PD

Pertemuan ke-4

Pokok Bahasan: PD linear tingkat 1

Bentuk umum: $\frac{dy}{dx}+P(x)y=Q(x)$ ……………………………………………(1)

Penyelesaian:

1. Metode faktor integral
2. Metode Lagrange (variasi parameter)
3. Metode Bernoulli

Metode faktor integral:

PD bentuk (1) dapat ditulis menjadi:

$$(P(x)y-Q(x))dx+dy=0$$

Misalkan $M(x,y)=P(x)y-Q(x), N(x,y)=1$

$$\frac{\partial M}{\partial y}=P(x)\,,\frac{\partial N}{\partial x}=0$$

jadi $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ dan $\frac{1}{N}(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x})=P(x)$ fungsi x saja sehingga

diperoleh faktor integral

$$u=e^{\int\frac{1}{N}(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x})dx}=e^{\int P(x)dx}$$

Persamaan (1) dapat ditulis dalam

$$dy+P(x)ydx=Q(x)dx$$

dikalikan dengan faktor integral, diperoleh

$$e^{\int P(x)dx}dy+e^{\int P(x)dx}P(x)ydx=e^{\int P(x)dx}Q(x)dx$$

$$d(e^{\int P(x)dx}y)=Q(x)e^{\int P(x)dx}dx$$

$$\int d(e^{\int P(x)dx}y) = \int Q(x)e^{\int P(x)dx}dx + c$$

$$e^{\int P(x)dx}y = \int Q(x)e^{\int P(x)dx}dx + c$$

PU: $y = e^{-\int P(x)dx}\int Q(x)e^{\int P(x)dx}dx + ce^{-\int P(x)dx}$ …………………………………(2)

PD Bernoulli:

Bentuk umum: $\frac{dy}{dx} + P(x)y = Q(x)y^n$

Diselesaikan dengan membawa ke PD linear tingkat 1 yaitu kedua ruas dibagi y^n diperoleh

$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$$

Substitusi $y^{1-n} = w$

Diturunkan menjadi $(1-n)y^{-n}\frac{dy}{dx} = \frac{dw}{dx}$

$$y^{-n}\frac{dy}{dx} = \frac{dw}{dx}\frac{1}{(1-n)}$$

$$\frac{1}{(1-n)}\frac{dw}{dx} + P(x)w = Q(x)$$

$\frac{dw}{dx} + (1-n)P(x)w = (1-n)Q(x)$ PD linear tingkat 1.

PU $w = e^{-\int(1-n)P(x)dx}\int(1-n)Q(x)e^{\int(1-n)P(x)dx}dx + ce^{-\int(1-n)P(x)dx}$

Dengan $w = y^{1-n}$

Contoh 1: Selesaikan PD $2(y-4x^2)dx + xdy = 0$

Jawab: diubah ke PD linear tingkat 1

$$x\frac{dy}{dx} + 2(y-4x^2) = 0$$

$$x\frac{dy}{dx} + 2y = 8x^2$$

$\frac{dy}{dx} + \frac{2}{x}y = 8x$ PD linear dengan $P(x) = \frac{2}{x}, Q(x) = 8x$

Faktor integral $u = e^{\int P(x)dx} = e^{\int \frac{2}{x}dx} = e^{\ln x^2} = x^2$

Kalikan PD $dy + \frac{2}{x}ydx = 8xdx$ dengan u, diperoleh

$$x^2(dy + \frac{2}{x}ydx) = 8x^3dx$$

$$x^2dy + 2xydx = 8x^3dx$$

$$d(x^2y) = 8x^3dx$$

$$\int d(x^2y) = \int 8x^3dx + c$$

$$x^2y = 2x^4 + c$$

$$x^2y - 2x^4 = c$$

Atau langsung dengan rumus (2)

$$y = e^{-\int P(x)dx}\int Q(x)e^{\int P(x)dx}dx + ce^{-\int P(x)dx}$$

$$y = e^{-\int \frac{2}{x}dx}\int 8xe^{\int \frac{2}{x}dx}dx + ce^{-\int \frac{2}{x}dx}$$

$$y = e^{\ln x^{-2}}\int 8xe^{\ln x^2}dx + ce^{\ln x^{-2}}$$

$$y = \frac{1}{x^2}\int 8x^3dx + \frac{c}{x^2}$$

$y = \frac{2x^4}{x^2} + \frac{c}{x^2}$

$x^2 y = 2x^4 + c$

$x^2 y - 2x^4 = c$

Contoh 2: selesaikan PD $\frac{dy}{dx} - \frac{1}{x} y = -\frac{y^2}{x^2}$

Jawab: PD tersebut PD Bernoulli dengan

$P(x) = \frac{-1}{x}, Q(x) = \frac{-1}{x^2}$ dan n = 2.

Selanjutnya diubah ke PD linear tingkat dengan substitusi

$y^{-1} = w$

Diperoleh PU

$$w = e^{-\int (1-2)P(x)dx} \int (1-2)Q(x) e^{\int (1-2)P(x)dx} dx + c e^{-\int (1-2)P(x)dx}$$

$$w = e^{\int \frac{-1}{x} dx} \int (1-2) \frac{-1}{x^2} e^{\int \frac{1}{x} dx} dx + c e^{\int \frac{-1}{x} dx}$$

$$w = \frac{1}{x} \int \frac{1}{x^2} x \, dx + \frac{c}{x}$$

$$w = \frac{1}{x} \ln x + \frac{c}{x}$$

$$xw = \ln x + c$$

$$xw - \ln x = \ln c_1$$

$$\frac{x}{y} - \ln x = \ln c_1$$

$$\ln e^{\frac{x}{y}} - \ln x = \ln c_1$$

$$\ln \frac{e^{\frac{x}{y}}}{x} = \ln c_1$$

$\frac{e^{\frac{x}{y}}}{x} = c_1$ sehingga PU: $e^{\frac{x}{y}} = c_1 x$

Latihan:

1. $ydx + (3x - xy + 2)dy = 0$
2. $\frac{dy}{dx} + \frac{3y}{x} = 6x^2$
3. $x^4 \frac{dy}{dx} + 2x^3 y = 1$
4. $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$
5. $\frac{dx}{dt} + \frac{x}{t^2} = \frac{1}{t^2}$
6. $\frac{dy}{dx} + 4xy = 8x$
7. $(u^2 + 1)\frac{dv}{du} + 4uv = 3u$
8. $x\frac{dy}{dx} + \frac{2x+1}{x+1} y = x - 1$
9. $(x^2 + x - 2)\frac{dy}{dx} + 3(x+1)y = x - 1$
10. $\frac{dr}{d\theta} + r \tan\theta = \cos\theta$