Designing Fuzzy Time Series Model Using Generalized Wang's Method and Its application to Forecasting Interest Rate of Bank Indonesia Certificate

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Abstract

Fuzzy time series is a dynamic process with linguistic values as its observations. Modelling fuzzy time series developed by some researchers used the discrete membership functions and table lookup scheme (Wang's Method) from training data. The Wang's Method is a simple method that can be used to overcome the conflicting rule by determining each rule degree. The weakness of fuzzy time series model based on the method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. This paper presents generalization of the Wang's method using the continuous membership function based on fuzzy time series data. Furthermore, this method is applied to forecast interest rate of Bank Indonesia Certificate (BIC) based on one-factor two-order fuzzy time series. The prediction of interest rate of BIC using the proposed method has a higher accuracy than that using the Wang's method.

Keywords: *fuzzy relation, fuzzy time series, generalized Wang's method, interest rate of BIC.*

1. Introduction

Fuzzy time series is a dynamic process with linguistic values as its observations. In recently, fuzzy time series model was developed by some researchers. Song and Chissom developed fuzzy time series by fuzzy relational equation using Mamdani's method [10]. In this modeling, determining the fuzzy relation need large computation. Then, Song and Chissom constructed first order fuzzy time series for time invariant and time variant cases

[11], [12]. This model need complexity computation for fuzzy relational equation. Furthermore, to overcome the weakness of the model, Chen designed fuzzy time series model by clustering of fuzzy relations [4].

Hwang constructed fuzzy time series model to forecast the enrollment in Alabama University [8]. Fuzzy time series model based on heuristic model gave more accuracy than its model designed by some previous researchers [7]. Then, forecasting for enrollment in Alabama University based on high order fuzzy time series resulted more accuracy prediction [5]. First order fuzzy time series model was also developed by Sah and Degtiarev [9] and Chen and Hsu [6]. Abadi [1] constructed fuzzy time series model using table lookup scheme (Wang's method) to forecast interest rate of Bank Indonesia certificate (BIC) and the result gave high accuracy. Then, forecasting inflation rate using singular value decomposition method have a higher accuracy than that using Wang's method [2], [3].

The weakness of the constructing fuzzy relations based on the Wang's method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. In this paper, we will design fuzzy time series model using generalized Wang's method to improve the prediction accuracy. Then, its result is used to forecast interest rate of BIC. The proposed method has a higher prediction accuracy than the Wang's method in application to forecasting interest rate of BIC.

The rest of this paper is organized as follows. In section 2, we present the Wang's method to construct fuzzy model. In section 3, we briefly review the definitions of fuzzy time series and its properties. In section 4, we present a generalization of Wang's method to construct fuzzy time series model based on training data. In section 5, we apply the proposed method to forecasting interest rate of BIC. We also compare the proposed method with the Wang's method in the forecasting interest rate of BIC. Finally, some conclusions are discussed in section 6.

2. Wang's method for designing fuzzy rules

In this section, we will introduce the Wang's method to construct fuzzy rules [13]. Suppose that we are given the following N input-output data: $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, $p = 1, 2, 3, ..., N$ where $x_{ip} \in [\alpha_i, \beta_i] \subset R$ and $y_p \in [\alpha_y, \beta_y] \subset R$, i = 1, 2, …, n. Designing fuzzy model using Wang's method is given by the following steps:

Step 1. Define fuzzy sets to cover the input and output domains.

For each space $[\alpha_i, \beta_i]$, i = 1, 2, …, n, define N_i fuzzy sets A_i^j, j = 1, 2, …, N_i which are complete in $[\alpha_i, \beta_i]$. Similarly, define N_y fuzzy sets B^j, j = 1, 2, …, N_y which are normal and complete in $[\alpha_y, \beta_y]$.

Step 2. Generate one rule from one input-output pair.

For each input-output pair $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, determine membership value of x_{ip}, i = 1, 2, …, n in fuzzy sets A_i^j, j = 1, 2, …, N_i and membership value of y_p in fuzzy sets B^j, j = 1, 2, …, N_y. Then, for each input variable x_{ip}, i = 1, 2, …, n, determine the fuzzy set in which x_{ip} has the largest membership value. In other word, determine A_i^{j*} such that $\mu_{A_i^{j*}}(x_{ip}) \geq \mu_{A_i^j}(x_{ip})$, $j = 1, 2, ..., N_i$. Similarly, determine B^{l*} such that $\mu_{B^{l*}}(y_p) \geq \mu_{B^l}(y_p)$, $l = 1, 2, ..., N_y$. Finally, we construct a fuzzy IF-THEN rule:

$$\text{IF } x_1 \text{ is } A_1^{j*} \text{ and } x_2 \text{ is } A_2^{j*} \text{ and ... and } x_n \text{ is } A_n^{j*}, \text{THEN } y \text{ is } B^{l*}$$

Step 3. Compute degree of each rule designed in step 2.

From step 2, one rule is generated by one input-output pair. If the number of input-output data is large, then it is possible that there are the conflicting rules. Two rules become conflicting rules if the rules have same IF parts but different THEN parts. To resolve this problem, we assign a degree to each rule designed in step 2. The degree of rule is defined as follows: suppose that the rules in Step 2 is constructed by the input-output pair $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, then its degree is defined as

$$D(rule) = \prod_{i=1}^{n} \mu_{A_i^{j*}}(x_{ip}) \mu_{B^{l*}}(y_p)$$

Step 4. Construct the fuzzy rule base.

The rule base consists of the following three sets of rules: (1) The rules designed in Step 2 that do not conflict with any other rules; (2) The rule from a conflicting group that has the maximum degree; (3) Linguistic rules from human experts.

Step 5. Construct the fuzzy model using the fuzzy rule base.

We can use any fuzzifier, inference engine and defuzzifier and combine with the fuzzy rule base to design fuzzy model.

If the number of training data is N and the number of all possible combinations of the fuzzy sets defined for the input variables is $\prod_{i=1}^{n} N_i$, then the number of fuzzy rules generated by Wang's method may be much less than both N and $\prod_{i=1}^{n} N_i$. Then, the fuzzy rule base generated by this method may not be complete so that the fuzzy rules can not cover all values in the input spaces.

3. Fuzzy time series

In this section, we introduce the following definitions and properties of fuzzy time series referred from Song and Chissom [10].

Definition 1. Let $Y(t) \subset \mathbf{R}$, t = ..., 0, 1, 2, ..., **,** be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1, 2, 3,...) are defined and $F(t)$ is the collection of $f_i(t)$, i = 1, 2, 3,...,then $F(t)$ is called **fuzzy time series** on $Y(t)$, t = ..., 0, 1, 2, 3, ...

In the Definition 1, $F(t)$ can be considered as a linguistic variable and $f_i(t)$ as the possible linguistic values of $F(t)$. The value of $F(t)$ can be different depending on time t so $F(t)$ is function of time t. The following procedure gives how to construct fuzzy time series model based on fuzzy relational equation.

Definition 2. Let I and J be indices sets for $F(t-1)$ and $F(t)$ respectively. If for any $f_j(t) \in F(t)$, j $\in J$, there exists $f_i(t-1) \in F(t-1)$, i $\in I$ such that there exists a fuzzy relation $R_{ij}(t,t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t,t-1)$, $R(t,t-1) = \bigcup_{i,j} R_{ij}(t,t-1)$ where $\cup$ is union operator, then $R(t,t-1)$ is called fuzzy relation between $F(t)$ and $F(t-1)$. This fuzzy relation can be written as

$$F(t) = F(t-1) \circ R(t,t-1). \quad (1)$$

where $\circ$ is max-min composition.

In the equation (1), we must compute all values of fuzzy relations $R_{ij}(t,t-1)$ to determine value of $F(t)$. Based on above definitions, concept for first order and m-order of fuzzy time series can be defined.

Definition 3. If $F(t)$ is caused by $F(t-1)$ only or by $F(t-1)$ or $F(t-2)$ or … or $F(t-m)$, then the fuzzy relational equation

$$F(t) = F(t-1) \circ R\ (t,t-1) \text{ or}$$
$$F(t) = (F(t-1) \cup F(t-2) \cup ... \cup F(t-m)) \circ R_0(t,t-m) \quad (2)$$

is called first order model of $F(t)$.

Definition 4. If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, ... and $F(t-m)$ simultaneously, then the fuzzy relational equation

$$F(t) = (F(t-1) \times F(t-2) \times ... \times F(t-m)) \circ R_a(t,t-m) \quad (3)$$

is called *m*-order model of $F(t)$.

From equations (2) and (3), the fuzzy relations $R(t,t-1)$, $R_a(t,t-m)$, $R_o(t,t-m)$ are important factors to design fuzzy time series model. Furthermore for the first order model of $F(t)$, for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists fuzzy relations $R_{ij}(t,t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t,t-1)$. This is equivalent to "if $f_i(t-1)$, then $f_j(t)$", and then we have the fuzzy relation $R_{ij}(t,t-1) = f_i(t-1) \times f_j(t)$. Because of $R(t,t-1) = \bigcup_{i,j} R_{ij}(t,t-1)$, then

$$R(t,t-1) = \text{maks}_{i,j}\{\min(f_j(t), f_i(t-1))\}. \quad (4)$$

For the relation $R_o(t,t-m)$ of the first order model, we get

$$R_o(t,t-m) = \underset{p}{\text{maks}}\{ \underset{k}{\text{maks}}\{\underset{ik,j}{\min}(f_{ik}(t-k), f_j(t))\}\}. \quad (5)$$

Based on *m*-order model of $F(t)$, we have

$$R_a(t,t-m) = \underset{p}{\text{maks}}\{ \underset{j,i1,i2,...,im}{\min}(f_{i1}(t-1) \times f_{i2}(t-2) \times ... \times f_{im}(t-m) \times f_j(t))\} \quad (6)$$

From equations (4), (5) and (6), we can compute the fuzzy relations using max-min composition.

Definition 5. If for $t_1 \neq t_2$, $R(t_1,t_1-1) = R(t_2,t_2-1)$ or $R_a(t_1,t_1-m) = R_a(t_2,t_2-m)$ or $R_o(t_1,t_1-m) = R_o(t_2,t_2-m)$, then $F(t)$ is called time-invariant fuzzy time series. Otherwise it is called time-variant fuzzy time series.

Time-invariant fuzzy time series models are independent of time *t*, those imply that in applications, the time-invariant fuzzy time series models are simpler than the time-variant fuzzy time series models. Therefore it is necessary to derive properties of time-invariant fuzzy time series models.

Theorem 1. If $F(t)$ is fuzzy time series and for any *t*, $F(t)$ has only finite elements $f_i(t)$, *i* = 1, 2, 3, ..., n, and $F(t) = F(t-1)$, then $F(t)$ is a time-invariant fuzzy time series.

Theorem 2. If $F(t)$ is a time-invariant fuzzy time series, then

$$R(t,t-1) = ... \cup f_{i1}(t-1) \times f_{j0}(t) \cup f_{i2}(t-2) \times f_{j1}(t-1) \cup ... \cup f_{im}(t-m) \times f_{jm-1}(t-m+1) \cup ...$$

where m is a positive integer and each pair of fuzzy sets is different.

Based on the Theorem 2, we should not calculate fuzzy relations for all possible pairs. We need only to use one possible pair of the element of $F(t)$ and $F(t-1)$ with all possible *t*'s. This implies that to construct time-invariant fuzzy time series model, we need only one observation for every *t* and we set fuzzy relations for every pair of observations in the different of time *t*. Then union of the fuzzy relations results a fuzzy relation for the model. Theorem 2 is very useful because we sometime have only one observation in every time *t*.

Let $F_1(t)$ be fuzzy time series on $Y(t)$. If $F_1(t)$ is caused by $(F_1(t-1), F_2(t-1))$, $(F_1(t-2), F_2(t-2)), ..., (F_1(t-n), F_2(t-n))$, then the fuzzy logical relationship is presented by $(F_1(t-n), F_2(t-n)), ..., (F_1(t-2), F_2(t-2)), (F_1(t-1), F_2(t-1)) \rightarrow F_1(t)$ and it is called two-factor *n*-order fuzzy time series forecasting model, where $F_1(t), F_2(t)$ are called the main factor and the secondary factor fuzzy time series respectively. If a fuzzy logical relationship is presented as

$$(F_1(t-n), F_2(t-n), ..., F_m(t-n)), ..., \ (F_1(t-2), F_2(t-2), ..., F_m(t-2)), \ (F_1(t-1), F_2(t-1), ..., \ F_m(t-1)) \rightarrow F_1(t) \quad (7)$$

then the fuzzy logical relationship is called m-factor n-order fuzzy time series forecasting model, where $F_1(t)$ are called the main factor fuzzy time series and $F_2(t), ..., F_m(t)$ are called the secondary factor fuzzy time series.

Because $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$ is equivalent to the fuzzy rule "IF $f_i(t-1)$, THEN $f_j(t)$", and the fuzzy relation $R_{ij}(t, t-1) = f_i(t-1) \times f_j(t)$, then we can view the fuzzy rule as the fuzzy relation and vice versa.

4. Designing fuzzy time series model using generalized Wang's method

Like in modeling traditional time series data, we use training data to set up the relationship among data values at different times. In fuzzy time series, the relationship is different from that in traditional time series. In fuzzy time series, we exploit the past experience knowledge into the model. The experience knowledge has form "IF … THEN …". This form is called fuzzy rules. Furthermore, main step to modeling fuzzy time series data is to identify the training data using fuzzy rules.

Let $A_{1,k}(t-i), ..., A_{N_i,k}(t-i)$ be N_i fuzzy sets with continuous membership function that are normal and complete in fuzzy time series $F_k(t-i)$, i =1, 2, 3,…, n, k = 1, 2, …, m, then the fuzzy rule:

$$R^j: \ IF \ (x_1(t-n) \ is \ A^j_{i_1,1}(t-n) \text{ and ...and } x_m(t-n) \text{ is } A^j_{i_m,m}(t-n)) \text{ and } \dots$$
$$\text{and} \ (x_1(t-1) \ is \ A^j_{i_1,1}(t-1) \text{ and ...and } x_m(t-1) \text{ is } A^j_{i_m,m}(t-1)), \ THEN \ x_1(t) \text{ is } A^j_{i_1,1}(t) \quad (8)$$

is equivalent to the fuzzy logical relationship (7) and vice versa. So (8) can be viewed as fuzzy relation in $U \times V$ where $U = U_1 \times ... \times U_{mn} \subset R^{mn}$, $V \subset R$ with

$$\mu_A(x_1(t-n), ..., x_1(t-1), ..., x_m(t-n), ..., x_m(t-1)) = \ \mu_{A_{i_1,1}}(x_1(t-n))...\mu_{A_{i_1,1}}(x_1(t-1))...\mu_{A_{i_m,m}}(x_m(t-n)...\mu_{A_{i_m,m}}(t-1),$$

where $A = A_{i_1,1}(t-n) \times ... \times A_{i_1,1}(t-1) \times ... \times A_{i_m,m}(t-n) \times ... \times A_{i_m,m}(t-1)$.

Let $F_1(t-1), F_2(t-1), ..., \ F_m(t-1) \rightarrow F_1(t)$ be m-factor one-order fuzzy time series forecasting model. Then $F_1(t-1), F_2(t-1), ..., \ F_m(t-1) \rightarrow F_1(t)$ can be viewed as fuzzy time series forecasting model with m inputs and one output. In this paper, we will design m-factor one-order time invariant fuzzy time series model using generalized Wang's method. But this method can be generalized to m-factor n-order fuzzy time series model.

Suppose we are given the following N training data: $(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$, $p = 1, 2, 3, ..., N$. We will introduce a method to construct fuzzy logical relationships from training data presented as follows:

Step 1. Define the universe of discourse for main factor and secondary factor. Let $U = [\alpha_1, \beta_1] \subset R$ be universe of discourse for main factor, $x_{1p}(t-1), x_{1p}(t) \in [\alpha_1, \beta_1]$ and V = $[\alpha_i, \beta_i] \subset R, i = 2, 3, ..., m$, be universe of discourse for secondary factors, $x_{ip}(t-1) \in [\alpha_i, \beta_i]$.

Step 2. Define fuzzy sets on the universes of discourse. Let $A_{1,k}(t-i), ..., A_{N_i,k}(t-i)$ be $N_{i,k}$ fuzzy sets in fuzzy time series $F_k(t-i)$ that are continuous, normal and complete in $[\alpha_k, \beta_k] \subset R$, $k = 1, 2, 3, ..., m$ i =0,1.

Step 3. Determine all possible antecedents of candidates of fuzzy logical relationships.

Based on the Step 2, there are $\prod_{k=1}^{m} N_{i,k}$ antecedents of candidates of fuzzy logical relationships. The antecedent has form: $A_{j_1,1}(t-1), A_{j_2,2}(t-1), ..., \ A_{j_m,m}(t-1)$, $j_i = 1, 2, ..., N_i$

Step 4. Determine consequence of each candidate of fuzzy logical relationship.

For each antecedent $A_{j_1,1}(t-1), A_{j_2,2}(t-1), ..., A_{jm,m}(t-1)$, we choose $A_{j_1^*,1}(t)$ as the consequence of the antecedent if there exists training data $(x_{1p^*}(t-1), x_{2p^*}(t-1), ..., x_{mp^*}(t-1); x_{1p^*}(t))$ such that

$\mu_{A_{j_1,1}}(x_{1p^*}(t-1))\mu_{A_{j_2,2}}(x_{2p^*}(t-1))...\mu_{A_{jm,m}}(x_{mp^*}(t-1))\mu_{A_{j_1^*,1}}(x_{1p^*}(t))$

$\geq \mu_{A_{j_1,1}}(x_{1p}(t-1))\mu_{A_{j_2,2}}(x_{2p}(t-1))...\mu_{A_{jm,m}}(x_{mp}(t-1))\mu_{A_{j_1,1}}(x_{1p}(t))$ for all training data

$(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$

From this step we have the following $M = \prod_{k=1}^{m} N_{1,k}$ collections of fuzzy logical relationships designed from training data:

$$R^l: \ (A^l_{j_1,1}(t-1), A^l_{j_2,2}(t-1), ..., A^l_{jm,m}(t-1)) \to A^l_{j_1^*,1}(t), \ l = 1, 2, 3, \ldots, M. \quad (9)$$

Step 5. Determine the membership function for each fuzzy logical relationship resulted in the Step 4. If we view each fuzzy logical relationship as fuzzy relation in $U \times V$ with $U = U_1 \times ... \times U_m \subset R^m$, $V \subset R$, then the membership function for the fuzzy logical relationship (9) is defined by

$\mu_{R^l}(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$

$= \mu_{A_{j_1,1}(t-1)}(x_{1p}(t-1))\mu_{A_{j_2,2}(t-1)}(x_{2p}(t-1))...\mu_{A_{jm,m}(t-1)}(x_{mp}(t-1))\mu_{A^l_{j_1^*,1}(t)}(x_{1p}(t))$

Step 6. For given input fuzzy set $A'(t-1)$ in input space U, compute the output fuzzy set $A'_l(t)$ in output space V for each fuzzy logical relationship (9) as

$\mu_{A'_l}(x_1(t)) = \sup_{x \in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t))))$ where $x(t-1) = (x_1(t-1), ..., x_m(t-1))$.

Step 7. Compute fuzzy set $A'(t)$ as the combination of M fuzzy sets $A'_1(t), A'_2(t), A'_3(t), . . . A'_M(t)$ by

$$\begin{aligned}\mu_{A'(t)}(x_1(t)) &= \max_{l=1}^{M}(\mu_{A'_1(t)}(x_1(t)), ..., \mu_{A'_M(t)}(x_1(t)))\\ &= \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t)))\\ &= \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))).\end{aligned}$$

Step 8. Calculate the forecasting outputs. Based on the Step 7, if we are given input fuzzy set $A'(t-1)$, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))). \quad (10)$$

Step 9. Defuzzify the output of the model. If the goal of output of the model is fuzzy set, then stop in the Step 8. We use this step if we want the real output. For example, if given the input fuzzy set $A'(t-1)$ with Gaussian membership function $\mu_{A'(t-1)}(x(t-1)) = \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^*(t-1))^2}{a_i^2})$, then the forecasting real output using the Step 8 and center average defuzzifier is

$$x_1(t) = f(x_1(t-1), ..., x_m(t-1)) = \frac{\sum_{j=1}^{M} y_j \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}{\sum_{j=1}^{M}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})} \quad (11)$$

where y_j is center of the fuzzy set $A^j_{i_1,1}(t)$.

From Step 4, the set of fuzzy logical relationships (9) constructed by this method contains fuzzy relations designed by the Wang's method. Therefore the proposed method is generalization of the Wang's method.

5. Application of the proposed method

In this section, we apply the proposed method to forecast interest rate of BIC based on one-factor two-order fuzzy time series model. The data are taken from January 1999 to February 2003. The data from January 1999 to December 2001 are used to training and the data from January 2002 to February 2003 are used to testing. We apply the procedure in Section 4 to predict interest rate of BIC of k^{th} month using data of $(k\text{-}2)^{th}$ and $(k\text{-}1)^{th}$ months. We use [10, 40] as universe of discourse of two inputs and one output and we define seven fuzzy sets $A_1, A_2, ..., A_7$ with Gaussian membership function on each universe of discourse of input and output. Then, we apply the Step 4 of the proposed method to yield forty nine fuzzy logical relationships. The fuzzy logical relationships generated by the proposed method are shown in Table I.

Table I. Fuzzy logical relationship groups for interest rate of BIC using generalized Wang's method

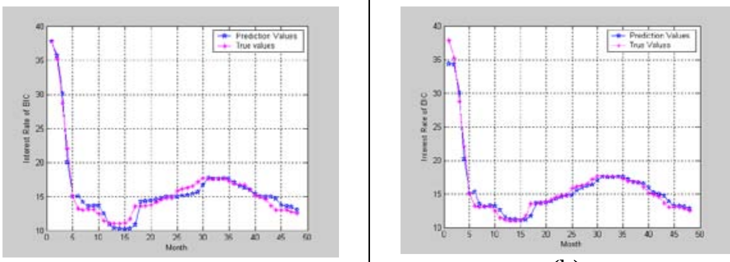
Number	$(x(t-2), x(t-1)) \to x(t)$	Number	$(x(t-2), x(t-1)) \to x(t)$	Number	$(x(t-2), x(t-1)) \to x(t)$
1	(A1, A1) → A1	17	(A3, A3) → A3	33	(A5, A5) → A3
2	(A1, A2) → A2	18	(A3, A4) → A3	34	(A5, A6) → A7
3	(A1, A3) → A2	19	(A3, A5) → A2	35	(A5, A7) → A7
4	(A1, A4) → A2	20	(A3, A6) → A2	36	(A6, A1) → A2
5	(A1, A5) → A3	21	(A3, A7) → A7	37	(A6, A2) → A2
6	(A1, A6) → A3	22	(A4, A1) → A2	38	(A6, A3) → A2
7	(A1, A7) → A3	23	(A4, A2) → A2	39	(A6, A4) → A3
8	(A2, A1) → A1	24	(A4, A3) → A2	40	(A6, A5) → A3
9	(A2, A2) → A2	25	(A4, A4) → A2	41	(A6, A6) → A5
10	(A2, A3) → A3	26	(A4, A5) → A2	42	(A6, A7) → A6
11	(A2, A4) → A3	27	(A4, A6) → A7	43	(A7, A1) → A2
12	(A2, A5) → A3	28	(A4, A7) → A7	44	(A7, A2) → A2
13	(A2, A6) → A3	29	(A5, A1) → A2	45	(A7, A3) → A3
14	(A2, A7) → A7	30	(A5, A2) → A2	46	(A7, A4) → A3
15	(A3, A1) → A2	31	(A5, A3) → A2	47	(A7, A5) → A3
16	(A3, A2) → A2	32	(A5, A4) → A2	48	(A7, A6) → A5
				49	(A7, A7) → A6

Based on the Table II, the average forecasting errors of interest rate of BIC using the Wang's method and the proposed method are 3.8568% and 2.7698%, respectively. So we can conclude that forecasting interest rate of BIC using the proposed method results more accuracy than that using the Wang's method.

Table II. Comparison of average forecasting errors of interest rate of BIC from the different methods

Method	Number of fuzzy relations	MSE of training data	MSE of testing data	Average forecasting errors (%)
Wang's method	12	0.98759	0.46438	3.8568
Generalized Wang's method	49	0.91623	0.24134	2.7698

The comparison of prediction and true values of interest rate of BIC using the Wang's method and the proposed method is shown in Figure 1.



(a) (b)

Figure 1. Prediction and true values of interest rate of BIC using
(a) Wang's method, (b) generalized Wang's method

6. Conclusions

In this paper, we have presented a generalization of Wang's method to construct fuzzy time series model based on the training data. We applied the proposed method to forecast the interest rate of BIC. The result is that forecasting interest rate of BIC using the proposed method has a higher accuracy than that using the Wang's method. It is important to determine the optimal number of fuzzy logical relationships to get efficient computations and to improve prediction accuracy. The precision of forecasting depends also to taking factors as input variables. In the future works, we will construct the optimal number of fuzzy logical relationships and select the important variables to improve prediction accuracy.

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