Mathematics Problem Solving and Problem-Based Learning for Joyful Learning in Primary Mathematics Instruction

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A. Introduction

This module is prepared for the training on *Joyful Learning in Primary School Mathematics* conducted by the SEAMEO QITEP in Mathematics in Yogyakarta, Indonesia. This module covers two topics: (1) Problem Solving, and (2) Problem-based Learning. However, in order the participants will have comprehensive understanding on both topics, the contents of the module will consist the following subtopics:

1. Problem solving in school mathematics curricula,
2. Classification of mathematical problems,
3. Heuristics and strategies for mathematics problem solving,
4. Problem-based learning for joyful learning in primary mathematics instruction, and
5. Some ideas of problem-based learning

The objective of this module is to provide some knowledge and skills that related to the mathematics problem solving and problem-based learning in primary mathematics instruction. To be more details, after learning this module, the participants are expected to be able to:

1. explain the roles of mathematics problem solving in primary school,
2. classify mathematical problems,
3. explain the steps of mathematics problem solving,
4. explain and use certain heuristics or strategies to solve certain mathematical problem,
5. explain the concept of problem-based learning, and
6. make a lesson plan for problem-based learning.

To achieve the objectives, participants are expected to read the explanation and ask some questions related to the topics, and then try to find the answer or discuss it with other participants or with the instructor during the sessions.

B. Problem Solving in School Mathematics Curricula

Problem solving in mathematics instruction is a fundamental means of developing mathematical knowledge at any level, including at primary school. Problem solving is one of the most important, if not the most important, aspect of doing mathematics. Everyone who learn or use mathematics will face any kind of mathematical problem to be solved. Therefore, developing the skills in problem solving should be part of the objectives in the school mathematics curricula.

Problem solving is one of the ten standards in the 2000 NCTM's Standards. As proposed in the 2000 NCTM's Principles and Standards, the standards in the school mathematics curriculum from prekindergarten through grade 12 consist of contents standards and processes standards. The content standards (the content that students should learn) are: (1) number and operations, (2) algebra, (3) geometry, (4) measurement, and (5) data analysis and probability. The process standards (ways of acquiring and using content knowledge) are: (1) problem solving, (2) reasoning and proof, (3) communication, (4) connections, and (5) representation.

Furthermore, the NCTM also stated that problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in the Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics.

Not only in the US school mathematics curricula, problem solving has been integral parts of school mathematics at any other countries, including Australia, Asian or even South East Asian countries such as Indonesia, and Singapore. According to Lenchner (2005: 2), the ultimate goal of school mathematics at all times is to develop in our students the ability to solve problems. Lenchner also argued that the ability to solve problems cannot always develop automatically from mastery of computational skills, but it need to be taught, and mathematics teachers must make a special effort to do so. Through problem solving, students acquire and apply mathematical concepts and skills, so they experience the power and usefulness of mathematics, both in mathematical contexts and everyday situations as well. This makes mathematics they are learning makes sense to them. Problems can also be used to introduce new concepts and extend previously learned knowledge.

In the NCTM's Principles and Standards it is described that:

> Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking. (NCTM, 2000. http://standardstrial.nctm.org/document/chapter3/prob.htm)

The NCTM's Principles and Standards also recommended that instructional programs from prekindergarten through grade 12 from prekindergarten through grade 12 should enable all students to:

- build new mathematical knowledge through problem solving,
- solve problems that arise in mathematics and in other contexts,
- apply and adapt a variety of appropriate strategies to solve problems, and
- monitor and reflect on the process of mathematical problem solving.

Integrating problem-solving experiences in the school mathematics curricula should also build and develop the innate curiosity of young children. Teachers need to value the thinking and efforts of their students as they develop a wide variety of strategies for tackling problems. In integrating problem solving in the mathematics lessons, the teacher should create an environment in which students' effort to discover strategies for solving a problem is appreciated. Such environment is conducive in promoting learning for all students and supports students with different learning styles (Ng Wee, 2008: 7).

Assignment 1

1. Look at your primary school mathematics curriculum. Explain the objectives of mathematics learning mention in your curriculum. Is problem solving one of the objectives in your mathematics curriculum?
2. Make a comparison among primary mathematics curricula from different schools or countries from which participants come. Is there any different about the objectives or the position of the problem solving in the curricula?

C. Classification for Mathematical Problems

Not every mathematical task is a problem for certain people or students. Any mathematical task can be classified as either an exercise or a problem. An exercise is a task for which a procedure for solving is already known; usually it can be solved by applying one or more computational procedures directly. A problem is more complex because the strategy to get the solution may not immediately apparent; solving a problem requires some degree of creativity or originality on the part of the problem solver (Lenchner, 2005: 2). This difference can be illustrated by the following questions.

Suppose there are twelve coins of 25 cents, seven coins of 50 cents, four coins of $1.
1. How many coins are there?
2. What is the total value of the coins?
3. Which set of coins has the greatest value, the 25-cent, 50-cent, or $1 set?
4. How many different amount of money can be made by using one or more coins from this collection?
5. How many different combinations of one or more coins can be made from this collection?

The first three questions can be answered directly by using a computational procedure, such as addition and multiplication. Therefore, these questions can be categorized as exercises. However, the last two questions cannot be answered immediately; there is no routine procedure is applicable and the person faced with these questions must determine an appropriate strategy in order to get the answer. These kinds of question are classified as problems.

In general, mathematics problems found in school mathematics textbooks are classified as exercises, not real problem. These mathematics problems are provided as practice and reinforcement at the end of the skill and concept development lessons for various topics in the syllabus. There are straight-forward procedures that students can apply to solve these exercises.

The real problem-solving tasks should demand higher level cognitive processes with feature that may require (Foong, in Lee, 2007:55):

- complex and non-algorithm thinking,
- analysis of task constraint and use of heuristics strategies,
- exploration of mathematical concepts, processes, or relationships, and

➢ awareness of the problem situation with an interest and the motivation to make a deliberate attempt to find a solution.

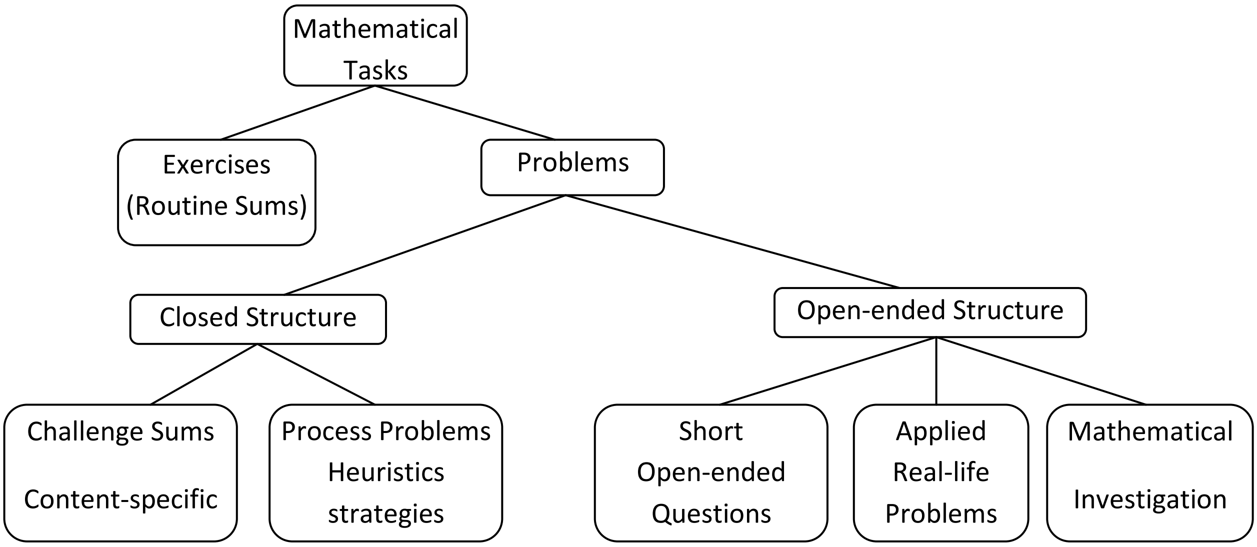


Figure 1 Classification Scheme for types of mathematical tasks (Foong, in Lee, 2007: 56)

Based on a systematic search of literature on problem solving and use of problems in research for his PhD (1990), Foong (in Lee, 2007: 56) proposed a classification scheme of different types of mathematical tasks, as shown in Figure 1.

1. Textbook **exercises** (routine sums): provided as practices at the end of a lesson and can be solved directly using already known procedures or skills that just learned.
2. **Problems**: has no immediately solution and the person who is confronted with it accepts it as a challenge and need to think for a while to tackle it.
3. **Closed problems**: well-structured, clearly formulated tasks, where the one correct answer always be determined in some fixed ways from the necessary data given in the problem situation. These closed problems include content-specific routine multiple-step challenge problems, non-routine heuristic-based problems. In order to solve these problems, the solver needs productive thinking rather than recall and must generate some process skills or some crucial steps.
4. **Challenge sums**: challenging problems that can be solved by using and after learning a particular mathematical topic such as arithmetic topic like whole numbers, fraction, ratio, percentage, with arithmetic operations such as addition, multiplication, or division, but it require higher-order analytical thinking skills.
5. **Non-routine problems** (process problem): problems that are unfamiliar or not domain-specific to any topic in the syllabus that require heuristic strategies to approach and solve it. These problems often contain a lot of cases for students to organize and consider. The mathematical content requirement should have been previously mastered by students in order to solve these problems.
6. **Open-ended problems** (often considered as 'ill-structured problems'): problems that lack clear formulation as there are missing data or assumptions and there is no fixed procedure that guarantees a correct solution. These include applied real-world problems, mathematical investigations and short open-ended questions.

7. **Applied real-world problems:** problems that are related to or come from everyday situations. To solve these kinds of problem, the individual need to begin with a real-world situation and then look for the relevant underlying mathematical ideas.
8. **Mathematical investigations**: open-ended activities for students to explore and extend a piece of pure mathematics for its own sake. The activities may develop in different ways for different students to provide them to develop their own system of generating results from exploration, tabulation of data to look for patterns, making conjectures and testing them, and justify and generalize their findings. Usually, these activities require alternative strategies, need to ask '*what if …'* questions and to observe changes.
9. **Short open-ended problems**: simple-structured problems that have many possible answers and can be solved in different ways. Usually these kinds of problem are used to develop deeper understanding of mathematical ideas and communication in students. Open-ended tasks require high-cognitive such as:
   - making own assumptions about missing data,
   - accessing relevant knowledge,
   - displaying number sense and equal grouping patterns,
   - using the strategy of systematic listing,
   - communicating argument using multiple modes of representation, and
   - displaying creativity in as many strategies and solution as possible.

In addition to the above classification, mathematical problems for primary school can also be classified based on mathematical contents or topics, such as arithmetic (number patterns, factors and multiples, divisibility, fractions), geometry and measurement, logic.

Assignment 2

1. Give an example of mathematics problem for each classification above. Explain that the problem related to the classification.
2. Give an example of mathematics problems for mathematics topic: number patterns, factors and multiples, divisibility, fractions), geometry and measurement, logic, or other topic. Explain each of your problems belongs to the above classification.

D. Heuristic and Strategies for Mathematics Problem Solving

The most widely adopted process of mathematics problem solving is the four-phase by Polya. This process consists of four steps, that are (Ng Wee, 2008: 2): (1) understanding the problem, (2) devising a plan to solve the problem, (3) implementing a solution plan, and (4) reflecting on the problem solution.

The first step in mathematics problem solving is to understand the problem. That is, students must understand what the problem means by identifying what the question needs to be answered, what information are already given, what information are missing, and also what assumptions and conditions that must be satisfied. One of ways to indicate students understanding about the problem is whenever they are able to describe the problem in their own words. When students have fully understand the problem, they are more likely to accept the problem as a challenge needs to be solve, so they start to devote themselves to find a solution.

In the second step, the students proceed to design a plan to solve the problem. In order to design a plan for solving the problem, the students need to have a general problem solving strategies, a so-called heuristics. They may have to select the apropriate strategy or to combine several strategies to solve the problem more effectively. Before implementing the "best" strategi(es), students should be encouraged to estimate the quantity, measure or magnitude of the solution. In this way, they can "see" a solution pattern without having to work through all the problem cases.

The terms "heuristics" and "strategies" in problem solving are commonly used to refer to certain aproaches or techniques used in the solution process (Foong, in Lee, 2007:62). A solution strategy is a useful technique for solving a wide variey of problems. A strategy consists of general steps to make a problem clear, simpler or manageable. Heuristic is a general strategy through which the solution to a problem is obtained. However, these two terms are often used interchangably and sometimes used togather as heuristics strategy (Foong, in Lee, 2007:62).

The next step is implementing the selected solution plan. This is the process of finding the actual solution of the problem by applying the heuristic (algorithm or computational procedure) that has been designed in the previous step. During this phase, many students may make computational mistakes. Therefore, they must have a good mastery of some basic algorithms and they have to check their solution throughout the process. In addition to the computational mistakes, students may also have difficulties in selecting the most appropriate heuristic for the problem. The use of inappropriate heuristic may result in the wrong solution. So, it is required to check solution found.

Therefore, the last step in problem solving process is to reflect on the problem solution. Students should check whether the answer obtained makes sense or reasonable. The error in a solution may be caused by a mistake in computation, wrong algorithm or heuristic. Compuational errors can be detected by using estimation, such as the result of multiplication 43×58 should be around 2400. Even if the answer seems reasonable, it is still required to check whether the answer satisfies all the given information and the required condition in the problem. In this phase, students should reflect on their chosen approaches to solve the problem. Some questions that are useful during the reflection phase are: (1) are all the given information used? (2) are all assumptions and conditions satisfied? (3) has the question in the problem been answered? (4) is the answer unique, or are there others? This reflection will examine whether the plan has resulted in the correct solution, or there is a need to seek another solution strategy, or even to realize the existence of other more efficient solution strategies to get the same result.

Mastering a number of heuristics will be very helpful for students in identifying and selecting the appropriate strategies to solve the facing problem. There are so many strategies that can be used in problem solving. However, the can be classified into four groups (Ng Wee, 2008: 9):

- **Giving a representation to the problem**: a diagram or a picture, a list/table
- **Making a calculated guess**: guess and check, look for pattern, make suppositions
- **Going through the process**: act it out, work backwards, **model methods (part-whole, comparison, before-after** concepts).
- **Modifying the question in the problem**: restate the problem, simplify the problem, solve part of the problem

The description togather with some examples of each startegy will be explained as follows. Howver, it should be considered the following notes (Ng Wee, 2008: 10).

- Not all problems require the use of heuristic(s) to get the solution, especially for simple, familiar, or routine problem.
- Though use of heuristic(s) may not guarantee that a solution will be found, the use of suitable heuristic usually enhances the chances of achieving a solution.
- A problem may be solved using different heuristics, enabling the choise of more efficient strategy.
- Some heuristics are more generic (such as gradwing a diagram, guess and check) and can be applied in many different problem situations, meanwhile certain heuristics can only be used for specific problem situations (e.g. work backwards, before-after concepts).
- Certain problems, especially more complex problems require the use of more than one heuristics to get the solution more efficiently.

1. Giving a Representation to the Problem

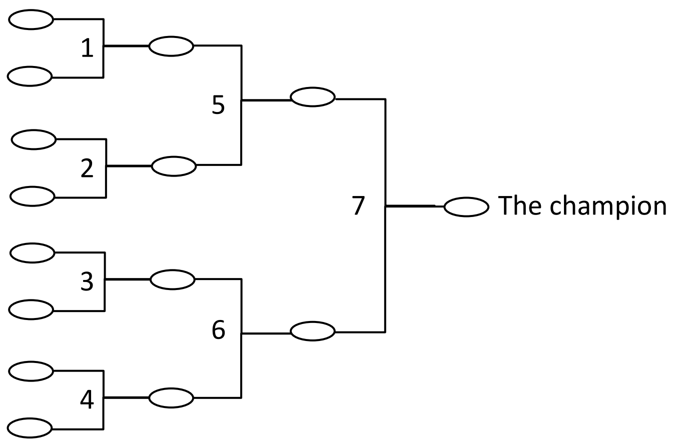
This is actually a part of ways to understand the problem. To better undestanding the problem, students should be able to represent the problem in various possible ways, such as diagrams or lists.

a. Drawing a Diagram

Drawing a diagram or a picture to model the event or relationships representing a problem is an effective strategy to help students visualize the problem, taht is to clarify what the elements are and what must be done to solve the problem. Drawing a diagram means transforming the problem into visual representation, representing the information given in the problem in the form of a diagram. In addtion to the better understanding of the problem, drawing a diagram is also one of startegies to solve certain problems. In some cases, the solution can be deduced directly from the diagram, or the appropriate strategies can be determined based on the representation.

Example
The eight teams of the City League will meet in this season's championship. The championship will apply a single-elimination tournament to determine the champion, that is e team will be out of the tournament after one loss. How many games will be required?

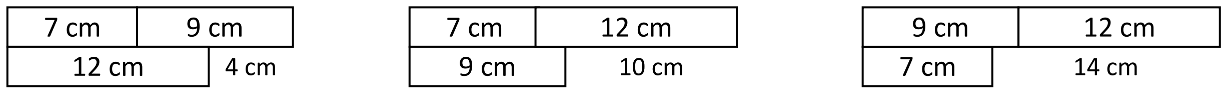
Solution: **1st round 2nd round 3rd round**



There will be seven games to be played to get the champion.

Example:
The length of three rods are 7 cm, 9 cm, and 12 cm. How can you use these rods to measure a length of 10 cm?

Solution:



(a) (b) (c)

The diagram (b) shows how to get the measure of 10 cm length.

b. Making a Systematics List

A systematic or an organized list in the form of table or chart is very useful tool to represent and classify information in the problem. A list, a chart or a table can be made systematically by certain characteristics to account possibilities and sort it. By using complete ordered lists students will be able to check for repeated answers or patterns and derive solutions. This strategy is very useful when the problem involves a great dela of data or sequence of posiible answers.

Example
There are a number of students and a number pens. If 3 pens are given to each students, 1 pen will be left over. If one students receive 5 pens, three students will not receive any pen. How many students and pens are there?

Solution:

No. of students	1	2	3	4	5	6	7	8	9	10	11
No. of pens (based on situation one)	4	7	10	13	16	19	22	25	28	31	34
No. of pens (based on situation two)	-	-	-	5	10	15	20	25	30	35	40

There are 8 students and 25 pens.

Example
There are some tables and chairs. Each table has four legs, while each chair has three legs. Someone looks there are 43 legs. How many tables and chairs are there?

No. tables	1	2	3	4	5	6	7	8	9	10	11
No. of table legs	4	8	12	16	20	24	28	32	36	40	-
No. of legs for chairs	39	35	31	27	23	19	15	11	7	3	-
No. of chairs	13	-	-	9	-	-	5	-	-	1	-

Notes: **- means impossible**

The possible answers are: one table and 13 chairs, 4 tables and 9 chairs, 7 tables and 5 chairs, and 10 tables and 1 chair.

2. Making a Calculated Guess

It is sometimes useful to make random guess when students facing a chalenging problem. Although it seldom happens, making random guess might led to a solution. Instead of making random guess, it is better to make calculated guess. At first, initial guess may be made randomly, then followed by the next guess chosen by revising the initial guess that better satisfies the problem conditions. Sometimes, students need to look for patterns among the data given or generated. In other cases they may make some suppositions about the problem situation.

c. Guessing and Checking

Guess and check is also known as trial and error. Students sometimes use this strategy to answer a question in a problem. Howver, trial and error is frequently unsuccessful in getting the solution. In using this strategy, students should make guesses systematically, use table to record it, check it against information or conditions required in the problem, then refine it if it is an error. Analyzing the unsuccessful guess can provide useful information about the problem and can help students decide how to improve further guesses.

Example

There are a number of students in a class. Each student has the same number of marbles. The total number of marbles of all students is 161. If every student has at least 2 marbles, how many students are there? (Assume that the number of students is larger than the number of marbles of each student).

Solution:

Since the last digit of 161 is 1, it is necessary to consider only the product of two numbers whose last digit is 1, 3, 7, or 9.

No. of marbles of each student	No. of students	Total number of marbles	Check: Equals 161?	Reasonable answer?
1	161	161	Yes	No, doesn't satisfy the condition
11	11	121	No	
11	21	231	No	
3	37	111	No	
3	47	141	No	
3	57	171	No	
13	17	191	No	
7	13	91	No	
7	23	161	Yes	Yes
9	9	81	No	
9	19	171	No	

The resasonable answer is that there 23 students in the class each has 7 marbles.

d. Looking for Patterns

Observing patterns and relationships is one of the strategies frequently used for solving mathematical problems. Patterns can appear in a number sequence, figures, systematic lists or tables. The ability to identify and recognize a pattern or a relationship among the elements in a given problem is sometimes very useful to simplify a problem-solving process and times. Students need to learn and improve their ability to identify and recognize many number patterns and predict or generalize the relationship between two data in mathematics. This can be achieved through examples and practice.

Examples:

1) Look at the following sequences. Write the next three numbers for each sequence.
   a) 1, 2, 4, 7, 11, 16, …
   b) 1, 2, 3, 5, 8, 13, …
   c) 1, 4, 9, 16, 25, …
   d) 2, 6, 12, 20, 30, …

Solution:

   a) **The pattern**: staring from the 2nd number, the nth number is the sum of the previous number and $(n-1)$. So, the next three numbers are **22**, **29**, **37**.
   b) **The pattern**: starting from the third number, every number is the sum of the last previous two numbers. So, the next three numbers are **21**, **34**, **55**.
   c) **The pattern**: each number is a square number. So, the next three numbers are **36**, **49**, **64**.
   d) **The pattern**: each number is a multiplication of two consecutive integers, starting from 1. SO, the next three numbers are **42, 56, 72.**

2) Complete the following tables.

a)

First number	1	2	3	4	5	6	7	8
Second number	5	9	13	17	21			

b)

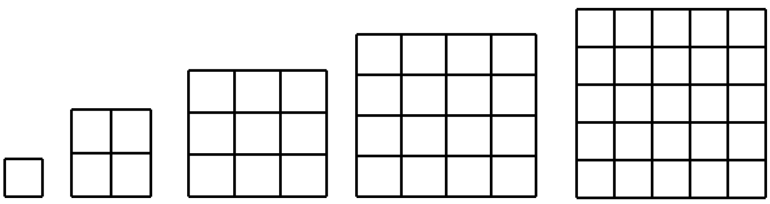
First number	2	3	4	5	6	7	8	9
Second number	1	3	6	10	15			

Solution:

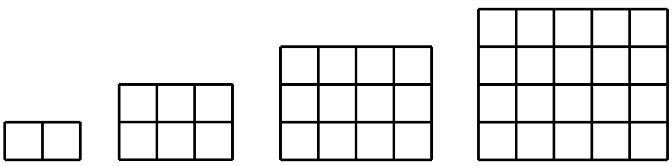
   a) **The rule**: the second number equals 4 times the first number (above) plus one. So, the next three fills on the table are: **25, 29, 33**.
   b) **The rule:** the second number equals the multiplication of the first number (above) and its previous value (i.e. the above number minus one) divided by two. So, the next three fills on the table are: **21, 28, 36**.

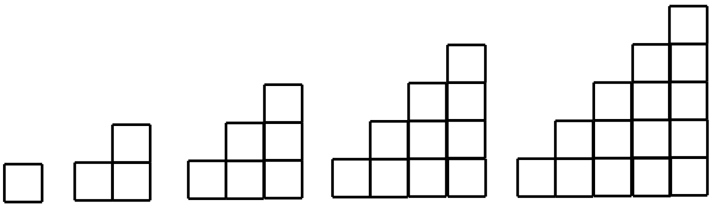
3) Look at the following sequence of pictures. If each sequence of pictures is made continually, how many unit squares are there in the 10th picture?

a)



b)



c) 

Solution:

a) The picture is always a square, and the nth picture has n^2 unit squares. So, the 10th picture has 100 unit squares.
b) The picture is always a rectangle, and the nth picture has $n \times (n+1)$ unit squares. So, the 10th picture has 110 unit squares.
c) The picture is always a triangular form, and the nth picture has $\frac{n\times(n+1)}{2}$ unit squares. So, the 10th picture has $\frac{110}{2} = 55$ unit squares.

4) What is the sum of $1 + 3 + 5 + 7 + \cdots + 99$?

Solution:

Instead of calculating directly the sum, students can make a few simpler calculations and observe the pattern happens, as follows.

The number of odds to be summed	The summation	The sum
1	1	$1 = 1^2$
2	1+3	$4 = 2^2$
3	1+3+5	$9 = 3^2$
4	1+3+5+7	$16 = 4^2$
5	1+3+5+7+9	$25 = 5^2$
⋮	⋮	⋮

It seems that the sum is always a square number, that is the square of the number of odds to be summed. Because $1 + 3 + 5 + 7 + \cdots. +99$ contains the first 50 odd numbers, it is equal to $50^2 = 2500$. Therefore, $1 + 3 + 5 + 7 + \cdots. +99 = 2500$.

e. Making Suppositions

The heuristic requires students to suppose something (assumption) about a given problem situation is true (although it may not be given explicitly in the problem or even not true) and then deduce useful information about the problem that lead to finding the solution. Suppositions may be made by adding or removing some element(s) from the problem. Making a supposition can help students find or better understand the relationship among elements of the problem. It also makes the problem easier to understand or solve. A supposition is also required for solving open-ended problems or problems with insufficient data or conditions are given.

Example

Every visitor comes to a mall with a car or a motorbike, in which every car is occupied by 4 visitors and every motorbike is occupied by 2 visitors. There are 20 vehicles in the parking area of the mall and there are 56 visitors. How many cars and motorbikes are there in the parking area?

Solution:

Suppose all 20 vehicles are motorbikes. The number of visitors using motorbikes is $2 \times 20 = 40$. The other $56 - 40 = 16$ visitors use cars. However, each car is occupied by 2 more visitors than a motorbike. So, there must be 16 other visitors using cars. So, the number of visitors using motorbike is $40 - 16 = 24$ and the number of visitors using cars is 32. Thus there are 12 motorbikes and 8 cars.

Is there any other solution? Use other heuristic, such as systematic lists, to check it.

Example

There are 32 animals consisting of some cows and goats that eat 200 kgs of grass. If each cow eats 10kgs of grass and each goat eats 5 kgs of grass, how many cows and goats are there?

Solution:

Suppose all animals are cows. The required grass is 320 kgs. The extra grass demanded is 120 kgs, which is enough for 24 goats. So, if there are 24 goats, that require 120 kgs, then there are 80 kgs of grass enough for 8 cows. Therefore, there are 8 cows and 24 goats.

Is there any other solution? Use other heuristic, such as systematic lists, to check it.

3. Going through the Process

In order to get the solution to a problem, students go through a process of synthesis by constructing the model based on the information given in the problem, then they analyze the model to develop a sequence of logical steps to achive the solution. In some cases, using physical objects as the model in the process of finding solution might be helpful to better understand the problem and obtain the solution more efficiently.

f. Acting it Out

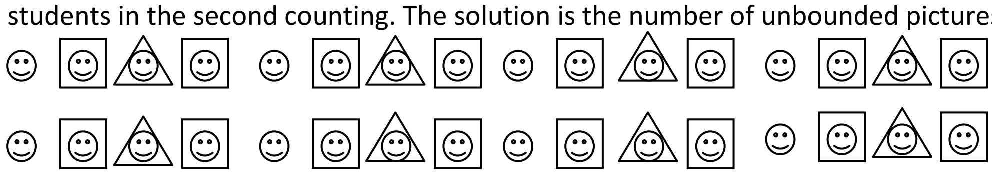
When students experience difficulty in visualizing a problem or the procedure necessary for its solution, they may find it helpful to physically act out the problem situation. They may use items such as physical objects or manipulative materials to represent the people or objects in the problem and attach conditions given in the problem. This acting out the problem makes mathematical concepts and problem more concrete and physical, so it may lead students to the answer, or to another strategy that will help them find the answer. This strategy is useful for dealing with a problem that is dynamic such as that involving multiple moving objects or changing variables. It is very effective for young students.

Example

In a class there are 32 students sitting on their chairs. They counted off by ones starting from number 1. Students who counted even numbers stood up. After that, the remaining students counted off by ones again and did the same thing. After the second counting was completed, how many students remained seated?

Solution:

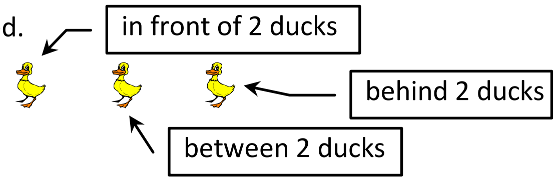
Lets make 32 pictures representing 32 students, as follows. Then, let the squared pictures represent the even numbered students in the first counting, and triangles are used to represent the even numbered students in the second counting. The solution is the number of unbounded pictures.



So, after the second counting, 8 students remained seated.

Example
There is a duck in front of two ducks, a duck behind two ducks, and a duck between two ducks. What is the least number of ducks that there could be in this group?

Solution:
Use some manipulative (or pictures) to represent ducks and arrange it such as all the conditions in the problem are satisfied.



There are three ducks.

g. Working Backwards

To solve a problem that involves a sequence of actions in which the final result of the action is known but the initial conditions are unknown, students can effectively consider the actions in reverse order. In this heuristic, students work from the end state of the problem and work backwards to the initial state by reversing the conditions given in the problem till a solution is found. The use of a diagram is often useful in the process of reversing the conditions.

Example
Andi gave Budi and Cindy as much money as each had. Then, Budi gave Andi and Cindy as much money as each already had. After that, Cindy gave Andi and Budi as much money as each had. At the end, each of the three people had $32. How much money did each have at the beginning?

Solution:
There are four stages in this problem:

1. Andi gave Budi and Cindy as much money as each had.
2. Budi gave Andi and Cindy as much money as each had.
3. Cindy gave Andi and Budi as much money as each had.
4. Each had $32.

To solve the problem, we start from the last stage, then work backwards to the stage 3, 2,and 1.By using the following table, it is easier to work backwards.

Stage	Actions	Andi	Budi	Cindy	Sum
4	Each ends with $32	$32	$32	$32	$96
3	Cindy gave Andi and Budi as much money as each had. *It means that Andi and Budi had $16 each, Cindy had $64*.	$16	$16	$64	$96
2	Budi gave Andi and Cindy as much money as each had. *It means that Andi had $8, Cindy had $32 each, Budi had $56*.	$8	$56	$32	$96
1	Andi gave Budi and Cindy as much money as each had. *It means that Cindy had $16, Budi had $28, and Andi had $52*.	$52	$28	$16	$96

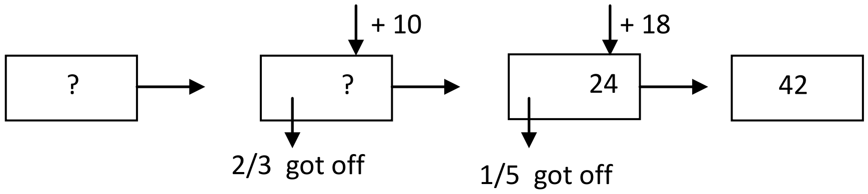
So, at the beginning, Andi had $52, Budi had $28, and Cindy had $16.

Example

There were some passengers in a bus. At the first bus stop, 2/3 of the passengers alighted and 10 other people came on board. At the second bus stop, 1/5 of the passengers alighted while other 18 people came on board. The number of passengers was 42. Hon may passengers were there at the first?

Solution:

Here we can use a diagram to show the situations in the problem and work backwards to get the solution.



(4) At the end, there are 42 passengers.
(3) At the second bus stop, before some new passenger came on board, there were 42-18 = 24 passengers. This is after 1/5 of the passengers got off.
So, just before reaching the second bus stop there were 24+24/4=30 passengers.
(2) At the first bust stop, before some new passenger came on board, there were 30-10=20 passengers. This is after 2/3 of the passengers got off.
(1) So, at the beginning there were 3x20 = 60 passengers.

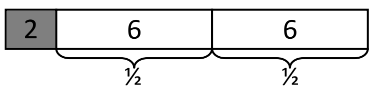
h. Model Methods (Part-Whole, Comparison, Before-After Concepts)

Part-whole model (or "part-part-whole" model)

A whole is divided into two or more parts. If the parts are known, the the whole can be determined by addition. When the whole and all but one part are known, then the unknown part can be determined by substraction. This model is also useful for concepts of multiplication or division and fraction when the whole is divided into equal parts.

Example
Yudi bought some oranges. He ate two of them and gave half of the remaining to Yuli. After that, he had 6 oranges. How many oranges did Yudi buy?

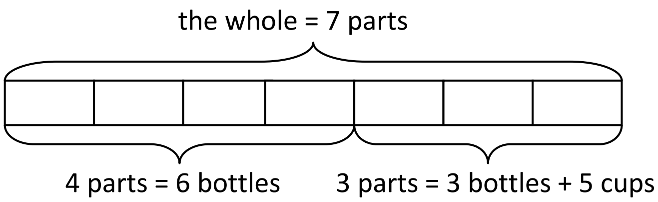
Solution:



Yudi bought 14 oranges.

Example
Six bottles of water can fill $\frac{4}{7}$ of a container. Another 3 bottles and 5 cups of water are needed to fullfil the container. How many cups of water can the container hold?

Solution:



4 parts = 6 bottles; 3 parts = 3 bottles + 5 cups

2 parts = 3 bootles; 3 parts = 3 bottles + 5 cups = 2 parts + 5 cups; so 1 part = 5 cups

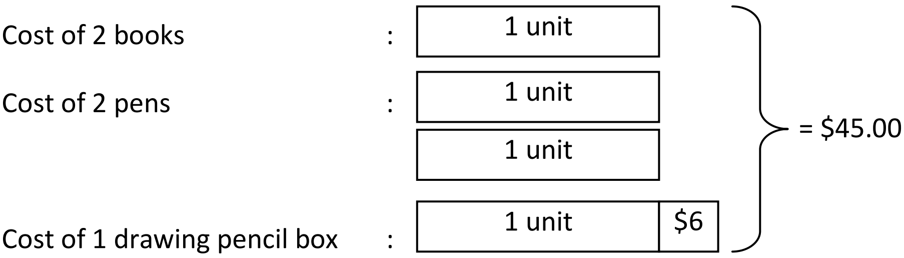
The container can hold $7 \times 5 = 35$ cups.

Comparison model

In this model two or more quantities are compared using "proportional" figure. Based on the comparison, when the quantities are known the difference or ratio can be determined.

Example
Two books, two pens and a drawing pencil box cost $45.00. A book costs twice as much as a pen. A drawing pencil box costs $6 more than a pen. What is the cost of a drawing pencil box?

Solution:



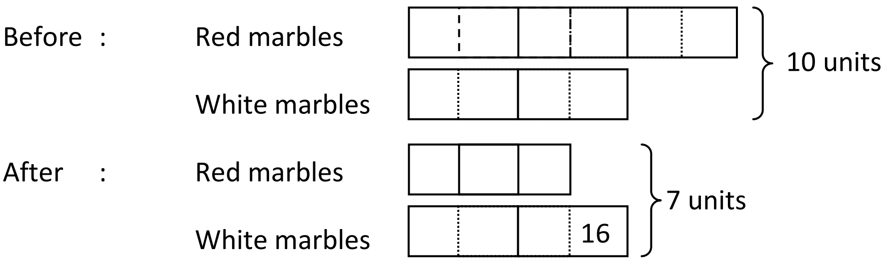
4 units + $6 = $45.00 ➔ 4 units = $39.00 ➔ 1 unit = $39.00 : 4 = $9.75

Cost of a drawing pencil box = 1 unit + $6 = $9.75 + $6 = $15.75

Before-after model

If the problem involves the relationship between new value of a quantity and its original value after increase or decrease. This model sometimes also involves the use of comparison model for complex structures.

Example
A box contains red and white marbles in ratio 3:2. After ½ of the red marbles are taken, there are 16 more white marbles than the number of red marbles. How many marbles are inside the box initially?



1 unit = 16 marbles. Initially, the box contains 10 units, that are 160 marbles.

4. Modifying the Question in the Problem

Solving a complicated problem can be very difficult for students. There are several approaches that can be used in attempting to solve complicated problem. They may try to see the problem in different way, to consider the simpler cases, or to break the problems into smaller, more manageable parts or identify and solve crucial parts of the problem. There are some heuristics that can be employed in this case. Among of them are: restate the problem, simplify the problem, solve part of the problem.

i. Restating the Problem

The given problem is looked from different points of view or perspectives. This allows students to restate in a way that is easier or simpler to solve and that is equivalent to the original problem. This can be done by using geometric or mathematical models derived logically from the original problem. Of course the identification of equivalent problems requires good knowledge, experience, and reasoning skills, and it might not easy for elementary students. Teachers can facilitate students by suitable worked examples and encourage them to tool at problems from a variety of perspectives, even unconventional ways, so they able to recognize various more commonly known equivalent problems.

Example

Five teams (A, B, C, D, E) representing five different clubs took part in a round-robin soccer tournament (i.e. every team played against every other team exactly once). There were no draws in this tournament. The score for each team is the number of wins. What is the total scores in this tournament?

Solution:

Since there was no draw in any of the games, each win corresponds to a game played. Therefore, the total scores in the tournament is the same as the number games played. Then games are:

A-B, A-C, A-D, A-E,	(4 games)
B-C, B-D, B-E,	(3 games)
C-D, C-E,	(2 games)
D-E	(1 game)

Thus, the number of games played in the tournament is: 4+3+2+1=10.

j. Simplifying the Problem

In this heuristic, the given problem is either divided into simpler cases and then each case is solved separately or simplified by restating it using smaller number(s) or a more familiar problem setting. This strategy is often related to both guess-and-check as well as look-for-pattern strategies. Students may consider a simple case to identify a solution process that can be generalized and applied to the more complex cases and the original problem. Solving a simpler case with simpler conditions that related to the original problem will help solving the original problem by applying the same approach and looking for a pattern in the simpler cases.

Example

The pages of a book are numbered from 1 to 200. How many pages whose number contains at least one digit 7?

Solution:

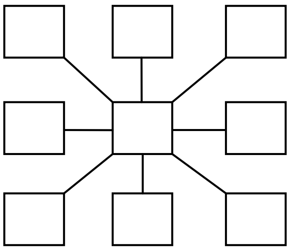
This problem can be solved by examining all page numbers, but this will take time and more work is necessary. Instead of doing this, the problem can be solved by looking at the simpler cases: (1) looking for the numbers contain one digit 7, and (2) looking for the numbers contain two digit 7. Because the numbers are from 1 to 200, the positions of digit 7 can only at the ones and tens places.

(1) For the case (1), one digit 7 at the ones place can happen every ten pages, i.e. 7, 17, 27, …, 197. There are 20 numbers.

(2) For the case (1), one digit 7 at tens place can happen for numbers 70, 71, 72, …, 76, 78, 79, 170, 171, 172, …, 176, 178, 179. There are 18 numbers.
(3) For the case (2), the only number is 77.

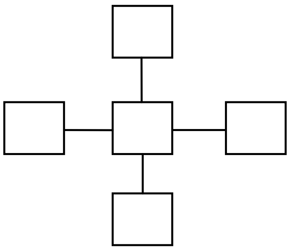
Therefore, there are 20+18+1=39 pages whose number contains at least one digit 7.

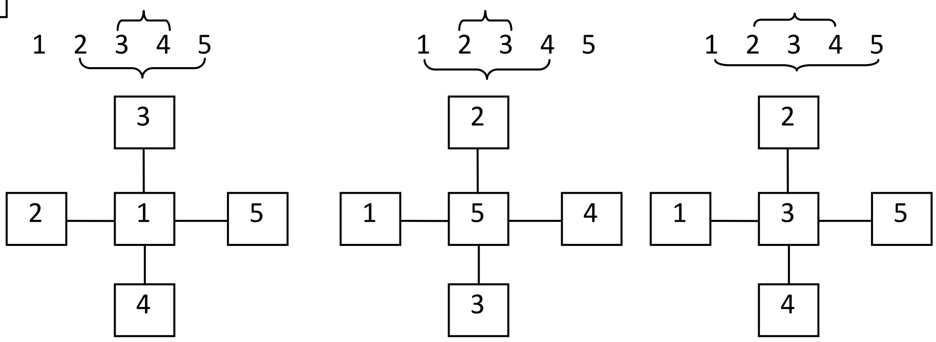
Example



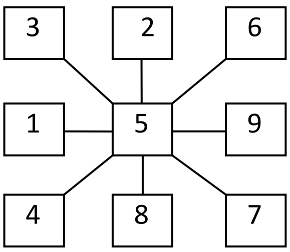
Arrange the numbers 1 through 9 in the following boxes in such a way that the sum of the numbers in each line is 15.

Solution:
Before solving the problem, we may consider the simpler problem using only numbers1 to 5 with five boxes as follows. The goal is to arrange the numbers 1 through 5 in the five boxes in such a way that the sum of the numbers on each line is the same. This simpler problem can be solve**d** by first leaving out an odd number and pairing the remaining 4 numbers so that the sum of the numbers in each pair is the same. The left out odd is then put on the middle box.

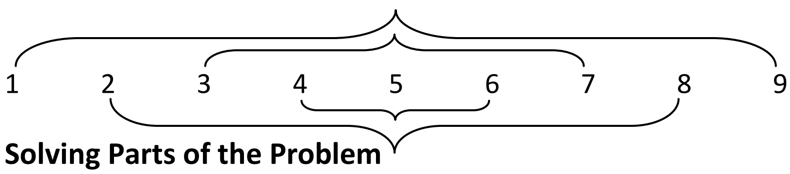




The sum is 8 The sum is 10 The sum is 9



The given problem now can be solved in a similar way, and the solution is as follows.



k. Solving Parts of the Problem

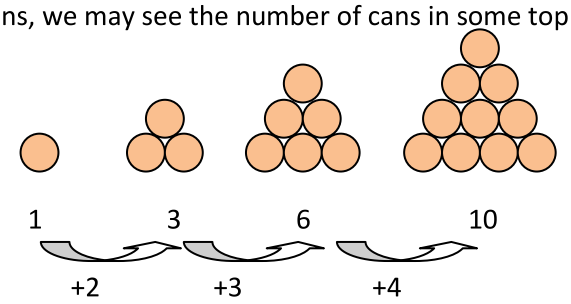
In this heuristic, the given problem is divided into successive sub-problems and each sub-problem is solved so that the question in the original problem is answered. This approach is useful in solving problems that is difficult or impossible to solve directly from the given data and conditions. This heuristics may be time-consuming, because each sub-problem may require different solution approach. Here, students need to indentify the sub-goals which are required to construct the solution of the given problem.

Example
A group of students is given a task to arrange some identical cylindrical cans in the form of triangular pyramid of height of at least 1.5 m. Each of the cans has a diameter of 12 cm. The cans must be arranged upright to form the triangular pyramid. At least how many cans are needed?

Solution:

The problem can be solved by first finding the number of levels in the cans arrangement to satisfy the requirement. Then, when the number of levels is found, the solution is found by determining the number of cans to form that level.

Since the required height of the pyramid is at least 1.5 m = 150 cm, and the diameter of each can is 12 cm, then the minimum number of levels is 150:12=12.5,that is 13. To determine the number of required cans, we may see the number of cans in some top levels of such a pyramid as below.



It is observed that every time going down one level, the number required cans increases by the next level's number counted from one from the top. Therefore, the minimum number of required cans is

$$1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 + 66 + 78 + 91 = 455.$$

Assignment 3

1. Solve each problem in the above examples using some other heuristics described in the examples.
2. Solve each problem in your examples in the **Assignment 2** using some heuristics described above.

E. Problem-Based Learning for Joyful Learning in Primary Mathematics Instruction

1. The Nature and Characteristics of Problem-based Learning

Problem-based learning (PBL) describes a learning environment where problems drive the learning. That is, learning begins with a problem to be solved, and the problem is posed so that the students discover that they need to learn some new knowledge before they can solve the problem. Rather than seeking a single correct answer, students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions. Proponents of mathematical problem solving insist that students become good problem solvers by learning mathematical knowledge heuristically (Roh, 2003).

PBL is focused experiential learning organized around the investigation and resolution of messy, real-world problems, ill-structured, problematic situation. PBL organizes curriculum around this holistic problem, enabling student learning in relevant and connected ways.

PBL creates a learning environment in which teachers coach student thinking and guide student inquiry, facilitating learning toward deeper levels of understanding while entering the inquiry as a co-investigator.

As cited by Roh, 2003 from (Krulik & Rudnick, 1999; Lewellen & Mikusa, 1999; Erickson, 1999; Carpenter et al., 1993; Hiebert et al., 1996; Hiebert et al., 1997), problem-based learning is a classroom strategy that organizes mathematics instruction around problem solving activities and affords students more opportunities to think critically, present their own creative ideas, and communicate with peers mathematically.

PBL sometimes refer to problem-based learning, but can also refer to project-based learning. There is overlap between problem-based learning and project-based learning, but both are not the same (IAE-pedia, 2009).

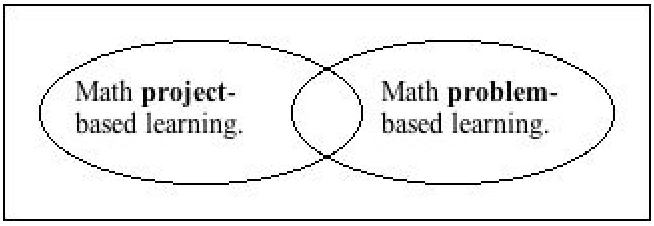


Figure 2 Overlap between problem-based learning and project-based learning (http://iae-pedia.org/Math_Problem-based_learning, 2009)

The following table illustrates both problem-based learning and project-based learning same (adopted from IAE-pedia, 2009).

Table 1 Comparison of Problem-based learning and Project-based learning

Problem-based Learning	**Project-based Learning**
➢ Students in a class are assigned a challenging math problem. ➢ They may work individually or perhaps in teams. ➢ While the assignment some times continues over several class periods, more typically it is done in one class period and/or as homework.	➢ Students are given a general project area, and they are asked to work on it individually or in teams. ➢ The project may lead to both a written and an oral presentation. ➢ Teacher might need to emphasize the idea of developing a paper that will be useful to other students or future students in the class, and students in other similar classes. The intended audience is much larger than just the teacher!
Example: Combine four 4s in various ways in order to make as many different integers as possible, using addition, subtraction, multiplication, division, and parentheses, concatenation, exponentiation, or other types of operation. E.g. 1. (4 + 4 + 4)/4 = 3 2. 444/4 = 111	**Example**: Select a topic in mathematics. Go to the library and/or Internet to find some articles or sources describe the topic and its applications.
➢ It illustrates that a math problem may have more than one solution. ➢ It illustrates the need for very careful definition of a problem. ➢ It is a problem that can engage individual students or a team of students over an extended period of time. ➢ It has some of the characteristics of a project. ➢ Some students are likely to produce more answers than others. In any case, there is the added challenge of trying to prove that one has found all possible answers. ➢ The problem can be varied based on the number and/or the types of operations allowed based on the students level.	➢ The "assignment" might include the requirement that students write a paper and do a presentation to the whole class. ➢ The assignment could well extend over a number of weeks, with much of the required work being done outside of class. ➢ If this assignment is given in a specific math class (such as geometry or algebra) then the assignment might include the requirement that the study focus on roles of the math being studied in the class be emphasized in the project.

Problem based learning has several distinct characteristics as mentioned by Park (2001) as follows.

1. It uses of real world problems - problems are relevant and contextual. It is in the process of struggling with actual problems that students learn content and critical thinking skills.
2. It reliance on problems to drive the learning - the problems do not test skills; they assist in development of the skills themselves.
3. The problems are truly ill-structured - there is not meant to be one solution, and as new information is gathered in a reiterative process, perception of the problem, and thus the solution, changes.
4. PBL is learner-centered - learners are progressively given more responsibility for their education and become increasingly independent of the teacher for their education.
5. PBL produces independent, life-long learners - students continue to learn on their own in life and in their careers.

It is clear that problem-based learning is very close to the problem solving itself. This is because PBL starts with a problem to be solved, and students working in a PBL environment must become skilled in problem solving, creative thinking, and critical thinking. PBL in mathematics classes would provide young students more opportunities to think critically, represent their own creative ideas, and communicate with their peers mathematically. A PBL environment provides students with opportunities to develop their abilities to adapt and change methods to fit new situations. Students in PBL environments typically have greater opportunity to learn mathematical processes associated with communication, representation, modeling, and reasoning (Smith, 1998; Erickson, 1999; Lubienski, 1999 as cited by Roh, 2003).

Although not as center for activities, the role of teachers in a PBL environment is very important. Within PBL environments, teachers' instructional abilities are more critical than in the traditional teacher-centered classrooms. Beyond presenting mathematical knowledge to students, teachers in PBL environments must engage students in organizing information and using their knowledge in applied settings. They should also have a deep understanding of mathematics that enables them to guide students in applying knowledge in a variety of problem situations. Teachers with little mathematical knowledge may contribute to student failure in mathematical PBL environments. Without an in-depth understanding of mathematics, teachers would neither choose appropriate tasks for nurturing student problem-solving strategies, nor plan appropriate problem-based classroom activities. In this context, they should also develop a broader range of pedagogical skills. Teachers pursuing problem-based instruction must not only supply mathematical knowledge to their students, but also know how to engage students in the processes of problem solving and applying knowledge to novel situations (Prawat, 1997; Smith III, 1997, as cited by Roh, 2003).

PBL has become popular because of its apparent benefits to student learning. Students engage in authentic experiences which require them to have and access all three forms of knowledge (domain specific content and skills, soft skills, and social and collaborative skills).

Through PBL, students learn (Park, 2001):

- ✓ Problem-solving skills,
- ✓ Self-directed learning skills,
- ✓ Ability to find and use appropriate resources,
- ✓ Critical thinking,
- ✓ Measurable knowledge base,

- ✓ Performance ability,
- ✓ Social and ethical skills,
- ✓ Self-sufficient and self-motivated,
- ✓ Facility with computer,
- ✓ Leadership skills,
- ✓ Ability to work on a team,
- ✓ Communication skills,
- ✓ Proactive thinking,
- ✓ Congruence with workplace skills.

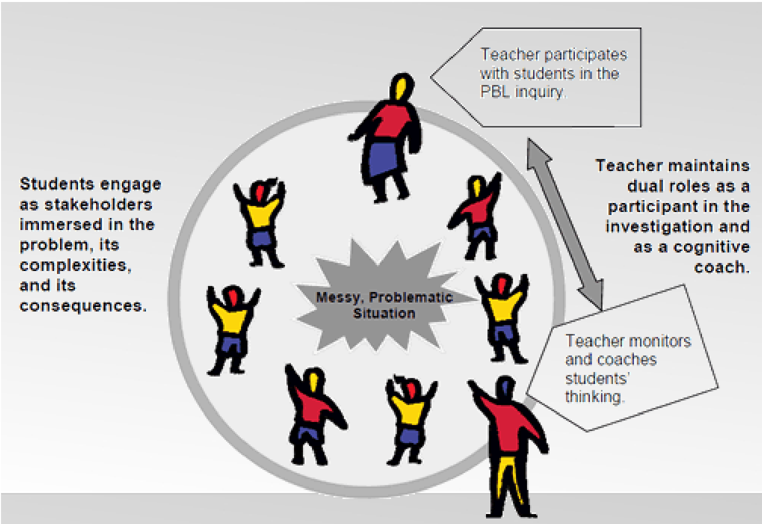


Figure 3 An Illustration of PBL situation (IMSA PBLNetwork, 2011)

The relationship between curriculum (specific content), problems, students and teacher is illustrated in the following figure (IMSA PBLNetwork, 2011).

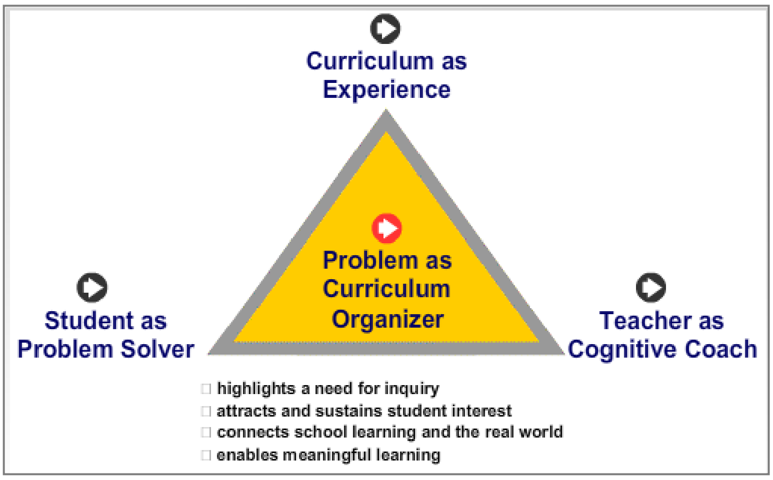


Figure 4 Curriculum, Problems, Students, and Teacher in PBL (IMSA PBLNetwork, 2011)

1) Curriculum as experience:
   - ➢ fosters active learning
   - ➢ supports knowledge construction
   - ➢ integrates content areas
   - ➢ provides relevance

2) Problem as curriculum organizer:
   - highlights a need for enquiry
   - attracts and sustain students interest
   - connects school learning and the real world
   - enable meaningful learning
3) Student as problem solver:
   - defines problems and conditions for resolution
   - establishes a context for learning
   - pursues meaning and understanding
   - becomes a self-directed learner
4) Teacher as cognitive coach:
   - models interest and enthusiasm for learning
   - coaches student thinking
   - exposes effective learning strategies
   - nurtures an environment that supports open inquiry

2. Version of PBL

In his book *Problem Based Learning for Administrators*, Edwin Bridges (1992: 6-8) suggests that there are two versions of PBL that have been implemented in the classroom: **Problem Stimulated PBL** and **Student Centered PBL**.

Problem Stimulated PBL (PS PBL)

Problem stimulated PBL uses role relevant problems in order to introduce and learn new knowledge. PS PBL emphasizes three major goals:

- development of domain-specific skills
- development of problem-solving skills
- acquisition of domain-specific knowledge

The PS PBL Process is as follows.

1) Students receive the following learning materials:
   - ✓ the problem
   - ✓ a list of objectives that the student is expected to master while working on the problem
   - ✓ a reference list of materials that pertain to the basic objectives
   - ✓ questions that focus on important concepts and applications of the knowledge base
2) Students work in teams to complete the project, resolve the problem, and accomplish the learning objectives.
   - ✓ each student has a particular role in the team - leader, facilitator, recorder, or team member
   - ✓ time allotted to each project is fixed
   - ✓ the team schedules its own activities and decides how to use the allotted time
3) Student performance is evaluated by instructors, peers, and self using questionnaires, interviews, observation, and other assessment methods.

Throughout the process, instructors serve as resources to the teams and provide guidance and direction if the team asks for it or becomes stymied in the project.

Student Centered PBL (SC PBL)

Student centered PBL is similar to PS PBL in some aspects. SC PBL has the same goals as PS PBL, but includes one more: fostering life-long learning skills.

The SC PBL Process is as follows.

1. Students receive the problem situation.
2. Students work on the problem in project teams.
3. Students are evaluated in multiple ways by instructors, peers, and self.

The process appears to be similar to that of PS PBL, but there are significant differences in each step, which are driven by the goal of fostering life-long learning skills. The major differences are in student responsbilities. In SC PBL,

- students identify the learning issues they wish to explore;
- students determine the content to be mastered;
- students determine and locate the resources to be used.

In short, students have self-defined learning issues.

As is the case with PS PBL, students decide how to appropriately use the newly acquired information and knowledge in order to solve the problem at hand.

3. PB Template

The IMSA (**Illinois Mathematics and Science Academy**) PBL Network provides the following template for implementation of PBL in classroom.

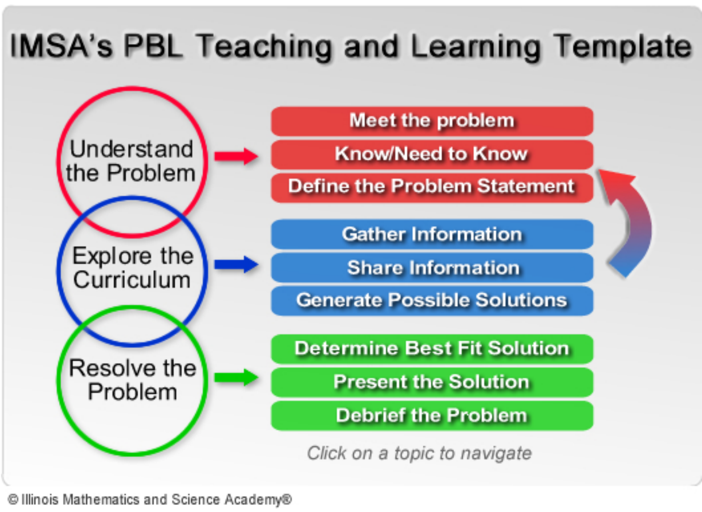


Figure 5 PBL Template by IMSA PBl Network

1) Understand the Problem
   a. Meet the problem: students encounter a messy problem that engages their interest and compels them to need to know more

b. Identify know and need to know: the group generates lists: (1) what they know, (2) what they need to know, (3) what they need to do
c. Define the problem statement: students list the task to be completed and the factors for successful completion.

2) Explore the Curriculum
   a. Gather information: learners plan how to gather information from multiple and varied resources: textbooks, newspapers, magazines, journals, Internet, surrounding, experiments, etc.
   b. Share information: students share information they have gathered with their group and discuss its relevance to the problem
   c. Generate possible solution: students synthesize information to generate several possible solutions.
3) Resolve the Problem
   a. Determine best fit solution: Students develop a graphic organizer to find a solution which fits the factors in their problem statement
   b. Present the solution: Students present their solution to and get feedback from a real world stakeholder in the problem
   c. Debrief the problem: Learners debrief the presentation to emphasize learning from other groups' presentation
   d. Debrief the problem: Learners debrief the problem and the process to emphasize the curriculum and group skills learned.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages. Students' successful experiences in managing their own knowledge also helps them solve mathematical problems well (Shoenfeld, 1985).

Mathematical ideas can be developed around problems posed by teachers and students. Real-life problem situations, such as farming (predicting daily/monthly expenditure and benefit), daily foods consumption (types of foods/drink/fruits, volumes, required nutrition, prices, etc.), shopping (types of goods, volumes, prices, discounts, etc.), building a house (area, volumes, types of geometric forms, budgets, types of materials, etc.), can be emerged into the primary classroom environment.

It is important to select appropriate problems to be posed in the problem-based instruction. The problems used in the problem-based instruction should be interesting and challenging that:

- stimulate students' interest and enthusiasm for problem solving in mathematics,
- broaden their mathematical intuition and develop their insight,
- introduce them to important mathematical ideas, and consequently
- provide opportunities to experience the fun, satisfaction, pleasure, and thrill of discovery associated with creative problem solving (Lenchner, 2005: 7).

There are many sources available for teachers to select appropriate problems that can be used as initiative in the problem-based learning. Among the sources are as follows:

i. The mathematics textbook that may contain a fairly sound collection of routine word problems related to the topic that will taught. These problems may be used as they occur, but also consider how it can be adapted or extended to meet the students need and interest. Special sections of the textbook or the teacher's guide may also contain non-routine problems that can be used for problem solving and problem-based learning.
ii. Other professional publications such as magazine, newspaper, collection of contest problems, web sites, puzzle books, are rich sources of mathematical problems from which appropriate problems can be selected directly or with adoption or extension. These resources can also become sources of inspiration to invent or to create similar other problems.
iii. Students themselves can be valuable resources when the teacher encourages them to create their own problems. To add students' enthusiasm in creating their own problems, teachers may use a bulletin board to display students' problems. Students may also be invited to share the fun of solving each other's problems. It will probably be found that problem *creating* helps to sharpen students' problem solving skills.
iv. Real life situations and contexts can also be used as sources of mathematical problems. There many mathematical problems that originally come from real life problems. These problems may relate to certain mathematics topics such arithmetic, geometry, measurement, logic.
v. Teachers' creativity and imaginative itself are best sources of problems. If a teacher has these abilities, he/she can create original and/or authentic problems that appropriate to their students learning.

Before a teacher uses a problem created by him/herself or taken or adapted from other sources, he/she needs to evaluate the problem to determine if it is a good problem for his/her students and appropriate as initiative in problem-based learning. Sometimes a problem may look appropriate at first, but it is actually inappropriate after closer inspection, because it may be too easy or too difficult, to routine or too complex, too simple or too time-consuming, and so on. A good problem should have the following characteristics (Lenchner, 2005: 8).

- It is sufficiently **interesting and challenging** to make the students want to solve it. Problems that deal with students' everyday experience or that in some way spark their curiosity may be interested and motivate them to solve it. Many students may be attracted to mathematical problems in the form of puzzles or brainteasers.
- It **can be approached through a variety of strategies**. Although a heuristic may be effective to solve certain problem, students generally benefit from the opportunity to consider several approaches from the viewpoint of relative directness and efficiency. For any given problem, different approaches may be more effective for different students.
- It **can be extended to related or other problems**. Students will benefit by considering new problems or recall past problems that require a similar approach to the just solved problem. Generalizing a result of accumulated problem solving experience will strengthen the students' problem solving skills.
- It **should be at the appropriate skill level for students' abilities**. Teachers need to solve by themselves the problems to be asked to their students. This will useful to determine the required skills and mathematical concepts related to the problems. By doing this, the problems may need to be changed its conditions, the language (vocabulary and sentences) to present, the level of difficulty, so that all students will understand the problem and will feel comfortable attempting the

solution. Students need not only to develop their ability to solve problem, but also to develop their confidence in this ability.

In addition, the problems used as initiative in problem-based learning should contain the concepts or skills that students must learn through the problem and the process of solving it. Therefore, teachers should consider the problem to be used does not only satisfy the good problem as described above, but they have also to evaluate the concepts and skills relate to the problem.

Presenting problems:

1. Using the chalkboard
   a. easy seen, the problem is readily available for reference throughout the problem solving process
   b. write the problem on the chalkboard before class or when students are engage in some other activity – students sometimes feel bored to wait the problem to be written;
   c. leave a blank or blanks in the problem in the place of an important word or piece of data, or provide all the data but omitting the final question; then have the students suggest appropriate words, data or questions;
   d. using chalkboard enable the problem to be left on display for more than a day.
2. Using the overhead projector or computer projector
   a. can be used to most of ideas for chalkboard
   b. using a transparency can eliminate concern about finding an appropriate time to prepare the display; a transparency is prepared together with the lesson plan;
   c. it requires less time to write a problem during the class
   d. the problem can be stored in a file and have it immediately available for use in other time.
   e. the problem can be displayed at once or line by line.
3. Using duplicated sheets
   a. the problem can be distributed to individual students
   b. save precious class time
   c. some students may simply find it easier to refer to a problem
   d. it eliminates the possibility of student error in copying the problem from the chalkboard or overhead projector
   e. enable students to physically cross out extraneous information and highlight essential information on the problem, or even complete a chart or graph required in the problem
   f. the sheet should leave enough blank space for students to write or work directly on the sheet.
4. Oral presentation
   a. reading aloud the problem that provide needed practice in screening out extraneous information
   b. help to sharpen students' listening and note-taking skills
   c. tell to the students that the problem will be read in a number of times, first ask them to listen, the ask them to take notes, stress the fact that their notes contain important information in the problem, and finally ask them to listen again to make sure that their notes accurate and they understand the question.

Assignment 4

1. Make a comparison between traditional teacher-centered learning and problem-based learning.

2. Identify the benefits of problem-based learning for students, teachers, and other stakeholders in education.
3. Design a mathematics lesson plan for certain grade at primary school using problem-based learning model. Chose one of the versions described above.

F. Some Ideas of Problem-Based Learning

Problem-based instruction provides a context that grounds abstract mathematics. It allows students to engage in mathematical reasoning and critical thinking that leads to questions and understanding of important and foundational mathematical ideas.

Before working on a mathematical problem, students are engaged in a short activity that leads mathematically to the problem. During problem-based mathematical instruction, students solve mathematical problems, often with a classmate or group of students. The solution illustrates what students know and understand. The teacher will encourage students to explore, notice patterns, develop efficient strategies, and generalize mathematical ideas. New learning is acquired as students view, discuss, make connections, and see mathematical relationships in the solutions that are presented.

The following are some ideas that could inspire teachers in planning a problem-based lesson.

1. **Poultry farming**

   You have a plan to raise chicken for eggs (egg-laying hens) at your free backyard of 7 x 8 m^2. You have to determine:

   (i) the number of chicken can be raised and the required capital
   (ii) the design of the production houses in accordance with the yard area
   (iii) the chicken feed and drugs requirement and maintenance for every month
   (iv) the raw incomes that can be expected
   (v) how long to get breakeven point (your capital will be back)

   These tasks will give experiences to students to:
   (a) calculate the number of chicken can be raised
   (b) compare the number of chicken that can be raised and the area of land
   (c) compare the number of hens and rooster
   (d) calculate the required capital
   (e) design the chicken production house
   (f) calculate the required materials to build the chicken house
   (g) calculate the budget required to build the chicken house
   (h) make the chicken house model
   (i) calculate the volumes of foods required per day
   (j) calculate the cost of food and drugs per day
   (k) calculate the maintenance cost
   (l) calculate the number eggs will be produced
   (m) calculate the total of selling eggs
   (n) draw diagrams for eggs production from day to day, week to week, or month to month

(o) draw diagrams for incomes from day to day, week to week, or month to month

Mathematical concepts and skills learned:

(i) addition, subtraction, multiplication
(ii) geometry (length, area, perimeter, volume)
(iii) statistics (collecting data, presenting data, interpreting data)

What are the required data and information in the problems? Students and teacher can discuss this and how to get the data and information.

2. **First grade mathematics**
   After a guest speaker from a local bank visits the class, a teacher coaches students to determine the need for a bank in their classroom. With the help of parent volunteers, students learn how to identify and describe the relative values and relationships among coins and solve addition and subtraction problems using currency.
3. **Fifth grade mathematics**
   The district budget for after school sports does not include 5th grade. The students develop a presentation for the school board stating their concerns. (Student will research, collect and analyze data and conduct interviews.)

G. Closing Remarks

In order to conduct problem-solving lessons effectively, teachers must have confidence in their own problem-solving abilities. Teachers develop confidence in the same way students do by having successful problem-solving experiences. The heuristics strategies for solving mathematical problems provided in this module are just as stimulates and may be insufficient for teachers to practice the skills. There are other resources available such mathematics problem-solving books, problem solving topics on the Internet that teachers can seek to develop their problem-solving skills and experiences.

Introducing problem solving into primary classroom must be done gradually, where the role of teachers are as facilitators who must foster a supportive atmosphere, giving suitable hints and guidance so that students would dare to make guesses without fear of being ridiculed (Foong, in Lee, 2007: 78-79).

Beyond teaching the strategies to solve problems, teachers should reflect on their roles while engaging students in the process of problem solving. The following questions can be used as teachers' reflection guides (Foong, in Lee, 2007: 79).

- Do the students show interest in the problem and accept the challenge? (<u>This is a matter of problem selection, problem formulation and problem posing or presentation</u>.)
- Do teachers build a supportive classroom atmosphere and encourage a variety of approaches? (<u>This is a matter of giving motivation and appreciation to the students' efforts</u>.)
- Do teachers provide a framework within which students can reflect on the process involved and learn from experience? (<u>This is a matter of problem discussions before solving a problem in order students to have understanding and discussion after the problem is solved in order students to reflect the process and the problem</u>.)

- How competent are teachers in planning their lesson? (This is a matter of checking the selected problem whether it is appropriate to the students' abilities, analyzing the problem and its solution for the appropriate hints and guiding questions for helping students to move ahead.)

As problem solving is a part of problem-based learning, in implementing PBL environments, teachers' instructional abilities become critically important as they take on increased responsibilities in addition to the presentation of mathematical knowledge. Beyond gaining proficiency in algorithms and mastering foundational knowledge in mathematics, students in PBL environments must learn a variety of mathematical processes and skills related communication, representation, modeling, and reasoning. Preparing teachers for their roles as managers of PBL environments presents new challenges both to novices and to experienced mathematics teachers (Smith, 1998; Erickson, 1999; Lubienski, 1999Lewellen & Mikusa, 1999, as cited by Roh, 2003).

Thus, teachers need to develop their abilities, knowledge and skills related to problem-solving strategies, problems selection or creation, and problem-based learning. This module is just a beginning. There are many resources available for teacher to do so. The following references list is just also some resources that can be extended by teachers to develop their competencies in those aspects.

Good luck!

H. References

Bridges, Edwin M.(1992). *Problem-based learning for administrators*. Eugene: ERIC Clearinghouse on Educational Management, University of Oregon. ERIC/CEM Accession Number: EA 023 722

IAE-Pedia.(2009). *Math Problem-based Learning*. Web: Information Age Education. Available at URL http://iae-pedia.org/Math_Problem-based_Learning (June 22nd , 2011).

IMSA (Illinois Mathematics and Science Academy). (2011). *IMSA's PBL Network, Collaborative Inquiry in Action*. http://pbln.imsa.edu/index.html (June 22nd, 2011)

Kyeong Ha Roh. (2003). *Problem-Based Learning in Mathematics*. ERIC Digest. ED482725. http://www.ericdigests.org/2004-3/math.html (June 22nd, 2011).

Lee Peng Yee(ed). (2007). *Teaching Primary School Mathematics, A Resource Book.* Singapore: McGrawHill.

Lenchner, George. (2005). *Creative Problem Solving in School Mathematics*. 2nd Ed. revised, extended. Bellmore, NY: Mathematical Olympiads for Elementary and Middle Schools, Inc.

Managing Basic Education. (2006). *Asyik Belajar dengan PAKEM: MATEMATIKA untuk Sekolah Dasar (SD-MI) dan Sekolah Menengah Pertama (SMP-MTS). available at http://www.mbeproject.net/downloade.html (June 13rd, 2011).*

Ng Wee Leng. (2008). *Problem Solving Heuristics for Primary School Mathematics, A Comprehensive Guide*. Singapore: Pearson Prentice Hall.

NCTM. (2000). *Principles and Standards for School Mathematics*. accessible on the URL: http://standardstrial.nctm.org/document/index.htm (June 22, 2011).

Park, Jee. (2001). Problem-based Learning. http://ldt.stanford.edu/~jeepark/jeepar+portfolio/PBL/skipintro.htm (June 22nd, 2011).

Schoenfeld, Alan H. (1985). *Mathematical Problem Solving*. New York: Academic Press, Inc.

Xia, Xiaogang, Chuanhan Lü and Bingyi Wang. (2008). "Research on Mathematics Instruction Experiment Based Problem Posing". *Journal of Mathematics Education*. , Vol. 1, No. 1, December 2008. pp.153-163

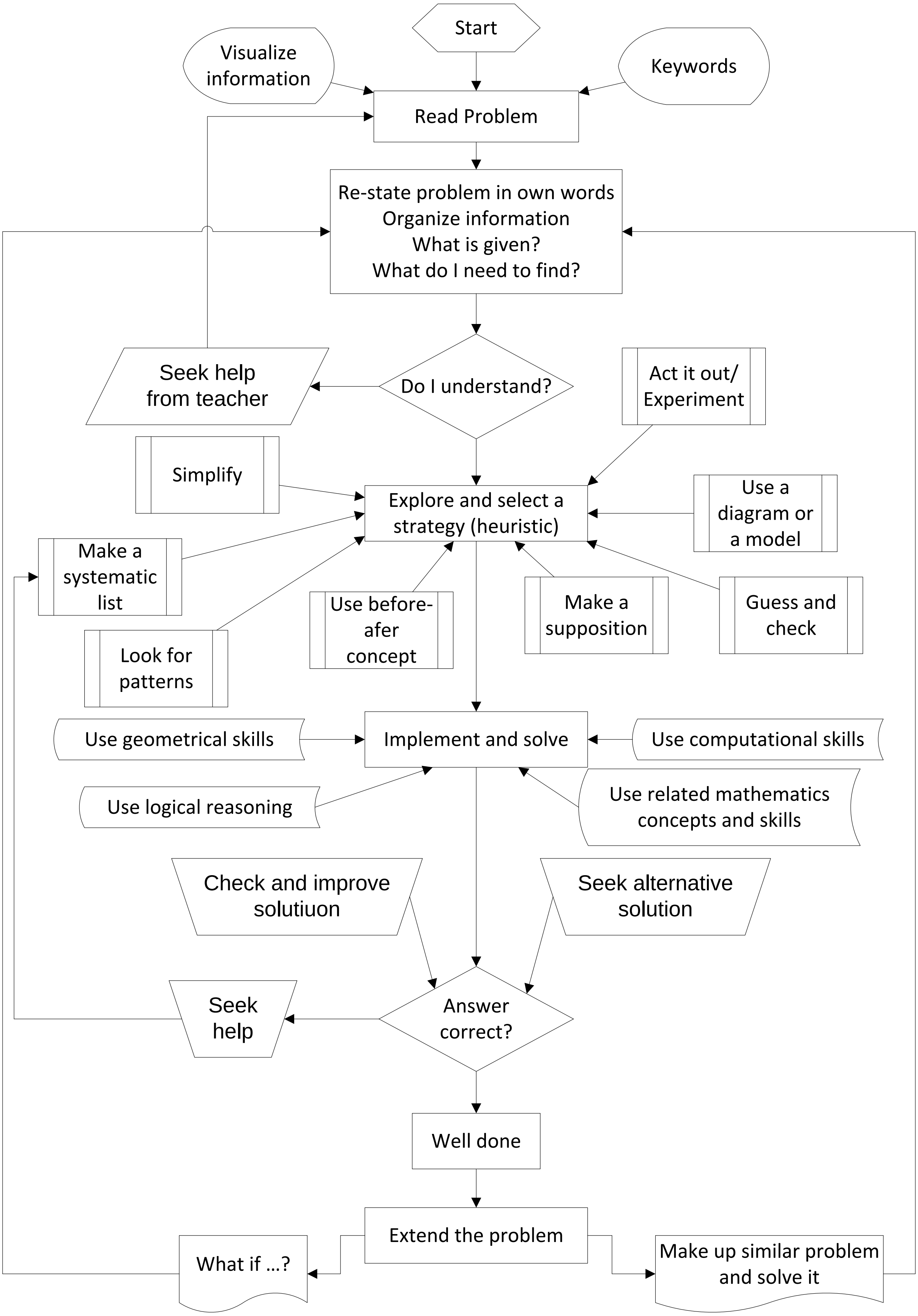
I. Appendices

1. A Guide to Problem-Solving Techniques (Schoenfeld, 1985: 24)

A Guide to Problem-Solving Techniques[a]

	UNDERSTANDING THE PROBLEM
First. You have to *understand* the problem.	*What is the unknown? What are the data? What is the condition?* Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?
	DEVISING A PLAN
Second. ind the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an nmediate connection cannot be found. You should obtain eventually a *plan* of the solution.	Have you seen it before? Or have you seen the same problem in a slightly different form? *Do you know a related problem?* Do you know a theorem that could be useful? *Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown. *Here is a problem related to yours and solved before Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? Could you restate the problem? Could you restate it still differently? Go back to definitions. If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?
	CARRYING OUT THE PLAN
Third. *Carry out* your plan.	Carrying out your plan of the solution, *check each step* Can you see clearly that the step is correct? Can you prove that it is correct?
	LOOKING BACK
Fourth. *Examine* the solution obtained.	Can you *check the result?* Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem?

2. Problem-solving Process Flowchart (Foong, in Lee, 2007: 80)



3. A Format of a Problem-solving Lesson Plan (Foong, in Lee, 2007: 81)

The Problem		
Teaching Actions – BEFORE		
1	Read/present the problem to the class or have a student read it (or distribute duplicated problem sheets to all students).	
2	Use whole class discussion. Add teachers own problem-specific comments or questions. a. …………………………………………………………………… b. …………………………………………………………………… c. ……………………………………………………………………	
3	(Optional) Use a whole class discussion about possible strategies.	
Teaching Actions – DURING		
4	Observe and ask students to identify their place in the problem-solving process. ………………………………………………………………………	
5	Provide hints as needed. Add teacher's problem-specific hints. a. …………………………………………………………………… b. …………………………………………………………………… c. ……………………………………………………………………	
6	Provide extensions as needed. Add teacher's problem-specific extensions. a. …………………………………………………………………… b. …………………………………………………………………… c. ……………………………………………………………………	
7	For students who obtain a solution, ask them to answer the problem. ………………………………………………………………………	
Teaching Actions – AFTER		
8	Discuss solutions and strategies; add teachers own problem-specific comments. a. …………………………………………………………………… b. …………………………………………………………………… c. ……………………………………………………………………	
Summary/Comments		
9	If possible relate the problem to previous problems and discuss or solve extensions.	
10	If appropriate, discuss special features of the problem. Add problem-specific comments. a. …………………………………………………………………… b. …………………………………………………………………… c. ……………………………………………………………………	