \subset R$, i = 1, 2, …, n. Designing fuzzy model using Wang's method is given by the following steps:

Step 1. Define fuzzy sets to cover the input and output domains.

For each space $[\alpha_i, \beta_i]$, i = 1, 2, …, n, define N_i fuzzy sets A_i^j, j = 1, 2, …, N_i which are complete in $[\alpha_i, \beta_i]$. Similarly, define N_y fuzzy sets B^j, j = 1, 2, …, N_y which are complete in $[\alpha_y, \beta_y]$.

Step 2. Generate one rule from one input-output pair.

For each input-output pair $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, determine the membership value of x_{ip}, i = 1, 2, …, n in fuzzy sets A_i^j, j = 1, 2, …, N_i and membership value of y_p in fuzzy sets B^j, j = 1, 2, …, N_y. Then, for each input variable x_{ip}, i = 1, 2, …, n, determine the fuzzy set in which x_{ip} has the largest membership value. In other word, determine A_i^{j*} such that $\mu_{A_i^{j*}}(x_{ip}) \geq \mu_{A_i^j}(x_{ip})$, $j = 1, 2, ..., N_i$. Similarly, determine B^{l*} such that $\mu_{B^{l*}}(y_p) \geq \mu_{B^l}(y_p)$, $l = 1, 2, ..., N_y$. Finally, we construct a fuzzy IF-THEN rule:

$$\text{IF } x_1 \text{ is } A_1^{j*} \text{ and } x_2 \text{ is } A_2^{j*} \text{ and ... and } x_n \text{ is } A_n^{j*}, \text{THEN } y \text{ is } B^{l*} \tag{1}$$

Step 3. Compute degree of each rule designed in step 2.

From step 2, one rule is generated by one input-output pair. If the number of input-output data is large, then it is possible that there are the conflicting rules. Two rules become conflicting rules if the rules have same IF parts but different THEN parts. To resolve this problem, we assign a degree to each rule designed in step 2. The degree of rule is defined as follows: suppose the rules (1) is constructed by the input-output pair $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, then its degree is defined as

$$D(rule) = \prod_{i=1}^{n} \mu_{A_i^{j*}}(x_{ip}) \mu_{B^{l*}}(y_p)$$

Step 4. Construct the fuzzy rule base.

The rule base consists of the following three sets of rules: (1) The rules designed in Step 2 that do not conflict with any other rules; (2) The rule from a conflicting group that has the maximum degree; (3) Linguistic rules from human experts.

Step 5. Construct the fuzzy model using the fuzzy rule base.

We can use any fuzzifier, fuzzy inference engine and defuzzifier combined with the fuzzy rule base to design fuzzy model. If the number of training data is N and the number of all possible combinations of the fuzzy sets defined for the input variables is $\prod_{i=1}^{n} N_i$, then the number of fuzzy rules generated by Wang's method may be much less than both N and $\prod_{i=1}^{n} N_i$. Then, the fuzzy rule base generated by this method may not be complete so that the fuzzy rules can not cover all values in the input spaces. To overcome this weakness, we will design the fuzzy rules covering all values in input spaces.

3. New method for constructing fuzzy rules

Given the following N training data: $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, $p = 1, 2, 3, ..., N$ where $x_{ip} \in [\alpha_i, \beta_i] \subset R$ and $y_p \in [\alpha_y, \beta_y] \subset R$, i = 1, 2, …, n. We will introduce a new method to design fuzzy rules. The method is given by the following steps:

Step 1. Define fuzzy sets to cover the input and output spaces.
For each space $[\alpha_i, \beta_i]$, i = 1, 2, …, n, define N_i fuzzy sets A_i^j, j = 1, 2, …, N_i which are complete and normal in $[\alpha_i, \beta_i]$. Similarly, define N_y fuzzy sets B^j, j = 1, 2, …, N_y which are complete and normal in $[\alpha_y, \beta_y]$.
Step 2. Determine all possible antecedents of fuzzy rule candidates.
Based on the Step 1, there are $\prod_{i=1}^{n} N_i$ antecedents of fuzzy rule candidates. The antecedent has form "x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n is $A_n^{j_n}$" simplified by "$A_1^{j_1}$ and $A_2^{j_2}$ and ... and $A_n^{j_n}$". For example, if we have two input and we define two fuzzy sets A_1, A_2 for first input space and C_1, C_2 for second input space, then all possible antecedents of fuzzy rule candidates are A_1 and C_1; A_1 and C_2; A_2 and C_1; A_2 and C_2.
Step 3. Determine consequence of each fuzzy rule candidate.
For each antecedent $A_1^{j_1}$ and $A_2^{j_2}$ and ... and $A_n^{j_n}$, the consequence of fuzzy rule is determined by firing strength of the rule $\mu_{A_1^{j_1}}(x_{1p})\mu_{A_2^{j_2}}(x_{2p})...\mu_{A_n^{j_n}}(x_{np})\mu_{B^j}(y_p)$ based on the training data. Choosing the consequence is done as follows: For any training data $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$ and for any fuzzy set B^j, choose B^{j*} such that $\mu_{A_1^{j_1}}(x_{1p*})\mu_{A_2^{j_2}}(x_{2p*})...\mu_{A_n^{j_n}}(x_{np*})\mu_{B^{j*}}(y_{p*}) \geq \mu_{A_1^{j_1}}(x_{1p})\mu_{A_2^{j_2}}(x_{2p})...\mu_{A_n^{j_n}}(x_{np})\mu_{B^j}(y_p)$, for some $(x_{1p*}, x_{2p*}, ..., x_{np*}; y_{p*})$. If there are at least two B^{j*} such that $\mu_{A_1^{j_1}}(x_{1p*})...\mu_{A_n^{j_n}}(x_{np*})\mu_{B^{j*}}(y_{p*}) \geq \mu_{A_1^{j_1}}(x_{1p})...\mu_{A_n^{j_n}}(x_{np})\mu_{B^j}(y_p)$, then choose one of B^{j*}. From this step, we have the fuzzy rule:

IF x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n is $A_n^{j_n}$, THEN y is B^{j*}

So if we continue this step for every antecedent, we get $\prod_{i=1}^{n} N_i$ complete fuzzy rules.
Step 4. Construct fuzzy rule base.
The fuzzy rule base is constructed by the $\prod_{i=1}^{n} N_i$ fuzzy rules designed by Step 3.
Step 5. Design fuzzy model using fuzzy rule base.
Fuzzy model is designed by combining the fuzzy rule base and any fuzzifier, fuzzy inference engine and defuzzifier. For example, if we use singleton fuzzifier, product inference engine and center average defuzzifier, then the fuzzy model has form:

$$f(x_1, x_2, ..., x_n) = \frac{\sum_{j=1}^{M} b_j \prod_{i=1}^{n} \mu_{A_i^j}(x_i)}{\sum_{j=1}^{M} \prod_{i=1}^{n} \mu_{A_i^j}(x_i)}$$

where M is the number of rules.

From Step 3, the set of fuzzy rules constructed by this method contains fuzzy rules designed by the Wang's method. Therefore the proposed method is the generalization of the Wang's method.

4. Applications of the proposed method

In this section, we apply the proposed method to forecast the Indonesian inflation rate and interest rate of BIC. The proposed method is implemented using Matlab 6.5.1.

4.1 Forecasting inflation rate

The data of inflation rate are taken from January 1999 to January 2003. The data from January 1999 to March 2002 are used to training and the data from April 2002 to January 2003 are used to testing. The procedure to forecasting inflation rate based on the proposed method is given by the following steps:
(1). Define the universe of discourse of each input. In this paper, we will predict the inflation rate of month k using inflation rate data of months k-1 and k-2 so we will build fuzzy model using 2-input and 1-output. The universe of discourse of each input is defined as [-2, 4].
(2). Define fuzzy sets on universe of discourse of each input such that fuzzy sets can cover the input spaces. We define thirteen fuzzy sets $A_1, A_2, ..., A_{13}$ that are normal and complete on [-2, 4] of each input space with Gaussian membership function.
(3). Determine all possible antecedents of fuzzy rule candidates. There are 169 antecedents of fuzzy rule candidates and the consequence of each antecedent is chosen using Step 3 of the proposed method. So we have 169 fuzzy rules in the form:

R^j: "IF x_{k-2} is A_{i_1} and x_{k-1} is A_{i_2}, THEN x_k is A^j"

where $j = 1, 2, \ldots, 169$, $i_1, i_2 = 1, 2, \ldots, 13$ and $A^j \in \{A_1, A_2, \ldots, A_{13}\}$.

(4). Construct fuzzy rule base from fuzzy rules designed in Step 3.

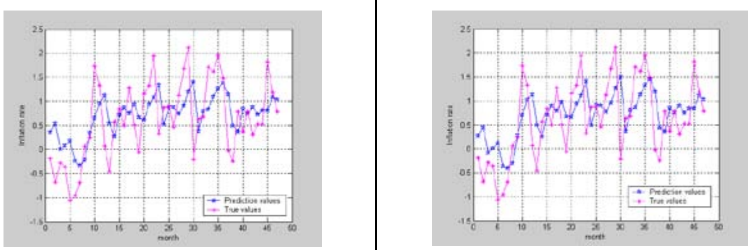
(5). Design fuzzy model combining the fuzzy rule base and certain fuzzifier, fuzzy inference engine and defuzzifier. In this paper, we use singleton fuzzifier, minimum and product inference engines, center average defuzzifier.

Table 1. Comparison of mean square errors of forecasting inflation rate using the Wang's method and proposed method

Methods	Number of fuzzy rules	MSE of training data		MSE of testing data	
		Minimum inference engine	Product inference engine	Minimum inference engine	Product inference engine
Wang's method	23	0.47860	0.45780	0.35309	0.35286
Proposed method	169	0.43436	0.40787	0.26097	0.26734

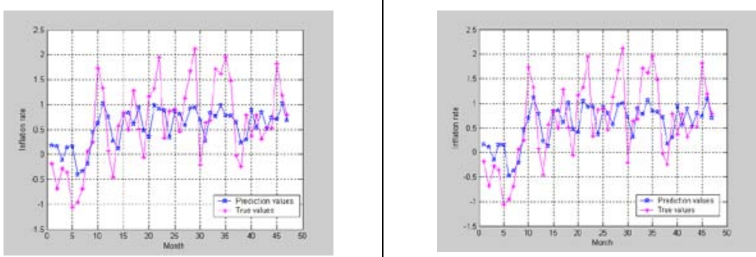
Table 1 shows a comparison of the MSE of training and testing data based on the Wang's method and proposed method. There are twenty three fuzzy rules generated by the Wang's method and there are 169 fuzzy rules constructed by the proposed method. The prediction of inflation rate by the Wang's method with singleton fuzzifier, product inference engine and center average defuzzifier has a higher accuracy than that with singleton fuzzifier, minimum inference engine and center average defuzzifier.

Table 1 illustrates that the forecasting inflation rate by the proposed method with singleton fuzzifier, minimum inference engine and center average defuzzifier has a higher accuracy than that with singleton fuzzifier, product inference engine and center average defuzzifier. The true values and prediction values of the inflation rate based on the Wang's method and the proposed method using minimum and product inference engines are shown in Figure 1 and Figure 2 respectively.



(a) (b)

Figure 1. The prediction and true values of inflation rate based on the Wang's method using: (a) minimum inference engine; (b) product inference engine



(a) (b)

Figure 2. The prediction and true values of inflation rate based on the proposed method using: (a) minimum inference engine; (b) product inference engine

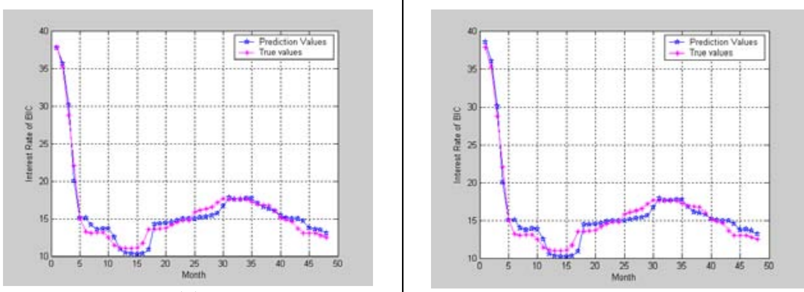
4.2 Forecasting interest rate of Bank Indonesia certificate

The data of the interest rate of BIC from January 1999 to December 2001 are used to training and the data from January 2002 to February 2003 are used to testing. The interest rate of BIC of k^{th} month will be predicted by data of interest rate of BIC of $(k\text{-}1)^{th}$ and $(k\text{-}2)^{th}$ months so we will build fuzzy model using 2-input and 1-output which universe of discourse of each input is [10, 40]. Seven fuzzy sets are defined in every input space with Gaussian membership function. The other steps to forecasting the interest rate of BIC use the same steps of the predicting inflation rate.

Table 2. Comparison of mean square errors of predicting interest rate of BIC using the Wang's method and proposed method

Method	Number of fuzzy rules	MSE of training data		MSE of testing data		Average forecasting errors of testing data (%)	
		Minimum inference engine	Product inference engine	Minimum inference engine	Product inference engine	Minimum inference engine	Product inference engine
Wang's method	12	0.98759	1.0687	0.46438	0.55075	3.8568	4.4393
Proposed method	49	0.95361	0.91682	0.28327	0.24098	2.8973	2.7813

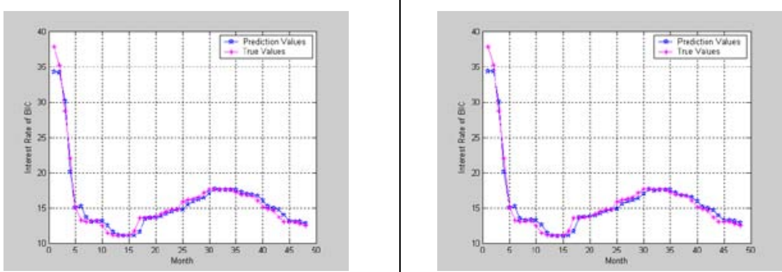
Table 2 shows that there are twelve fuzzy rules generated by Wang's method and there are forty nine fuzzy rules constructed by proposed method. The average forecasting errors of the prediction of the interest rate of BIC by Wang's method and proposed method are 3.8568 % and 2.7813 % respectively.



(a) (b)

Figure 3. The prediction and true values of interest rate of BIC based on the Wang's method using: (a) minimum inference engine; (b) product inference engine

From Table 2, forecasting the interest rate of BIC based on the Wang's method using minimum inference engine has a good accuracy. Based on the proposed method, forecasting of the interest rate of BIC using product inference engine has a good accuracy. The MSE values and average forecasting errors based on the proposed method are smaller than those based on the Wang's method. Figure 3 and Figure 4 show the true values and prediction values of the interest rate of BIC based on the Wang's method and proposed method respectively.



(a) (b)

Figure 4. The prediction and true values of interest rate of BIC based on the proposed method using: (a) minimum inference engine; (b) product inference engine

5. Conclusions

In this paper, we have presented a new method for designing complete fuzzy rules from training data. The method uses the firing strength of rule to construct complete fuzzy rules. The resulted fuzzy rules can cover all values in the input spaces. The set of the fuzzy rules generated by the proposed method contains all fuzzy rules designed by the Wang's method. In other word, the proposed method is generalization of the Wang's method. We apply the proposed method to forecast the Indonesian inflation rate and interest rate of BIC. The result is that forecasting Indonesian inflation rate and interest rate of BIC using the proposed method has a higher accuracy than that using the Wang's method. To increase the prediction accuracy, we can define more fuzzy sets in the input and output spaces but defining more fuzzy sets can imply complexity of computations. So determining the optimal number of fuzzy rules is important to get efficient computations. In the next works, we will design the optimal number of fuzzy rules.

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