Fuzzy Model for Estimating Inflation Rate

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Abstract

Inflation rate is one of some indicators that could be used to taking policy for government. If the inflation rate is not controlled, then it could cause the instability of Indonesian economy. The aim in this paper is to estimate the inflation rate using fuzzy system. Interest rate, exchange rate of rupiah to US dollar, gross national product and money supply are considered as fuzzy input sets and the inflation rate as fuzzy output set. Fuzzy system maps the input set to output set using fuzzifier, fuzzy rule, fuzzy inference engine and defuzzifier. We chose the singleton fuzzifier, product inference engine, Mamdani's implication, center average defuzzifier and Gaussian membership function to construct fuzzy system. The fuzzy model derived can match all input-output pairs in training data to arbitrary accuracy. Furthermore parameter of the model should be chosen to provide a balance matching and generalization.

Key words: inflation rate, fuzzy model

1. Introduction

The Inflation rate is often seen as an important indicator for the performance of a central bank. Therefore inflation forecast is one of some indicators that could be used to taking policy for government. If the inflation rate is not controlled, then it could cause the instability of Indonesian economy.

Future inflation depends on other economic variables, like interest rate, output and exchange rate [2]. The economic variables that can influence the inflation are interest rate, exchange rate of rupiah to US dollar, gross national product and money supply, [6]. In [6], Muchson presented the model of inflation rate using enter method of regression analysis. Stock and Watson used the factor model to forecast the inflation, [7]. In [5], Moser used the factor, VAR and ARIMA models to forecast the Austrian inflation. Thus Mitze did the forecasting EMU inflation rate using fuzzy rule approach, [4]. In this paper we propose the fuzzy model for estimating Indonesian inflation rate. We use fuzzy system developed by Wang because it is good approximator for nonlinear function, [1], [8].

2. Fuzzy system

Fuzzy system consists of fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. The procedure to constructing the fuzzy system is described by the following steps:

a. Determining the fuzzy input and output data pairs

To generate the fuzzy rule from the available data, the first step is to define the membership function for input dan output data.

b. Constructing fuzzification

Fuzzifier maps the real values of input in $\boldsymbol{R}^n$ into fuzzy sets. There are three kinds of fuzzifier, those are singleton, Gaussian and trianguler. Because of simple calculation, [3], singleton fuzzifier is used in this fuzzy system.

The singleton fuzzifier maps a real-valued point x^* $\in U$ (universal set) into a fuzzy singleton A^l in U which its membership function is

$$\mu_{A^l}(x) = \begin{cases} 1, & if \;\; x = x^* \\ 0, & otherwise \;\; x \neq x^* \end{cases} \tag{1}$$

c. Determining fuzzy rule base

Fuzzy rule base described relationship between input and output variables. Fuzzy rule base consists of sets of fuzzy IF-THEN rules (Mamdani) in the form:

IF x_1 is A_1^l and x_2 is A_2^l and…and x_n is A_n^l, THEN y is B^l (2)

with A_i^l, B^l are fuzzy sets in $U_i \subset \boldsymbol{R}$ and $V \subset \mathbf{R}$ respectively, (x_1, x_2, …, x_n) and y are the input and output respectively, M is number of rules, l = 1, 2, …, M.

d. Determining fuzzy inference engine

Fuzzy inference engine combining the fuzzy rule base maps a fuzzy set to a fuzzy set. The fuzzy inference engines used in fuzzy system are product, minimum, Lukasiewics, Sadeh and Dienes-Rescher inference engine. Because of simple computation and continuity, the product inference engine is used to construct the fuzzy system, [3].

The product inference engine is defined as:

$$\mu_{B^i}(y) = \underset{l=1}{\overset{M}{mak}} \left[\sup_{x \in U} \left((\mu_{A^l}(x) \prod_{i=1}^{n} \mu_{A_i^l}(x_i) \mu_{B^l}(y)) \right) \right] \tag{3}$$

where A^l is fuzzy set in U and B^l is fuzzy set in V.

e. Designing defuzzification

Defuzzifier will map the fuzzy set to the real number (output). The defuzzifiers commonly used are center of gravity, center average and maximum defuzzifier.In this paper, the center average defuzzifier is used. Let $\bar{y}^l$ be the center of the l'*th* fuzzy set and w_l be its height, the center average defuzzifier determines y* as

$$y^* = \frac{\sum_{l=1}^{M} \bar{y}^l w_l}{\sum_{l=1}^{M} w_l} \tag{4}$$

f. Constructing fuzzy system

If the fuzzy set B^l is normal with center $\bar{y}^l$, then fuzzy system designed by singleton fuzzifier, fuzzy rule base (2), product inference engine and center average defuzzifier is the following form

$$f(x) = \frac{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)} \tag{5}$$

with input $x \in U \subset R^n$ and f(x) $\in V \subset R$.

The fuzzy system (5) is the nonlinear mapping that maps $x \in U \subset R^n$ to $f(\mathrm{x}) \in V \subset R$. If we choose the different membership functions for $\mu_{A_i^l}$ and μ_{B^l}, then we have the different fuzzy systems. Let $\mu_{A_i^l}$ and μ_{B^l} be the Gaussian membership function:

$$\mu_{A_i^l}(x_i) = a_i^l \exp\left(-\left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l} \right)^2 \right) \tag{6}$$

$$\mu_{B^l}(x_i) = \exp\left(-(y - \bar{y}^l)^2 \right) \tag{7}$$

with $a_i^l \in (0,1]$, $\sigma_i^l \in (o, \infty)$, $\bar{x}_i^l$, $\bar{y}^l \in R$, then the fuzzy systems (5) become

$$f(x) = \frac{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} a_i^l \exp\left(-\left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l} \right)^2 \right) \right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} a_i^l \exp\left(-\left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l} \right)^2 \right) \right)} \tag{8}$$

Suppose that we are given N input-output pairs (x_0^l, y_0^l), l = 1, 2, 3,…,N. Furthermore we want to design the fuzzy system f(x) matching for all pairs for any given accuracy; that is, for $\varepsilon > 0$, we construct f(x) with $\left| f(x_0^l) - y_0^l \right| < \varepsilon$, l = 1, 2, 3, …, N.

If we choose $a_i^l = 1$, $\sigma_i^l = \sigma$ and $\left| x - x_0^l \right|^2 = \sum_{i=1}^{s} \left(x_i - x_{0i}^l \right)^2$, then the fuzzy system (8) become

$$f(x)=\frac{\sum_{l=1}^{N} y_0^l \exp\left(-\frac{\left|x-x_0^l\right|^2}{\sigma^2}\right)}{\sum_{l=1}^{N} \exp\left(-\frac{\left|x-x_0^l\right|^2}{\sigma^2}\right)} \qquad (9)$$

with y_0^l is center of fuzzy set B^l.

Theorem 1: For any $\varepsilon>0$, there exists $\sigma^*>0$ such that the fuzzy system (9) with $\sigma=\sigma^*$ has property that $\left|f(x_0^l)-y_0^l\right|<\varepsilon$, for all l = 1, 2, …,N.

Proof: If we take arbitrary $\varepsilon>0$ and set $\sigma^*=\sigma>0$ for k = 1, 2,…, N, then fuzzy system $f(x_0^k)$ (9) become

$$f(x_0^k)=\frac{\sum_{l=1}^{N} y_0^l \exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)}{\sum_{l=1}^{N} \exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)}$$

$$=\frac{y_0^k+\sum_{l=1,\, l\neq k}^{N} y_0^l \exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)}{1+\sum_{l=1,\, l\neq k}^{N} \exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)}$$

then $\left|f(x_0^k)-y_0^k\right|=$

$$\frac{\sum_{l=1,\, l\neq k}^{N} (y_0^l-y_0^k)\exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)}{1+\sum_{l=1,\, l\neq k}^{N} \exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)}.$$

If $x_0^k\neq x_0^l$, for $l\neq k$, then $\exp\left(-\frac{\left|x_0^k-x_0^l\right|^2}{\sigma^2}\right)$ tends to 0, for small enough σ so that $\left|f(x_0^k)-y_0^k\right|<\varepsilon$, and if $x_0^k=x_0^l$ for $l\neq k$, then $\left|f(x_0^k)-y_0^k\right|<\varepsilon$.

Based on the theorem 1, the smaller σ, the smaller error $\left|f(x_0^l)-y_0^l\right|$ but the graphic f(x) becomes less smooth function. If the graphic f(x) is not smooth function, then it may not generalize well for the data not in the training data. Therefore, it is necessary to choose σ such that f(x) can minimize the error for training data and not in training data.

3. Estimating inflation rate

In this paper, the fuzzy model of Indonesian inflation rate is based on the variables of interest rate, exchange rate of rupiah to US dollar, gross national product and money supply. The training data are taken from the annual report of Bank Indonesia from 1980 to 1999.

The money supply, interest rate, exchange rate of rupiah to US dollar, gross national product are considered as fuzzy inputs x_1, x_2, x_3, x_4 respectively. The inflation rate is as a fuzzy output y. Then $(x_{01}^l, x_{02}^l, x_{03}^l, x_{04}^l, y_{ol})$ is the *l-th* input-output data pair with l = 1, 2, 3, ..., 20.

The fuzzy model of Indonesian inflation rate constructed by the singleton fuzzifier, fuzzy rule base, product inference engine and center average defuzzifier has the form

$$f(x)=\frac{\sum_{i=1}^{20} y_{0i} \exp\left(-\sum_{j=1}^{4}\frac{\left(x_j-x_{0j}^i\right)^2}{\sigma^2}\right)}{\sum_{i=1}^{20} \exp\left(-\sum_{j=1}^{4}\frac{\left(x_j-x_{0j}^i\right)^2}{\sigma^2}\right)} \qquad (10)$$

In this paper, the parameter σ is determined by trial and error. The error of the function $f(x)$ with σ^2 = 1000 is smaller than that with σ^2 = 1000000 for the training data. But the error of $f(x)$ for σ^2 =1000 is bigger than that for σ^2 =1000000 if $f(x)$ is applied to not in training data. Then the parameter σ must be choose such that the model minimizes the error of training data and not in training data.

Figure 1 views the prediction of Indonesian inflation rate using multiple regression method, [6]. The graphic of estimation of Indonesian inflation rate done by fuzzy model is viewed in Figure 2.

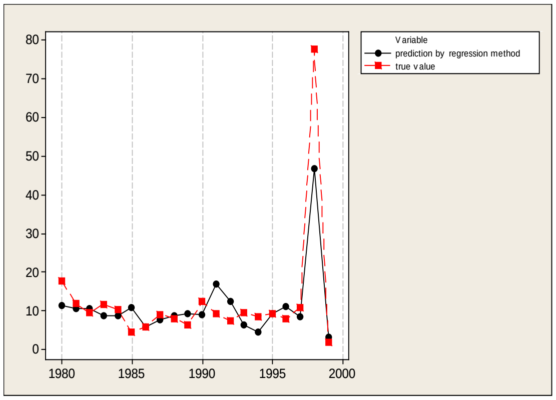

Figure 1: The prediction and true values of inflation rate using regression method.

The root mean square error (RMSE) of prediction using regression method is 7.72 and using fuzzy method, the RMSE is 0.39.

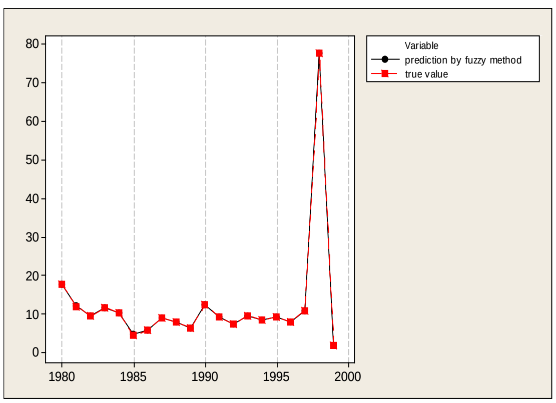

Figure 2: The prediction and true values of inflation rate using fuzzy method.

4. Conclusions

In this paper, a fuzzy model for estimating inflation rate has been proposed. The value of parameter σ for fuzzy model must be determined to get the good model for arbitrary accuracy. In this paper, the choosing for σ is done by trial and error. The fuzzy model is more accurate than regression model. In the future research, we will develop a model of inflation rate based on fuzzy time series to get a higher forecasting accuracy.

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