Proceedings of "The 6th SEAMS-UGM Conference 2011"
Contribution of Fuzzy Systems for Time Series Analysis, pp. xx–xx.

CONTRIBUTION OF FUZZY SYSTEMS FOR TIME SERIES ANALYSIS

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Abstract. A time series is a realization or sample function from a certain stochastic process. The main goals of the analysis of time series are forecasting, modeling and characterizing. Conventional time series models i.e. autoregressive (AR), moving average (MA), hybrid AR and MA (ARMA) models, assume that the time series is stationary. The other methods to model time series are soft computing techniques that include fuzzy systems, neural networks, genetic algorithms and hybrids. That techniques have been used to model the complexity of relationships in nonlinear time series because those techniques is as universal approximators that capable to approximate any real continuous function on a compact set to any degree of accuracy. As a universal approximator, fuzzy systems have capability to model nonstationary time series. Not all kinds of series data can be analyzed by conventional time series methods. Song & Chissom [19] introduced fuzzy time series as a dynamic process with linguistic values as its observations. Techniques to model fuzzy time series data are based on fuzzy systems. In this paper, we apply fuzzy model to forecast interest rate of Bank Indonesia certificate that gives better prediction accuracy than using other fuzzy time series methods and conventional statistical methods (AR and ECM).
Keywords: *soft computing, fuzzy systems, time series, fuzzy time series, fuzzy relation*

1. INTRODUCTION

A time series is a realization or sample function from a certain stochastic process. To understanding of systems based on time series, some researchers adopt time series analysis methods. Those methods are based on many assumptions. Conventional statistical methods have been used to analysis time series data such in modeling for economic problems using parametric methods. The main goals of the analysis of time series are forecasting, modeling and characterizing. Conventional statistical models for time series analysis can be classified into linear models and non-linear models. Linear models are autoregressive (AR), moving average (MA), hybrid AR and MA (ARMA) models. This model assumes that the time series is stationary.

Soft computing, defined in Wikipedia, is a term applied to a field within computer science which is characterized by the use of inexact solutions to computationally-hard tasks. Soft computing deals with imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low solution cost. Soft computing techniques include fuzzy systems, neural networks, genetic algorithms and hybrids (Zadeh, [25]). That techniques have been used to model the complexity of relationships in nonlinear time series because those techniques is as universal approximators that capable to approximate any real continuous function on a compact set to any degree of accuracy (Wang, [22]). Soft computing techniques, as universal approximators,

2010 Mathematics Subject Classification: 94D05;03E72

make no assumptions about the structure of the data.

Fuzzy systems are systems combining fuzzifier, fuzzy rule bases, fuzzy inference engine and defuzzifier (Wang, [22]). The systems have advantages that the developed models are characterized by linguistic interpretability and the generated rules can be understood, verified and extended. As a universal approximator, fuzzy systems have capability to model nonstationary time series and give effect of data pre-processing on the forecast performance (Zhang, et.al, [26]; Zhang & Qi, [27]). Studying on data pre-processing using soft computing method has been done. Popoola [16] has analyzed effect of data pre-processing on the forecast performance of subtractive clustering fuzzy systems. Then, Popoola [16] has developed fuzzy model for time series using wavelet-based pre-processing method. Wang [23] and Tseng, et al [20] applied fuzzy model to analyze financial time series data.

Not all kinds of series data can be analyzed by conventional time series methods. Song & Chissom [19] introduced fuzzy time series as a dynamic process with linguistic values as its observations. Techniques to model fuzzy time series data are based on fuzzy systems. Some researchers have developed fuzzy time series model. Hwang et al. [12] used data variants to modeling, Huarng [11] constructed fuzzy time series model by determining effective intervals length. Then, Sah and Degtiarev [18] and Chen and Hsu [8] established fuzzy time series 1-order. Lee, et al [14] and Jilani et al [13] developed fuzzy time series high order. Abadi, et al ([1], [2], [3], [4]) developed fuzzy model for fuzzy time series data that optimize the fuzzy relations. This method was applied to forecast interest rate of Bank Indonesia certificate and gave better prediction accuracy than using other fuzzy time series methods and conventional statistical method (AR and ECM).

The rest of this paper is organized as follows. In section 2, we briefly review the conventional time series model. In section 3, we introduce fuzzy systems and its properties. In section 4, construction of fuzzy model for time series data using table lookup scheme (Wang’s method) is introduced. Optimization of fuzzy model for time series data is discussed in section 5. We also give example of application of fuzzy systems for forecasting interest rate of Bank Indonesia Certificate based on time series data in section 6. Finally, some conclusions are discussed in section 7.

2. TIME SERIES MODELS

A time series can be expressed as $\{X_t : t = 1, 2, ..., N\}$ where t is time index and N is the total number of observations and X_t is the function of components with

$$X_t = f(T_t, S_t, C_t, I_t)$$

where T_t, S_t, C_t, I_t represent the trend, seasonal, cyclical and irregular components respectively.

The main goals of the analysis of time series are forecasting, modeling and characterizing. Conventional statistical models for time series analysis can be classified into linear models and non-linear models. Linear models are autoregressive (AR), moving average (MA), hybrid AR and MA (ARMA) models. The linear models assume that the underlying data generation process is time invariant. The orders of simple autoregressive and moving average models can be determined by the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the time series data and the models can be identified from those functions (Makridakis, et.al, [15]). Box and Jenkins [6] introduced a model combining both AR and MA models called ARMA model. An ARMA model with order (p,q) is expressed as ARMA(p, q) where p is order of moving average (MA) and q is order of autoregressive (AR). The models assume that the time series data is stationary. If the time series data is nonstationary, then the modified model, integrated ARMA or ARIMA model, is used to generate a model (Chatfield, [7]). If the dependence is nonlinear where variance of a time series increases with time i.e. the time series is heteroskedastic, then the series is modeled by autoregressive conditional heteroskedastic (ARCH) model (Engle, [9]). Bollerslev [5] introduced the generalization of ARCH model called generalized ARCH (GARCH) model.

3. FUZZY SYSTEMS

In this section, we introduce some basic definitions and properties of fuzzy systems.

Definition 3.1 (Zimmermann, [28]) *Let U be universal set. Fuzzy set A in universal set U is a set* $A = \{(x, \mu_A(x)) \mid x \in U\}$ *where* μ_A *is function from U to* [0, 1].

Let $U_1, U_2, ..., U_n$ be subsets of $\mathbb{R}$. A classical relation among $U_1, U_2, ..., U_n$ is subset of $U_1 \times U_2 \times ... \times U_n$. The definition of classical relation can be generalized to fuzzy relation in $U_1 \times U_2 \times ... \times U_n$ as follow.

Definition 3.2 (Wang, [22]) *A fuzzy relation Q in* $U_1 \times U_2 \times ... \times U_n$ *is defined as the fuzzy set* $Q = \{((u_1, u_2, ..., u_n), \mu_Q(u_1, u_2, ..., u_n)) \mid (u_1, u_2, ..., u_n) \in U_1 \times U_2 \times ... \times U_n\}$ *where* $\mu_Q : U_1 \times U_2 \times ... \times U_n \to [0,1]$.

Based on the definition of fuzzy relation, the concept of compositions of fuzzy relation can be generated.

Definition 3.3 (Wang, [21]) *Let* A *and B be fuzzy relations in* $U \times V$ *and* $V \times W$, *respectively. Composition of fuzzy relations A and B, denoted by* $A \circ B$, *is defined as a fuzzy relation in* $U \times W$ *whose membership function is defined by*

$\mu_{A \circ B}(x, z) = \sup_{y \in V} t[\mu_A(x, y), \mu_B(y, z)]$ *for every* $(x, z) \in U \times W$.

A *l*-th fuzzy rule of fuzzy rule bases can be represented by:

$$Ru^{(l)} : \text{If } x_1 \text{ is } A_1^l \text{ and } \ldots\ x_n \text{ is } A_n^l, \text{ then y is } B^l \tag{3.1}$$

where A_i^l and B^l are fuzzy sets in $U_i \subseteq \mathbb{R}$ and $V \subseteq \mathbb{R}$, respectively and $x = (x_1, x_2, \ldots, x_n)^T \in U$ and $y \in V$ are linguistic variables.

The fuzzy rule (3.1) can be represented by fuzzy relation in *UxV* where the membership function is defined by $\mu_{Ru^{(l)}}(x_1, x_2, ..., x_n, y) = \mu_{A_1^l}(x_1) * ... * \mu_{A_n^l}(x_n) * \mu_{B^l}(y)$.

Fuzzy system is a system combining fuzzifier, fuzzy rule bases, fuzzy inference engine and defuzzifier. In fuzzy inference engine, fuzzy logic principles are used to combine fuzzy rule in fuzzy rule bases into a mapping from a fuzzy set *A* in *U* to a fuzzy set *B* in *V*. In applications, if the input and output of fuzzy system are real numbers, then a fuzzifier and defuzzifier can be used. Supposed $U \subseteq \mathbb{R}^n$, A' is fuzzy set in *U* and real input $x^* \in U$. A fuzzifier is defined by a mapping from *U* to fuzzy set A' that map $x^* \in U$ to fuzzy set A' in

U. There are three kinds of fuzzifier i.e. singleton fuzzifier, Gaussian fuzzifier and triangular fuzzifier. A d*efuzzifier* is defined as a mapping from fuzzy set B' in $V \subseteq \mathbb{R}$, the output of fuzzy inference engine, to real number $y^* \in V$. There are three kinds of defuzzifier i.e. center of gravity, center average and maximum.

Definition 3.4 (Wang, [22]) *Let A' be fuzzy set in U. A fuzzy inference engine based on individual rule inference with union combination, Mamdani's product implication, algebraic product for all t-norm operators, maximum for all s-norm operators, gives output of fuzzy set B' in V whose membership function as*

$$\mu_{B'}(y) = \max_{l=1}^{K}\left[\sup_{x\in U}\left((\mu_{A'}(x)\prod_{i=1}^{n}\mu_{A_i^l}(x)\mu_{B^l}(y))\right)\right]. \tag{3.2}$$

If fuzzy set B^l is normal with center $\bar{y}^l$, then fuzzy system using Mamdani implication, fuzzy inference engine, singleton fuzzifier and center average defuzzifier is

$$f(x) = \frac{\sum_{l=1}^{K}\bar{y}^l\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)}{\sum_{l=1}^{K}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)} \tag{3.3}$$

with input $x \in U \subset \mathbb{R}^n$ and $f(x) \in V \subset \mathbb{R}$.

The advantage of fuzzy system (3.3) is that the computation of the system is simple. The fuzzy system (3.3) is non linear mapping that maps $x \in U \subset \mathbb{R}^n$ to $f(x) \in V \subset \mathbb{R}$. Different membership functions of $\mu_{A_i^l}$ and μ_{B^l} give the different fuzzy system. If the membership functions of $\mu_{A_i^l}$ and μ_{B^l} is Gaussian, then the fuzzy system (3.3) becomes

$$f(x) = \frac{\sum_{l=1}^{M}\bar{y}^l\left(\prod_{i=1}^{n}a_i^l \exp\left(-\left(\frac{x_i-\bar{x}_i^l}{\sigma_i^l}\right)^2\right)\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}a_i^l \exp\left(-\left(\frac{x_i-\bar{x}_i^l}{\sigma_i^l}\right)^2\right)\right)} \tag{3.4}$$

where $a_i^l \in (0, 1]$, $\sigma_i^l \in (o,\infty)$, $\bar{x}_i^l$, $\bar{y}^l \in R$.

Theorem 3.5 (Wang, [22]) *Let U be compact set in $\mathbb{R}^n$. For any real continuous function g(x) on U and for every $\varepsilon > 0$, there exists a fuzzy system f(x) in the form of (3.4) such that* $\sup_{x\in U}|f(x)-g(x)| < \varepsilon$.

Based on the Theorem 3.5, fuzzy system can be used to approximate any real continuous function on compact set with any degree of accuracy. In applications, not all of values of function are known so it is necessary to construct fuzzy system based on sample data. Supposed there are *N* input-output pairs (x_0^l, y_0^l), $x_0^l \in \mathbb{R}^s$, $y_0^l \in \mathbb{R}$, $l = 1, 2,3,\ldots, N$. If chosen $a_i^l = 1$, $\sigma_i^l = \sigma$ and $\left\|x - x_0^l\right\|^2 = \sum_{i=1}^{s}\left(x_i - x_{0i}^l\right)^2$, then fuzzy system (3.4) has the following property.

Theorem 3.6 (Wang, [22]) *For arbitrary* $\varepsilon > 0$, *there exists* $\sigma^* > 0$ *such that fuzzy system (3.4) with* $\sigma = \sigma^*$ *has the property that* $\left| f(x_0^l) - y_0^l \right| < \varepsilon$, *for all* $l = 1, 2, \ldots, N$.

4. CONSTRUCTION OF FUZZY MODEL FOR TIME SERIES DATA USING TABLE LOOKUP SCHEME (WANG'S METHOD)

In this section, construction of fuzzy model for time series data using table lookup scheme will be introduced. Suppose given the following N training data: $(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$, $p = 1, 2, 3, ..., N$. Construction of fuzzy relations to modeling time series data from training data based on the table lookup scheme is presented as follows:

Step 1. Define the universal set for main factor and secondary factors. Let $U = [\alpha_1, \beta_1] \subset \mathbb{R}$ be universal set for main factor, $x_{1p}(t-1), x_{1p}(t) \in [\alpha_1, \beta_1]$ and $V = [\alpha_i, \beta_i] \subset \mathbb{R}, i = 2, 3, ..., m$, be universal set for secondary factors, $x_{ip}(t-1) \in [\alpha_i, \beta_i]$.

Step 2. Define fuzzy sets on the universal sets. Let $A_{1,k}(t-i), ..., A_{N_i,k}(t-i)$ be N_i fuzzy sets in time series $F_k(t-i)$. The fuzzy sets are continuous, normal and complete in $[\alpha_k, \beta_k] \subset \mathbb{R}$, $i = 0, 1$, $k = 1, 2, 3, ..., m$.

Step 3. Set up fuzzy relations using training data. From this step we have the following M collections of fuzzy relations designed from training data:

$$(A^l_{j_1^*,1}(t-1), A^l_{j_2^*,2}(t-1), ..., A^l_{j_m^*,m}(t-1)) \rightarrow A^l_{i_1^*,1}(t),\ l = 1, 2, 3, \ldots, M. \quad (4.1)$$

Step 4. Determine the membership function for each fuzzy relation resulted in the Step 3. The fuzzy relation (4.1) can be viewed as a fuzzy relation on $U \times V$ with $U = U_1 \times ... \times U_m \subset \mathbb{R}^m$, $V \subset \mathbb{R}$ and the membership function for the fuzzy relation is defined by $\mu_{R^l}(x_{1p}(t-1), x_{2p}(t-1), ..., x_{mp}(t-1); x_{1p}(t))$

$$= \mu_{A_{j_1^*,1}(t-1)}(x_{1p}(t-1))\mu_{A_{j_2^*,2}(t-1)}(x_{2p}(t-1))...\mu_{A_{j_m^*,m}(t-1)}(x_{mp}(t-1))\mu_{A^l_{i_1^*,1}(t)}(x_{1p}(t))$$

Step 5. For given fuzzy set input $A'(t-1)$ in input space U, establish the fuzzy set output $A'_l(t)$ in output space V for each fuzzy relation (4.1) defined as $\mu_{A'_l}(x_1(t)) = \sup_{x\in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t)))$, where $x(t-1) = (x_1(t-1), ..., x_m(t-1))$.

Step 6. Find out fuzzy set $A'(t)$ as the combination of M fuzzy sets $A'_1(t), A'_2(t), A'_3(t), ..\ A'_M(t)$ defined as $\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\mu_{A'_1(t)}(x_1(t)), ..., \mu_{A'_M(t)}(x_1(t))) =$

$$\max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t))) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f}(t-1)}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))).$$

Step 7. Calculate the forecasting outputs. Based on the Step 6, if fuzzy set input $A'(t-1)$ is given, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\sup_{x\in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f}(t-1)}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))). \quad (4.2)$$

Step 8. Defuzzify the output of the model. If the aim of output of the model is fuzzy set, then we stop at the Step 7. We use this step if we want the real output. If fuzzy set input $A'(t-1)$ is given with Gaussian membership function $\mu_{A'(t-1)}(x(t-1)) = \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1) - x_i^*(t-1))^2}{a_i^2})$, then membership function of forecasting output $A'(t)$ in (4.2) is

$$\mu_{B^l}(y) = \max_{l=1}^{K}\left[\prod_{i=1}^{m}\exp\left(-\frac{\left(x_i^*(t-1)-\bar{x}_i^l(t-1)\right)^2}{a_i^2+(\sigma_i^l)^2}\right)\mu_{B^l}(y)\right] \qquad (4.3)$$

With $y\in[\alpha_1,\beta_1]$. If given real input $(x_1(t-1),...,x_m(t-1))$, then the forecasting real output using the Step 7 and center average defuzzifier is

$$x_1(t) = f(x_1(t-1),...,x_m(t-1)) = \frac{\sum_{j=1}^{M} y_j \exp\left(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2}\right)}{\sum_{j=1}^{M}\exp\left(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2}\right)} \qquad (4.4)$$

where y_j is center of the fuzzy set $A_{i_1,1}^j(t)$.

5. OPTIMIZATION OF FUZZY MODELING FOR TIME SERIES DATA

In this section, a procedure to get optimal time series model is developed. The procedure uses the following steps: (1) Determine significant input variables, (2) Construct complete fuzzy relations, (3) Reduce the complete fuzzy relations to get the number of optimal fuzzy relations. In this paper, optimization of fuzzy model is measured by values of Mean Squared Error (MSE) and Mean Absolute Percent Error (MAPE) from testing data.

5.1 Selection of Input Variables

Given M fuzzy relations where the l^{th} fuzzy relation is expressed by:

"If x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n *is* $A_n^{j_n}$, then y is B_i"

and the output of fuzzy model is defined by $f=\frac{\sum_{r=1}^{M} y_r w_r}{\sum_{r=1}^{M} w_r}$, where y_r is center of fuzzy set B_r, $w_r = A_1^r\times A_2^r\times...\times A_n^r$, and $A_i^r(x_i)=\exp\left(-\frac{(x_i-\bar{x}_i^r)^2}{\sigma_{ir}^2}\right)$. Saez and Cipriano [17] defined the sensitivity of input variable x_i by $\xi_i(x)=\frac{\partial f(x)}{\partial x_i}$ with $x=(x_1,x_2,...,x_n)$. Sensitivity $\xi_i(x)$ depends on input variable x and computation of the sensitivity is based on training data. Thus, we compute $I_i=\mu_i^2+\sigma_i^2$ for each variable where μ and σ are mean and standard deviation of sensitivity of variable x_i respectively. Then, input variable with the smallest value I_i is discarded. Based on this procedure, to choose the important input variables, we must take some variables having the biggest values I_i.

5.2 Construction of Complete Fuzzy Relations Using Method of Degree of Fuzzy Relation

In modeling time series, if there are less number of training data, then fuzzy relations resulted may not cover all values in input domain. So in this paper, a procedure to construct complete fuzzy relations will be introduced. Given the following N input-output data $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$, $p = 1, 2, 3, ..., N$ where $x_{ip} \in [\alpha_i, \beta_i] \subset \mathbb{R}$ and $y_p \in [\alpha_y, \beta_y] \subset \mathbb{R}$, i = 1, 2, …, n. The method to design complete fuzzy relations is given by the following steps:

Step 1. Define fuzzy sets to cover the input and output spaces.

For each space $[\alpha_i, \beta_i]$, i = 1, 2, …, n, define N_i fuzzy sets A_i^j, j = 1, 2, …, N_i which are complete and normal in $[\alpha_i, \beta_i]$. Similarly, define N_y fuzzy sets B^j, j = 1, 2, …, N_y which are complete and normal in $[\alpha_y, \beta_y]$.

Step 2. Determine all possible antecedents of fuzzy relation candidates.

Based on the Step 1, there are $\prod_{i=1}^{n} N_i$ antecedents of fuzzy relation candidates. The antecedent has form: x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n is $A_n^{j_n}$ simplified by $A_1^{j_1}$ and $A_2^{j_2}$ and ... and $A_n^{j_n}$.

Step 3. Determine consequence of each fuzzy relation candidate.

For each antecedent $A_1^{j_1}$ and $A_2^{j_2}$ and ... and $A_n^{j_n}$, the consequence of fuzzy relation is determined by degree of the rule as

$$\mu_{A_1^{j_1}}(x_{1p})\mu_{A_2^{j_2}}(x_{2p})...\mu_{A_n^{j_n}}(x_{np})\mu_{B^j}(y_p)$$

based on the training data. Choosing the consequence is done as follows: For any training data $(x_{1p}, x_{2p}, ..., x_{np}; y_p)$ and for any fuzzy set B^j, choose B^{j*} such that $\mu_{A_1^{j_1}}(x_{1p*})\mu_{A_2^{j_2}}(x_{2p*})...\mu_{A_n^{j_n}}(x_{np*})\mu_{B^{j*}}(y_{p*}) \geq \mu_{A_1^{j_1}}(x_{1p})\mu_{A_2^{j_2}}(x_{2p})...\mu_{A_n^{j_n}}(x_{np})\mu_{B^j}(y_p)$, for some $(x_{1p*}, x_{2p*}, ..., x_{np*}; y_{p*})$. If there are at least two B^{j*} such that $\mu_{A_1^{j_1}}(x_{1p*})...\mu_{A_n^{j_n}}(x_{np*})\mu_{B^{j*}}(y_{p*}) \geq \mu_{A_1^{j_1}}(x_{1p})...\mu_{A_n^{j_n}}(x_{np})\mu_{B^j}(y_p)$, then choose one of some B^{j*}. From this step, we have the fuzzy relations in form:

IF x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n is $A_n^{j_n}$, THEN y is B^{j*}

So if this process is continued for every antecedent, there are $\prod_{i=1}^{n} N_i$ complete fuzzy relations.

Theorem 5.1 *If A is a set of fuzzy relations constructed by Wang's method and B is a set of fuzzy relations generated by method of degree of fuzzy relation, then* $A \subseteq B$.

Based on the Theorem 5.1, the method of degree of fuzzy relation is generalization of the Wang's method.

5.3 Reduction of Fuzzy Relations Using Singular Value Decomposition Method

If the number of training data is large, then the number of fuzzy relations may be large too. So increasing the number of fuzzy relations will add the complexity of computations. To overcome that, we will apply singular value decomposition method (Yen, at.al [24]). Reduction of fuzzy relations is done by the following steps referring to Abadi, et.al [4]:

Step 1. Set up the firing strength of the fuzzy relation for each training datum $(x;y)$ = $(x_1(t-1), x_2(t-1), ..., x_m(t-1); x_1(t))$ as follows

$$L_l(x;y) = \frac{\prod_{f=1}^{m} \mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A_{1,1}^l}(x_1(t))}{\sum_{k=1}^{M}\prod_{f=1}^{m} \mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A_{1,1}^k}(x_1(t))}$$

Step 2. Construct N x M firing strength matrix $\boldsymbol{L} = (\boldsymbol{L_{ij}})$ where L_{ij} is firing strength of j-th fuzzy relation for i-th datum, i = 1, 2, …, N, j = 1, 2, …, M.
Step 3. Compute singular value decomposition of $\boldsymbol{L}$ as $\boldsymbol{L} = \boldsymbol{USV^T}$.
Step 4. Determine the biggest r singular values with $r \le \text{rank}(\boldsymbol{L})$.
Step 5. Partition V as $\boldsymbol{V} = \begin{pmatrix} \boldsymbol{V}_{\varepsilon 11} & \boldsymbol{V}_{\varepsilon 12} \\ \boldsymbol{V}_{\varepsilon 21} & \boldsymbol{V}_{\varepsilon 22} \end{pmatrix}$, where $\boldsymbol{V}_{\varepsilon 11}$ is r x r matrix, $\boldsymbol{V}_{\varepsilon 21}$ is (M-r)x r matrix, and construct $\boldsymbol{V}_1^T = (\boldsymbol{V}_{\varepsilon 11}^T, \boldsymbol{V}_{\varepsilon 21}^T)$.
Step 6. Apply QR-factorization to $\boldsymbol{V}_1^T$ and find M x M permutation matrix $\boldsymbol{P}$ such that $\boldsymbol{V}_1^T \boldsymbol{P} = \boldsymbol{QR}$ where $\boldsymbol{Q}$ is r x r orthogonal matrix, $\boldsymbol{R} = [\boldsymbol{R}_{11}, \boldsymbol{R}_{12}]$, and $\boldsymbol{R}_{11}$ is r x r upper triangular matrix.
Step 7. Assign the position of entries one's in the first r columns of matrix $\boldsymbol{P}$ that indicate the position of the r most important fuzzy relations.
Step 8. Construct time series forecasting model (4.3) or (4.4) using the r most important fuzzy relations.
Step 9. If the model is optimal, then stop. If it not yet optimal, then go to Step 4.

6. FORECASTING INTEREST RATE OF BANK INDONESIA CERTIFICATE USING FUZZY MODEL

In this section, singular value decomposition method is applied to forecast interest rate of Bank Indonesia Certificate (BIC) based on time series data. First, the method of sensitivity input is applied to select input variables. Second, singular value decomposition method is applied to select the optimal fuzzy relations. The initial fuzzy model with 8 input variables (x(k-8), x(k-7), …, x(k-1)) from data of interest rate of BIC will be considered. The universal set of 8 inputs and 1 output is [10, 40] and 7 fuzzy sets $A_1, A_2, ..., A_7$ are defined on each universal set of input and output with Gaussian membership function. Then the procedure in Section 5.1 is applied to find significant inputs. The distribution of sensitivity of input variables I_i is shown in Figure 1(a). We choose the biggest two sensitivity of input variables I_i and three sensitivity of input variables I_i. Based on selecting the biggest two sensitivity of input variables and three sensitivity of input variables, the selected input variables are x(k-8), x(k-1) and x(k-8), x(k-3), x(k-1), respectively.

Then time series model constructed by two input variables x(k-8) and x(k-1) has better prediction accuracy than time series model constructed by three input variables x(k-8), x(k-3), x(k-1). So we choose x(k-8) and x(k-1) as input variables to predict value x(k). Then the method of degree of fuzzy relation is applied to yield 49 fuzzy relations showed in Table 1.

The singular value decomposition method in Section 5.3 is applied to get optimal fuzzy relations. The singular values of firing strength matrix are shown in Figure 1(b). There are 10 optimal fuzzy relations. The positions of the 10 most important fuzzy relations are known as 1, 2, 8, 9, 10, 15, 17, 29, 37, 44 printed bold in Table 1. The resulted fuzzy relations are used to design time series forecasting model (4.3) and (4.4).

Table 1. Fuzzy relations for interest rate of BIC using method of degree of fuzzy relation based on time series data

Rule	$(x(t-8), x(t-1)) \rightarrow x(t)$	Rule	$(x(t-8), x(t-1)) \rightarrow x(t)$	Rule	$(x(t-8), x(t-1)) \rightarrow x(t)$
1	**(A1, A1) → A1**	**17**	**(A3, A3) → A2**	33	(A5, A5) → A1
2	**(A1, A2) → A2**	18	(A3, A4) → A2	34	(A5, A6) → A2
3	(A1, A3) → A2	19	(A3, A5) → A2	35	(A5, A7) → A2
4	(A1, A4) → A3	20	(A3, A6) → A2	36	(A6, A1) → A2
5	(A1, A5) → A3	21	(A3, A7) → A2	**37**	**(A6, A2) → A2**
6	(A1, A6) → A3	22	(A4, A1) → A1	38	(A6, A3) → A2
7	(A1, A7) → A3	23	(A4, A2) → A1	39	(A6, A4) → A2
8	**(A2, A1) → A1**	24	(A4, A3) → A2	40	(A6, A5) → A2
9	**(A2, A2) → A2**	25	(A4, A4) → A2	41	(A6, A6) → A2
10	**(A2, A3) → A3**	26	(A4, A5) → A2	42	(A6, A7) → A2
11	(A2, A4) → A3	27	(A4, A6) → A2	43	(A7, A1) → A2
12	(A2, A5) → A3	28	(A4, A7) → A2	**44**	**(A7, A2) → A2**
13	(A2, A6) → A3	**29**	**(A5, A1) → A1**	45	(A7, A3) → A2
14	(A2, A7) → A3	30	(A5, A2) → A1	46	(A7, A4) → A2
15	**(A3, A1) → A1**	31	(A5, A3) → A1	47	(A7, A5) → A2
16	(A3, A2) → A2	32	(A5, A4) → A1	48	(A7, A6) → A2
				49	(A7, A7) → A2

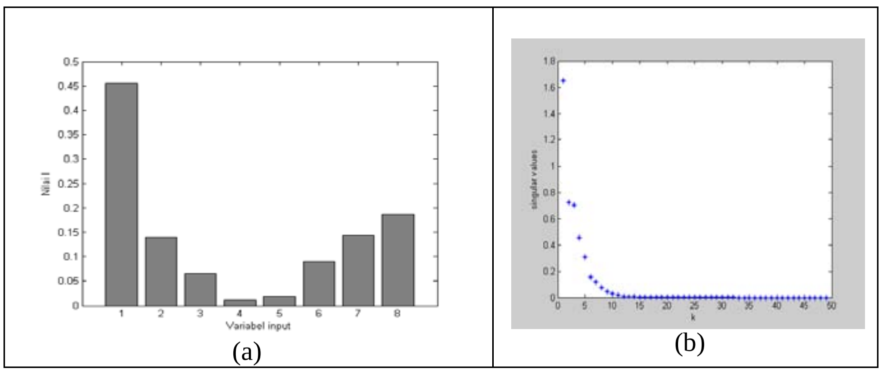

Figure 1. (a) Distribution of sensitivity of input variables I_i; (b) Distribution of singular values of firing strength matrix based on time series data of interest rate of BIC

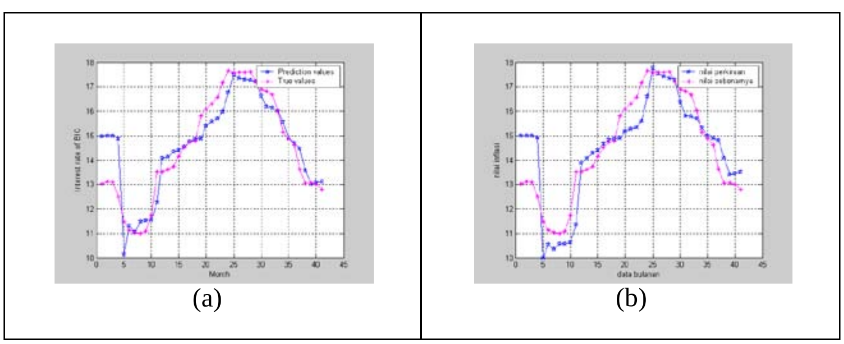

Figure 2. Prediction and true values of interest rate of BIC using: (a) singular value decomposition method (b) degree of fuzzy relation method

Table 2. Comparison of MSE and MAPE for forecasting interest rate of BIC using different methods

Method	Number of fuzzy set	Number of fuzzy relations	MSE of testing data	MAPE of testing data (%)
Singular value decomposition (selected 2-input variables), Abadi, et. al., [4]	7	10	0.14180	1.8787
Degree of fuzzy relation (2-order model), Abadi, et. al., [3]	7	49	0.38679	3.7750
Wang (2-order model), [22]	7	12	0.55075	4.4393
Wang (1-order, 6-factor model), [22]	16	36	0.26990	3.1256
Song & Chissom (1-order model), [19]	6	8	2.62025	9.8409
Chen, S.M & Hsu, C.C (1-order model), [8]	6	8	2.62025	9.8409
Chen, S.M & Hsu, C.C (2-order model), [8]	6	10	2.99333	11.029
Chen, S.M & Hsu, C.C (1-order model), [8]	10	12	1.50179	7.3517
Chen, S.M & Hsu, C.C (2-order model), [8]	10	14	1.58718	7.7343
Lee et.al. (-1order, 6-factor model), [14]	16	36	1.07718	6.1832
Jilani et.al. (1-order, 6-factor model), [13]	16	36	0.62164	5.0642
AR(7)			31.64529	35.0970
ECM			1.10010	6.3881

7. CONCLUSIONS

In this paper, we have presented capability of fuzzy systems to model time series data. As a universal approximator, fuzzy systems have capability to model non stationary time series. The uniqueness of fuzzy system is that the system can formulate problems based on expert knowledge or empirical data. We also presented a method to select input variables and reduce fuzzy relations of time series model based on training data. The method was used to get significant input variables and optimal number of fuzzy relations. We applied the proposed method to forecast the interest rate of BIC. The result was that forecasting interest rate of BIC using the proposed method has a higher accuracy than that using conventional time series methods.

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