Abstract Algebra

4^{th} meeting

Materials: subring and its examples

Motivation:

1. $\mathbb{Z}$, $\mathbb{R}, \mathbb{Q}, \mathbb{C}$ with ordinary operations additive (+) and multiplicative (.) are ring and $\mathbb{Z} \subset \mathbb{Q}$, $\mathbb{Z} \subset \mathbb{R}$, $\mathbb{Z} \subset \mathbb{C}$, $\mathbb{Q} \subset \mathbb{R}$, $\mathbb{Q} \subset \mathbb{C}$, $\mathbb{R} \subset \mathbb{C}$.

2. $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\}$ with additive (+) and multiplicative (x) operations on matrix is ring.

3. $N = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \middle| \, a, b \in \mathbb{R} \right\}$ with additive (+) and multiplicative (x) operations on matrix is ring.

4. $K = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \middle| \, a \in \mathbb{R} \right\}$ with additive (+) and multiplicative (x) operations on matrix is ring.

We know that $K \subset N \subset M$.

From this fact, the concept of subring of ring is defined as follows:

Definition 1: Let $(R, +, \bullet)$ be a ring, $S \neq \phi$ and $S \subset R$. S is called subring of R if $(S, +, \bullet)$ is also ring.

Examples:

1. $\mathbb{Z}$, $\mathbb{R}, \mathbb{Q}, \mathbb{C}$ with ordinary operations additive (+) and multiplicative (.) are ring and $\mathbb{Z} \subset \mathbb{Q}$, $\mathbb{Z} \subset \mathbb{R}$, $\mathbb{Z} \subset \mathbb{C}$, $\mathbb{Q} \subset \mathbb{R}$, $\mathbb{Q} \subset \mathbb{C}$, $\mathbb{R} \subset \mathbb{C}$. So we conclude that $\mathbb{Z}$ is subring of $\mathbb{R}$, $\mathbb{Q}$, and $\mathbb{C}$. Then, $\mathbb{Q}$ is subring of $\mathbb{R}$ *and* $\mathbb{C}$. And $\mathbb{R}$ is subring of $\mathbb{C}$.
2. K is subring of N and M. Then, N is subring of M.
3. Let $\mathbb{Z}_{15}$ be set of integer classes of modulo 15. Find all subring of $\mathbb{Z}_{15}$!
4. Let $\mathbb{Z}_7$ be set of integer classes of modulo 7. Find all subrings of $\mathbb{Z}_7$!

Theorem 1: Let $(R, +, \bullet)$ be a ring, $S \neq \phi$ and $S \subset R$. S is called subring of R if and only if for every $a, b \in S$ (i). $a - b \in S$ (ii). $ab \in S$

Proof: (see sukirman, 2006, page: 36)

Theorem 2: If S and T are subrings of ring R, then $S \cap T$ is subring of R.

Proof: (see sukirman, 2006, page: 42)

Let S and T be subrings of ring R. Is $S \cup T$ subring of R? explain.