INTEGRAL
(ANTI TURUNAN)

I. INTEGRAL TAK TENTU
A. Rumus Integral Bentuk Baku

	Derifatif	*Integral*
1.	$d/dx\ X^n = nX^{n-1}$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c$
2.	d/dx cos x = - sin x	$\int \sin x\,dx = -\cos x + c$
3.	d/dx sin x = cos x	$\int \cos x\,dx = \sin x + c$
4.	$d/dx\ tg\ x = sec^2 x$	$\int sec^2 x\,dx = tg\ x + c$
5.	$d/dx\ ctg\ x = -cosec^2 x$	$\int cosec^2 x\,dx = -ctg\ x + c$
6.	$d/dx \ln x = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln x + c$
7.	$d/dx\ a^x = a^x \ln a$	$\int a^x\,dx = \frac{a^x}{\ln a} + c$
8	$d/dx\ e^x = e^x$	$\int e^x\,dx = e^x + c$
9.	$d/dx\ arc\sin x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = arc\sin x + c$ $= -arc\cos x + c$
10.	$d/dx\ arc\cos x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{1-x^2}}\,dx = arc\cos x + c$ $= -arc\sin x + c$
11.	$d/dx\ arc\ tg\ x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = arc\ tg\ x + c$ $= -arc\ ctg\ x + c$
12.	$d/dx\ arc\sec x = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}}\,dx = arc\sec x + c$ $= -arc\ cosec\ x + c$

13.	d/dx cosh x = sinh x	$\int$ sinh x dx = cosh x + c
14.	d/dx sinh x = cosh x	$\int$ cosh x dx = sinh x + c
15.	d/dx tgh x = sech^2x	$\int$ sech^2 x dx = tgh x + c
16.	d/dx ctgh x = - cosech^2x	$\int$ cosech^2 x dx = -ctgh x + c
17.	d/dx arc sinh x = $\frac{1}{\sqrt{x^2+1}}$	$\int \frac{1}{\sqrt{x^2+1}}$ dx = arc sinh x + c
18.	d/dx arc cosh x = $\frac{1}{\sqrt{x^2-1}}$	$\int \frac{1}{\sqrt{x^2-1}}$ dx = arc cosh x + c
19.	d/dx arc tgh x = $\frac{1}{1-x^2}$	$\int \frac{1}{1-x^2}$ dx = arc tgh x + c
20.	d/dx arc ctgh x = $\frac{1}{1-x^2}$	$\int \frac{1}{1-x^2}$ dx = arc ctgh x + c

Rumus selengkapnya dapat lihat di Hasyim Baisuni : 150

Contoh:

1. $\int x^5 \, dx = \frac{1}{5+1} x^{5+1} + c = \frac{1}{6} x^6 + c$

2. $\int e^{5x} \, dx = \frac{1}{5} e^{5x} + c$

3. $\int \sqrt{x} \, dx = \int x^{1/2} \, dx = \frac{1}{3/2} x^{3/2} + c$

4. $\int \frac{5}{x} \, dx = 5 \ln x + c$

5. $\int 5^x \, dx = \frac{5^x}{\ln 5} + c$ (rumus 7)

6. $\int 2 \sin x \, dx = 2 \int \sin x \, dx = -2 \cos x + c$

7. $\int \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x \right) dx = \int \frac{x^3}{3} dx - \int \frac{x^2}{2} dx - \int 6x\, dx$

$$= \frac{1}{3} \int x^3\, dx - \frac{1}{2} \int x^2\, dx - 6 \int x\, dx$$

$$= \frac{1}{3} . \frac{1}{4} x^4 - \frac{1}{2} . \frac{1}{3} x^3 - 6 . \frac{1}{2} x^2 + c$$

$$= \frac{1}{12} x^4 - \frac{1}{6} x^3 - 3x^2 + c$$

RUMUS TAMBAHAN (PENUNJANG)

1. $\int a\, du = a \int du$
2. $\int (du + dv) = \int du + \int dv$

Keterangan : a=Konstanta

B. INTEGRAL DENGAN CARA SUBSTITUSI

Maksudnya adalah mengintegrasikan fungsi-fungsi yang bentuknya seperti pada integral baku, melalui substitusi.

Sebagai ilustrasi sbb:

$$\int x^n\, dx = \frac{1}{n+1} x^{n+1} + c$$

$$\int z^n\, dz = \frac{1}{n+1} z^{n+1} + c$$

$$\int (3 + 5x)^4\, d(3 + 5x) = \tfrac{1}{5} (3 + 5x)^5 + c$$

tetapi bagaimana yang ini :

$$\int (\underbrace{3 + 6x)^7\, dx}_{\text{tidak sama}} =$$

Agar sama, maka x diganti dengan (3 + 6x), yaitu dengan cara mendeferensialkan fungsi yang ada dalam kurung.

$Y = (3 + 6x) \longrightarrow dy/dx = 6$

$$\frac{d(3+6x)}{dx} = 6$$

$$dx = 1/6 \, d(3 + 6x)$$

sehingga

$$\int (3 + 6x)^7 \, dx = \int (3 + 6x)^7 \frac{1}{6} d(3 + 6x)$$

$$= \frac{1}{6} \int (3 + 6x)^7 \, d\,(3 + 6x)$$

sudah sama

$$= \frac{1}{6} \cdot \frac{1}{8} (3 + 6x)^8 + c$$

$$= \frac{1}{48} (3 + 6x)^8 + c$$

Catatan : substitusi dipakai bila kesulitan dengan rumus baku

Contoh 2.

Carilah $\int \sin(2x - 3)\, dx$

Jawab :

$(2x - 3)$ dideferensialkan $\longrightarrow \dfrac{d(2x-3)}{dx} = 2 \longrightarrow dx = 1/2 d(2x - 3)$

Sehingga

$$\int \sin(2x - 3)\, dx = \int \sin(2x - 3) \, ½ \, d(2x - 3)$$

$$= 1/2 \int \sin(2x - 3)\, d(2x - 3)$$

$$= -1/2 \cos(2x - 3) + c$$

Contoh 3.

Hitunglah $\int \sqrt{2x+3}\, dx$

Jawab :

$$\int \sqrt{2x+3}\ dx \quad = \int (2x+3)^{1/2}\, dx$$

$$\frac{d(2x+3)}{dx} = 2 \longrightarrow dx = ½.d(2x+3)$$

$$\int (2x+3)^{1/2}\, dx = \int (2x+3)^{1/2}.\ ½.d(2x+3)$$

$$= 1/2 \int (2x+3)^{1/2}\, d(2x+3)$$

$$= \frac{1}{2} . \frac{1}{1/2+1} (2x+3)^{\frac{1}{2}+1} + c$$

$$= \frac{1}{2} . \frac{2}{3} (2x+3)^{\frac{3}{2}} + c$$

$$= \frac{1}{3} (2x+3)^{\frac{3}{2}} + c$$

Dari contoh-contoh tersebut dapat dibuat rumus integral dengan cara substitusi sbb

$$\int (ax+b)^n\, dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c$$

$$\int \cos(ax+b)\, dx = \frac{1}{a} \sin(ax+b) + c$$

$$\int \sin(ax+b)\, dx = -\frac{1}{a} \cos(ax+b)^{n+1} + c$$

Keterangan :

Rumus no.1 di atas hanyalah penjabaran dari rumus baku yang sudah kita pelajari, yaitu :

$$\int x^n\, dx = \frac{1}{n+1} x^{n+1} + c$$

Pembuktian :

Hitunglah $\int 4x^2\,dx$

1. Dikerjakan dengan rumus baku

$$\int 4x^2\,dx = 4\int x^2 dx = 4.\frac{1}{3}x^3 + c = \frac{4}{3}x^3 + c$$

2. Dikerjakan dengan rumus 1 di atas

$$\int 4x^2\,dx = \int (2x)^2\,dx = \int (2x+0)^2\,dx$$

dari rumus diketahui :

$$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c$$

$$\int (2x+0)^2\,dx = \frac{1}{2(2+1)}(2x+0)^{2+1} + c$$

$$= \frac{1}{6}(2x)^3 + c$$

$$= \frac{1}{6}.2^3.x^3 + c$$

$$= \frac{1}{6}.8.x^3 + c$$

$$= \frac{8}{6}.x^3 + c$$

$$= \frac{4}{3}.x^3 + c$$

Jadi terbukti bahwa rumus no. 1 tersebut merupakan penjabaran dari rumus bakunya.

C. INTEGRAL TRIGONOMETRI

Rumus-rumus penunjang untuk mengerjakan integral trigonometri adalah sbb:

1. $\sin^2 x + \cos^2 x = 1$
2. $1 + tg^2 x = \sec^2 x$
3. $1 + ctg^2 x = cosec^2 x$
4. $\sin^2 x = ½ (1 – \cos 2x)$
5. $\cos^2 x = ½ (1 + \cos 2x)$
6. $\sin x. \cos x = ½ \sin 2x$
7. $\sin x. \cos y = ½ \left[\sin(x+y)+\sin(x-y)\right]$
8. $\sin x. \sin y = ½ \left[\cos(x+y)-\cos(x+y)\right]$
9. $\cos x. \cos y = ½ \left[\cos(x-y)+\cos(x+y)\right]$
10. $1 – \cos x = 2 \sin^2 \frac{1}{2} x$
11. $1 + \cos x = 2 \cos^2 \frac{1}{2} x$

contoh 1.

$\int \sin^2 x \, dx = \int 1/2 \, (1 - \cos 2x) \, dx \rightarrow$ rumus no. 4

$= \int (1/2 - 1/2 \cos 2x) \, dx$

$= \int 1/2 \, dx - \int 1/2 \cos 2x \, dx$

$= \int 1/2 \, dx - \int 1/2 \cos 2x \; 1/2 \; d (2x)$

$= 1/2 \, x – ¼ \sin 2x + c$

ingat $\frac{d(2x)}{dx}$ = 2, sehingga dx = ½ d (2x)

contoh 2.

$$\int \cos^2 3x\, dx = \int 1/2\ (1 + \cos 6x)\, dx \quad \rightarrow \quad \text{rumus no. 5}$$

$$= \int (½ + ½ \cos 6x)\, dx$$

$$= \int 1/2\, dx + \int 1/2\ \cos 6x\, dx$$

$$= \int 1/2\, dx + \int 1/2 \cos 6x\ 1/6\ d(6x)$$

$$= ½ \int dx + 1/12 \int \cos 6x\ d(6x)$$

$$= ½\, x + 1/12 \sin 6x + c$$

ingat $\frac{d(6x)}{dx}$ **= 6 →** **dx = 1/6 d (6x)**

D. Integral dengan bentuk f1 (x) / f (x) dan f1 (x). f (x)

Contoh ∫ f^1 (x) / f (x):

1. Tentukan harga dari $\int \frac{(2x+3)}{(x^2+3x-5)}\, dx$

Jawab : misal z = (x^2 + 3x – 5)

$$\frac{dz}{dx} = 2x + 3$$

sehingga dz = (2x + 3). dx

$$\int \frac{(2x+3)}{(x^2+3x-5)}\, dx = \int \frac{dz}{z}$$

dapat ditulis $= \int \frac{1}{z} .\, dz$

Sehingga

$$\int \frac{1}{z} .\, dz = \ln z + c$$

$$= \ln (x^2 + 3x - 5) + c$$

2. Tentukan $\int \frac{3x^2}{(x^3-4)}$ dx

Jawab : sesuai dengan rumus diatas, maka

$$\int \frac{3x^2}{(x^3-4)} = \ln (x^3 - 4) + c$$

3. Hitunglah $\int \frac{2x^2}{(x^3-4)}$ dx

Jawab: $\int \frac{2x^2}{(x^3-4)}\,dx = \frac{2}{3}\int \frac{3x^2}{x^3-4}\,dx \rightarrow$ dikalikan $\frac{3}{3}$

$$= \frac{2}{3}\ln (x^3 - 4) + c$$

Contoh $\int f^1 (x). f (x)$

1. Tentukan harga $\int$ tg x. $\sec^2$ x dx

Jawab :

misal z = tg x

Maka $\frac{dz}{dx} = \sec^2 x$

Sehingga dz = $\sec^2$ x. dx

jadi $\int$ tg x. $\sec^2$ x dx = $\int$ z. dz

$$\int z.\,dz = ½\,z^2 + c$$

$$= ½\,(\text{tg } x)^2 + c$$

2. Tentukan harga $\int (x^2 + 7x - 4)(2x + 7)\,dx$

Jawab : misal z $= (x^2 + 7x - 4)$

Maka $\frac{dz}{dx} = (2x + 7)$

Sehingga dz $= (2x + 7).\,dx$

Jadi $\int (x^2 + 7x - 4)(2x + 7)\,dx$

$= \int z.\,dz$

$= \frac{1}{2} z^2 + c$

$= \frac{1}{2}(x^2 + 7x - 4)^2 + c$