Meeting 1:

Motivation:

Let $\mathbb{Z}$ be set of all integers. Set $\mathbb{Z}$ with ordinary operations additive (+) and multiplicative (.) has the following properties:

For every $a,b,c \in \mathbb{Z}$

I. (i) $a+b \in \mathbb{Z}$ (closed to additive)

(ii). $(a+b)+c = a+(b+c)$ (associative)

(iii). There is an element 0 in $\mathbb{Z}$ such that $a+0=0+a=a$. The element 0 is called identity element.

(iv). There exists an element -a such that a + -a = -a + a = 0. (-a is called inverse of a).

(v). a + b = b + a. (commutative)

II. (i). $ab \in \mathbb{Z}$ (closed to multiplicative)

(ii). $(ab)c = a(bc)$ (associative)

III. (i). $a(b+c) = ab+ac$ (left distributive law)

(ii). $(a+b)c = ac+bc$ (right distributive law)

We conclude that

I. ($\mathbb{Z}$, +) is abelian group

II. ($\mathbb{Z}$, .) is semigroup

III. Left distributive and right distributive laws are hold.

Definition 1(ring):

Let R be a nonempty set. A Ring (R, +, .) is set R with two binary operations + and . (called additve and multiplicative) defined on R such that the following axioms are satisfied:

I. (R, +) is abelian group. For every $a,b,c \in R$

(i). $(a+b)+c = a+(b+c)$ (associative)

(ii). There is an element 0 in R such that $a+0=0+a=a$. The element 0 is called identity element.

(iii). There exists an element -a such that a + -a = -a + a = 0. (-a is called inverse of a).

(iv). a + b = b + a. (commutative)

II. (R, .) is semigroup. For every $a,b,c \in R$

(i). $(ab)c = a(bc)$ (associative)

III. Left distributive and right distributive laws are hold. For every $a,b,c \in R$

(i). $a(b+c) = ab + ac$ (left distributive law)

(ii). $(a+b)c = ac + bc$ (right distributive law)

Definition 2:

1. If (R, +, .) is ring, then an identity element under additive operation is called zero element, denoted by z.

2. If there exists an element u such that $u \neq z$ and u is identity element under multiplicative operation, then u is called unity.

3. If ring R has unity, then ring R is called ring with unity.

4. If u, $a \in R$ there exists $a^{-1} \in R$ such that $a\,a^{-1} = a^{-1}a = u$, then a is called unit.

5. If ring R is commutative under multiplicative operation, then R is called commutative ring.

Definition 3:

1. Let a and b be nonzero elements of ring R such that ab = z, then a are called left zero diviso and b is called right zero divisor. If a is left zero divisor and right zero divisor, then a is called zero divisor.
2. If R is commutative ring with unity and no zero divisor, then R is called integral domain.
3. If R is commutative ring with unity and every nonzero element of R has inverse under multiplicative, then R is called field.

Examples:

1. Set of $\mathbb{Z}$ with ordinary operations additive (+) and multiplicative (.) is a ring.

2. How about $\mathbb{R}, \mathbb{Q}, \mathbb{C}$ with ordinary operations additive (+) and multiplicative (.)?

3. The set $\mathbb{Z}_4 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ under additive ($+_4$) and multiplicative ($\times_4$) modulo 4 is a ring.

4. The set $\mathbb{Z}_5 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ under additive ($+_5$) and multiplicative ($\times_5$) modulo 5 is a ring.

5. Identify set $Z_n = \{\overline{0}, \overline{1}, \overline{2}, ..., \overline{n-1}\}$ under additive ($+_n$) and multiplicative ($\times_n$) modulo *n* where *n* is positive integer. Is $Z_n = \{\overline{0}, \overline{1}, \overline{2}, ..., \overline{n-1}\}$ ring, integral domain, field? Explain!

6. Identify set $Z_p = \{\overline{0}, \overline{1}, \tilde{2}, ..., \overline{p-1}\}$ under additive ($+_p$) and multiplicative ($\times_p$) modulo p where p is prime number. Is $Z_p = \{\overline{0}, \overline{1}, \overline{2}, ..., \overline{p-1}\}$ ring, integral domain, field? Explain!

7. Let *n* be positive integer. Is set $n\mathbb{Z} = \{nk \mid k \in \mathbb{Z}\}$ with ordinary operations additive (+) and multiplicative (.) a ring, integral domain, field?

8. $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$ with additive (+) and multiplicative (x) operations on matrix. Is N a ring, integral domain, field? Explain!

9. $N = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$ with additive (+) and multiplicative (x) operations on matrix. Is N a ring, integral domain, field? Explain!

10. $K = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in \mathbb{R} \right\}$ with additive (+) and multiplicative (x) operations on matrix. Is K a ring, integral domain, field? Explain!