Optimization of Fuzzy Relations of Fuzzy Time Series Model Using Combination of Singular Value Decomposition and *QR* Factorization Methods and Its Application to Forecasting Inflation Rate

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ABSTRACT

Modelling fuzzy time series based on table lookup scheme (Wang's method) from training data was done. The Wang's method is a simple method that can be used to overcome the conflicting rule by determining each rule degree. The weakness of fuzzy time series model based on the method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. Generalization of the Wang's method has been developed to construct completely fuzzy relations. But too many fuzzy relations result complex computations. This paper presents a method to optimize fuzzy relations using combination of singular value decomposition and *QR* factorization methods. Then, this method is applied to forecast inflation rate. The prediction of inflation rate using the proposed method has a higher accuracy than that using the Wang's method.

Keywords: *fuzzy relation, fuzzy time series, singular value decomposition, QR factorization, inflation rate.*

1. Introduction

Fuzzy time series is a dynamic process with linguistic values as its observations. In recently, fuzzy time series model was developed by some researchers. Song and Chissom developed fuzzy time series by fuzzy relational equation using Mamdani's method[1]. In this modeling, determining the fuzzy relations needs large computation. Then, Song and Chissom constructed first order fuzzy time series for time invariant and time variant case[2,3]. This model needs complexity computation for fuzzy relational equation. Furthermore, to overcome the weakness of the model, Chen designed fuzzy time series model by clustering of fuzzy relations[4]. Hwang constructed fuzzy time series model to forecast the enrollment in Alabama University[5]. Fuzzy time series model based on heuristic model gave more accuracy than its model designed by some previous researchers[6]. Then, forecasting for enrollment in Alabama University based on high order fuzzy time series resulted higher accuracy [7]. First order fuzzy time series model was also developed by Sah and Degtiarev[8] and Chen and Hsu[9].

Abadi constructed fuzzy time series model using table lookup scheme (Wang's method) and QR factorization method to forecast interest rate of Bank Indonesia certificate (BIC) and the result gave high accuracy[10,11]. Then, forecasting inflation rate using singular value decomposition method had a higher accuracy than that using Wang's method[12,13]. The weakness of the constructing fuzzy relations based on the Wang's method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. To overcome this weakness, Abadi[14] designed generalized Wang's method. Furthermore, Abadi[15] constructed complete fuzzy relations of fuzzy time series model based on training data. Too many fuzzy relations result complex computations and too few fuzzy relations cause less powerful of fuzzy time series model in prediction accuracy.

In this paper, we will design optimal fuzzy relations of fuzzy time series model using singular value decomposition and *QR* factorization methods to improve the prediction accuracy. Then, its result is used to forecast inflation rate.

2. Fuzzy time series

In this section, we introduce the following definitions and properties of fuzzy time series referred from Song and Chissom[1]. Let $Y(t) \subset \mathbf{R}$, t = ..., 0, 1, 2, ..., **,** be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1, 2, 3,...) are defined and $F(t)$ is the collection of $f_i(t)$, i = 1, 2, 3,...,then $F(t)$ is called **fuzzy time series** on $Y(t)$, t = ..., 0, 1, 2, 3, ...

In the definition, $F(t)$ can be considered as a linguistic variable and $f_i(t)$ as the possible linguistic values of $F(t)$. The value of $F(t)$ can be different depending on time *t* so $F(t)$ is function of time *t*.

Let $F_1(t)$ be fuzzy time series on $Y(t)$. If a fuzzy relation is presented as

$$(F_1(t-n), F_2(t-n),..., F_m(t-n)),..., \ (F_1(t-2), F_2(t-2),..., F_m(t-2)), \ (F_1(t-1), F_2(t-1),..., \ F_m(t-1)) \rightarrow F_1(t) \qquad (1)$$

then the fuzzy relation is called *m*-factor *n*-order fuzzy time series forecasting model, where $F_1(t)$ are called the main factor fuzzy time series and $F_2(t),..., F_m(t)$ are called the secondary factor fuzzy time series.

Let $A_{1,k}(t-i),..., A_{N_i,k}(t-i)$ be N_i fuzzy sets with continuous membership function that are normal and complete in fuzzy time series $F_k(t-i)$, *i* =1, 2, 3,…, *n*, *k* = 1, 2, …, *m*, then the fuzzy rule:

$$R^j: \ IF \ (x_1(t-n) \ is \ A^j_{i_1,1}(t-n) \text{ and ...and } x_m(t-n) \text{ is } A^j_{i_m,m}(t-n)) \text{ and } \ldots$$

$$\text{and } (x_1(t-1) \ is \ A^j_{i_1,1}(t-1) \text{ and ...and } x_m(t-1) \text{ is } A^j_{i_m,m}(t-1)), \ THEN \ \ x_1(t) \text{ is } A^j_{i_1,1}(t) \qquad (2)$$

is equivalent to the fuzzy relation (1) and vice versa. So (2) can be viewed as fuzzy relation in $U \times V$ where $U = U_1 \times ... \times U_{mn} \subset R^{mn}$, $V \subset R$ with $\mu_A(x_1(t-n),..., x_1(t-1),..., x_m(t-n),..., x_m(t-1)) =$

$\mu_{A_{i_1,1}}(x_1(t-n))...\mu_{A_{i_1,1}}(x_1(t-1))...\mu_{A_{i_m,m}}(x_m(t-n)...\mu_{A_{i_m,m}}(t-1)$,

where $A = A_{i_1,1}(t-n) \times ... \times A_{i_1,1}(t-1) \times ... \times A_{i_m,m}(t-n) \times ... \times A_{i_m,m}(t-1)$.

Designing m-factor one-order time invariant fuzzy time series model using generalized Wang's method can be seen in[15]. But this method can be generalized to m-factor n-order fuzzy time series model. Suppose we are given the following N training data: $(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$, $p = 1, 2, 3, ..., N$. Based on a method developed in[15], we have complete fuzzy relations designed from training data:

$$R^l:\ (A^l_{j_1,1}(t-1), A^l_{j_2,2}(t-1), ..., A^l_{j_m,m}(t-1)) \rightarrow A^l_{j_1^*,1}(t),\ l = 1, 2, 3, \ldots, M. \quad (3)$$

If we are given input fuzzy set $A'(t-1)$, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f}(t-1)}(x_f(t-1))\mu_{A^l_{j_1,1}}(x_1(t)))). \quad (4)$$

For example, if given the input fuzzy set $A'(t-1)$ with Gaussian membership function $\mu_{A'(t-1)}(x(t-1)) = \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^*(t-1))^2}{a_i^2})$, then the forecasting real output with center average defuzzifier is

$$x_1(t) = f(x_1(t-1), ..., x_m(t-1)) = \frac{\sum_{j=1}^{M} y_j \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}{\sum_{j=1}^{M}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})} \quad (5)$$

where y_j is center of the fuzzy set $A^j_{j_1,1}(t)$.

3. Optimization of fuzzy relations

If the number of training data is large, then the number of fuzzy relations may be large too. So increasing the number of fuzzy relations will add the complexity of computations. To overcome that, we will apply singular value decomposition and QR factorization methods to get the optimal fuzzy relations using the following steps.

Step 1. Set up the firing strength of the fuzzy relation (3) for each training datum $(x;y)$ = $(x_1(t-1), x_2(t-1), ..., x_m(t-1); x_1(t))$ as follows

$$L_l(x;y) = \frac{\prod_{f=1}^{m} \mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A_{i_1,1}^{l}}(x_1(t))}{\sum_{k=1}^{M}\prod_{f=1}^{m} \mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A_{i_1,1}^{k}}(x_1(t))}$$

Step 2. Construct N x M matrix $L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_M(1) \\ L_1(2) & L_2(2) & \cdots & L_M(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(N) & L_2(N) & \cdots & L_M(N) \end{pmatrix}$.

Step 3. Compute singular value decomposition of L as $L = USV^T$, where U and V are N x N and M x M orthogonal matrices respectively, S is N x M matrix whose entries $s_{ij} = 0, i \neq j$, $s_{ii} = \sigma_i \quad i = 1, 2, ..., r$ with $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq 0$, $r \leq \min(N, M)$.

Step 4. Determine the number of fuzzy relations that will be taken as t with $t \leq \text{rank}(L)$.

Step 5. Partition V as $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, where V_{11} is t x t matrix, V_{21} is $(M\text{-}t)$ x t matrix, and construct $\bar{V}^T = \begin{pmatrix} V_{11}^T & V_{21}^T \end{pmatrix}$.

Step 6. Apply QR-factorization to $\bar{V}^T$ and find M x M permutation matrix E such that $\bar{V}^T E = QR$ where Q is t x t orthogonal matrix, $R = [R_{11}\ R_{12}]$, and R_{11} is t x t upper triangular matrix.

Step 7. Assign the position of entries one's in the first t columns of matrix E that indicate the position of the t most important fuzzy relations.

Step 8. Construct fuzzy time series forecasting model (4) or (5) using the t most important fuzzy relations.

4. Application of the proposed method

In this section, we apply the method to forecast the inflation rate in Indonesia based on six-factors one-order fuzzy time series model. The main factor is inflation rate and the secondary factors are the interest rate of Bank Indonesia certificate, interest rate of deposit, money supply, total of deposit and exchange rate. The universes of discourse of interest rate of Bank Indonesia certificate, interest rate of deposit, exchange rate, total of deposit, money supply, inflation rate are defined as [10, 40], [10, 40], [6000, 12000], [360000, 460000], [40000, 90000], [-2, 4] respectively.

It is defined fuzzy sets that are continuous, complete and normal on the universe of discourse such that the fuzzy sets can cover the input spaces. We define sixteen fuzzy sets $B_1, B_2, ..., B_{16}$, sixteen fuzzy sets $C_1, C_2, ..., C_{16}$, twenty five fuzzy sets $D_1, D_2, ..., D_{25}$, twenty one fuzzy sets $E_1, E_2, ..., E_{21}$, twenty one fuzzy sets $F_1, F_2, ..., F_{21}$, thirteen fuzzy sets $A_1, A_2, ..., A_{13}$ on the universes of discourse of the interest rate of Bank Indonesia certificate, interest rate of deposit, exchange rate, total of deposit, money supply, inflation rate respectively. All fuzzy sets are defined by Gaussian membership function. Then, we set up fuzzy relations based on training data resulting 36 fuzzy relations as the following Table 1.

Table 1. Six-factors one-order fuzzy relation groups for inflation rate using Wang's method

rule	$((x_2(t-1), x_3(t-1), x_4(t-1), x_5(t-1), x_6(t-1), x_1(t-1))$	$\to x_1(t)$	rule	$((x_2(t-1), x_3(t-1), x_4(t-1), x_5(t-1), x_6(t-1), x_1(t-1))$	$\to x_1(t)$
1	(B14, C14, D13, E12, F2, A11)	→ A8	19	(B3, C1, D13, E3, F7, A8)	→ A6
2	(B15, C14, D12, E13, F2, A8)	→ A5	20	(B3, C2, D10, E2, F7, A6)	→ A4
3	(B15, C14, D12, E13, F3, A5)	→ A4	21	(B3, C2, D12, E3, F8, A4)	→ A7
4	(B14, C13, D10, E15, F2, A4)	→ A4	22	(B3, C2, D15, E6, F8, A7)	→ A8
5	(B10, C11, D9, E16, F2, A4)	→ A4	23	(B3, C2, D15, E7, F8, A8)	→ A9
6	(B7, C8, D4, E13, F2, A4)	→ A3	24	(B3, C2, D15, E7, F14, A9)	→ A6
7	(B4, C5, D5, E13, F2, A3)	→ A3	25	(B3, C2, D15, E9, F9, A6)	→ A7
8	(B3, C3, D7, E11, F3, A3)	→ A4	26	(B3, C3, D16, E11, F9, A7)	→ A7
9	(B3, C2, D11, E10, F4, A4)	→ A5	27	(B4, C3, D19, E13, F9, A7)	→ A6
10	(B3, C2, D5, E4, F4, A5)	→ A5	28	(B4, C3, D24, E15, F10, A6)	→ A7
11	(B3, C2, D7, E9, F4, A5)	→ A8	29	(B4, C3, D21, E14, F10, A7)	→ A8
12	(B2, C2, D5, E6, F8, A8)	→ A8	30	(B4, C3, D23, E14, F11, A8)	→ A9
13	(B2, C2, D7, E7, F5, A8)	→ A5	31	(B5, C3, D15, E10, F12, A9)	→ A5
14	(B2, C2, D7, E7, F5, A5)	→ A4	32	(B5, C3, D12, E10, F13, A5)	→ A6
15	(B2, C1, D7, E7, F5, A4)	→ A6	33	(B5, C5, D16, E12, F13, A6)	→ A6
16	(B1, C1, D9, E7, F5, A6)	→ A7	34	(B5, C5, D19, E16, F12, A6)	→ A8
17	(B2, C1, D11, E8, F6, A7)	→ A6	35	(B5, C5, D19, E17, F14, A8)	→ A8
18	(B2, C1, D12, E4, F7, A6)	→ A8	36	(B5, C5, D19, E18, F16, A8)	→ A9

To know the optimal fuzzy relations, we apply the singular value decomposition method to matrix of firing strength of the fuzzy relations in Table 1 for each training datum. The distribution of the singular values of the matrix can be seen in Figure 1.

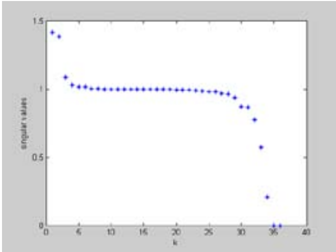

Figure 1. Distribution of singular values of firing strength matrix

There are thirty four nonzero singular values of the matrix. The singular values of the matrix decrease strictly after the first twenty nine singular values (Figure 1). Based on properties of singular value decomposition, the error of training data can be decreased by taking more singular values of the matrix but this may cause increasing error of testing data.

Table 2. Comparison of MSE of training and testing data based on different methods

Methods	Number of fuzzy relations	MSE of training data	MSE of testing data
Wang's method	36	0.063906	0.30645
Proposed method	35	0.123720	0.28040
	34	0.137980	0.21162
	33	0.138000	0.21162
	32	0.140420	0.21162
	31	0.181040	0.21162
	30	0.181220	0.21162
	29	0.191000	0.21162
	28	0.190450	0.21162
	27	0.206390	0.21162
	26	0.205660	0.21162
	25	0.180630	0.21162
	24	0.190440	0.21162
	23	0.197300	0.21162
	22	0.199100	0.21162
	21	0.203250	0.21162
	20	0.483350	0.21162
	8	0.485210	0.66290
Neural network	-	0.757744	0.42400

Then we must choose the appropriate t singular values to reduce the number of fuzzy relations. To get permutation matrix E, we apply the QR factorization and then we assign the position of entries one's in the first t columns of matrix E that indicate the position of the *t* most important fuzzy relations. The mean square errors (MSE) of training and testing data from the different number of reduced fuzzy relations are shown in Table 2.

Based on Table 2, fuzzy time series model (5) or (6) designed by 34 fuzzy relations gives a better accuracy than that designed by other different number of fuzzy relations. The predicting inflation rate using the proposed method results a better accuracy than that using the Wang's method and neural network method (Table 2). The comparison of prediction and true values of inflation rate using Wang's method and the proposed method is shown in Figure 2.

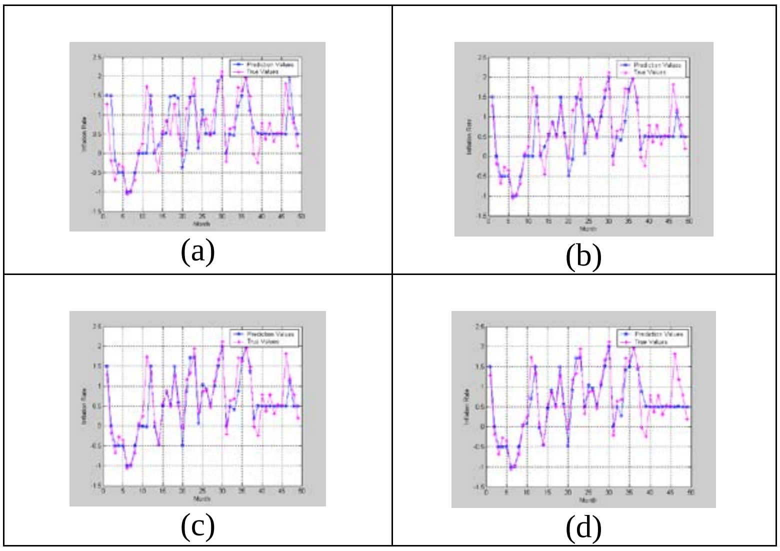

Figure 2. Prediction and true values of inflation rate using proposed method with: (a) twenty fuzzy relations, (b) twenty nine fuzzy relations, (c) thirty four fuzzy relations, (d) using Wang's method with thirty six fuzzy relations

Conclusions

In this paper, we have presented a method to design fuzzy time series model. The method combines singular value decomposition and *QR* factorization methods. The methods are used to remove the less important fuzzy relations. Then, the proposed method is applied to forecast the inflation rate. Furthermore, predicting inflation rate using the proposed method gives more accuracy than that using Wang's method and neural network.

The precision of forecasting depends also to taking factors as input variables and the number of defined fuzzy sets. In the next research, we will design how to select the important variables to improve prediction accuracy.

Acknowledgements

The authors would like to thank to LPPM Gadjah Mada University, Department of Mathematics Gadjah Mada University, Department of Economics and Business Gadjah Mada University, Department of Mathematics Education Yogyakarta State University. This work is the part of our works supported by LPPM UGM under Grant No: LPPM-UGM/1187/2009.

References

1. Song, Q and Chissom, B.S., Forecasting enrollments with fuzzy time series, part I, *Fuzzy Sets and Systems*, 1993a, **54**: 1-9.
2. Song, Q and Chissom,B.S., Fuzzy time series and its models, *Fuzzy Sets and Systems*, 1993b, **54**: 269-277.
3. Song, Q. and Chissom, B.S., Forecasting enrollments with fuzzy time series, part II, *Fuzzy Sets and Systems*, 1994, **62**: 1-8.
4. Chen, S.M., Forecasting enrollments based on fuzzy time series, *Fuzzy Sets and Systems*, 1996, **81**: 311-319.
5. Hwang, J.R., Chen, S.M., and C.H. Lee, C.H., Handling forecasting problems using fuzzy time series, *Fuzzy Sets and Systems*, 1998, **100**: 217-228.
6. Huarng, K., Heuristic models of fuzzy time series for forecasting, *Fuzzy Sets and Systems*, 2001, **123**: 369-386.
7. Chen, S.M., Forecasting enrollments based on high-order fuzzy time series, *Cybernetics and Systems Journal*, 2002, **33**: 1-16.
8. Sah, M., and Degtiarev, K.Y., Forecasting enrollments model based on first-order fuzzy time series, *Transaction on Engineering, Computing and Technology VI, Enformatika*, 2004, **VI**: 375-378.
9. Chen, S.M., and Hsu, C.C., A new method to forecasting enrollments using fuzzy time series, *International Journal of Applied Sciences and Engineering*, 2004, **2:3**: 234-244.

10. Abadi, A.M., Subanar, Widodo and Saleh, S., Forecasting interest rate of Bank Indonesia certificate based on univariate fuzzy time series, *International Conference on Mathematics and Its applications SEAMS*, 2007, Gadjah Mada University, Yogyakarta.
11. Abadi, A.M., Subanar, Widodo and Saleh, S., Reducing fuzzy relations of fuzzy time series model using QR factorization and its application to forecasting interest rate of Bank Indonesia certificate, *Proceedings of The 5th International Conference on Mathematics, Statistics and Their Applications (ICMSA)*, 2009b, Andalas University, Padang.
12. Abadi, A.M., Subanar, Widodo and Saleh, S., Constructing complete fuzzy rules of fuzzy model using singular value decomposition, *Proceeding of International Conference on Mathematics, Statistics and Applications (ICMSA)*, 2008a, Syiah Kuala University, Banda Aceh.
13. Abadi, A.M., Subanar, Widodo and Saleh, S., Designing fuzzy time series model and its application to forecasting inflation rate, *7Th World Congress in Probability and Statistics*, 2008b, National University of Singapore, Singapore.
14. Abadi, A.M., Subanar, Widodo and Saleh, S., A new method for generating fuzzy rule from training data and its application in financial problems, *The Proceedings of The 3rd International Conference on Mathematics and Statistics (ICoMS-3)*, 2008c, Institut Pertanian Bogor, Bogor.
15. Abadi, A.M., Subanar, Widodo and Saleh, S., Designing fuzzy time series model using generalized Wang's method and its application to forecasting interest rate of Bank Indonesia certificate, *Proceedings of The First International Seminar on Science and Technology*, 2009a, Islamic University of Indonesia, Yogyakarta.