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Philosophical Grounds for Mathematics Research

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ABSTRACT

Some mathematicians have, for a long time, repeatedly been engaged in debates over paradoxes and difficulties they have seen emerging from the midst of their strongest and most intuitive mathematical research. From the rise of non-Euclidean geometry, to present-day problems in the analytic theory of the continuum, and from Cantor's discovery of a transfinite hierarchy to the fall of Frege's system, mathematicians have also voiced their concern at how we research our everyday intuitions in unfamiliar domains, and wildly research our mathematics where intuition either has given out, or becomes prone to new and hitherto unforeseen pitfalls, or outright contradiction. At the heart of mathematical research lies the task of isolating precisely what it is that our intuition provides us with, and deciding when we should be particularly circumspect about applying it; nevertheless, those who research an epistemologically satisfying account of the role of intuition in mathematics are often faced with an unappealing choice, between the smoky metaphysical research of Brouwer, and the mystical affidavit of Gödel and the Platonists that we can intuitively research the realm of mathematical truth.

Key Words: philosophical grounds, mathematics research

A. BACKGROUND

It was indicated that, in term of the research of its foundations, mathematics is perceived as logical science, cleanly structured, and well-founded or in short mathematics is a highly structured logical science; however if we dig deep enough and in depth research, we still find some sand that makes the discursion involves mathematical philosophy. It is the fact that, in term of the research of its foundations, an assortment of historical came, starting in ancient Greece, running through the turbulent present into an exiting future; while in term of logical foundation systems, the methods of mathematics are deductive, and logic therefore has a fundamental role in the development of mathematics. Suitable logical frameworks in which mathematics can be researched can therefore be called logical foundation systems for mathematics.

Some problems still arises: in term of meaning, we are wondered about the use of special languages for researching mathematics, whether they strange things or out of this world and what does it all mean?; and then, in the sense of ontology, we may wonder whether mathematicians talk about strange thing, whether they really exist, and how they can we tell or does it matter? In epistemological research, mathematics has often been presented as a paradigm of precision and certainty, but some writers have suggested that this is an illusion. How can we research the truth of mathematical propositions?; and in term of application, how can knowledge of abstract mathematics be applied in the real world?; what are the implications for mathematics of the information revolution?; and what can mathematics contribute?.

Thompson, P.,1993, insisted that the analysis combines a cognitive, psychological account of the great "intuitions" which are fundamental to research in mathematics, with an epistemic account of what role the intuitiveness of mathematical propositions should play in their justification. He examined that the extent to which our intuitive research are limited both by the nature of our sense-experience, and by our capacity for conceptualization

B. Mathematical Research among the Un-stability of Mathematics Foundations

Litlangs, 2004, confronted that Aristotle disagreed with Plato; according to Aristotle, forms were not entities remote from appearance but something which entered into objects of the world. Aristotle claimed that when we can abstract oneness or circularity, it does not mean that these abstractions represent something remote and eternal. For Aristotle, mathematics was simply reasoning about idealizations; and he looked closely at the structure of mathematics, distinguishing logic, principles used to demonstrate theorems, definitions and hypotheses. Plato also reflected on infinity, perceiving the difference between a potential infinity e.g. adding one to a number ad infinitum and a complete infinity e.g. number of points into which a line is divisible.

Bold, T., 2004, claimed that both the intuitionist and the formalist assured that research in mathematics are just inventions and do not inform us with anything about the world; both take this approach to explain the absolute certainty of mathematics and reject the use of infinity. Bold noted that intuitionist researchers admit this major similarity the formalist and note the difference as a disagreement on where mathematical exactness exist; the intuitionist says in the human intellect and the formalist says on paper. According to Arend Heyting, mathematics research is a production of the human mind; he claimed that intuitionism claims mathematical research inherit their certainty from human knowledge that is based on empirical experience.

Bold maintained that since, infinity can not be experienced, the intuitionist refuses to push application of mathematics beyond finite; while Heyting declared that faith in transcendental existence, unsupported by concepts, must be rejected as a means of mathematical proof. Similarly, Bold found that Hilbert wrote that for logical inferences to be reliable it must be possible to research these objects completely in their parts; since there is no such survey for infinity a reliable inference can only be based on a finite system. According to the formalist researchers, the whole of mathematics consists of only arbitrary rules like those of chess. Further, Bold, T., 2004, indicated that, on the other hand, the logicist researchers came close to proving that mathematics was a branch of logic.

According to Bold, the logicist researchers want to define mathematical concepts in terms of logical concepts and deduct mathematical propositions from logical axioms; as the basic elements of logic are sets and their properties, the logicists use sets to define mathematical concepts. Hilbert actually put a structure on the intuitive part of mathematics, essentially that of finitary thought and formal systems; with Gödel's work. Thompson, P.,1993, argued that the Gödelian brand of Platonism, in particular, takes its research from the actual experience of doing mathematics, and Gödel accounts for the obviousness of the elementary set-theoretical axioms by positing a faculty of mathematical intuition, analogous to sense-perception in physics, so that, presumably, the axioms 'force themselves upon us' much as the assumption of 'medium-sized physical objects' forces itself upon us as an explanation of our physical experiences.

However, Thompson stated that counterintuitive researcher has acquired an ambiguous role in our language use that is when applied to a strange but true principle; counterintuitive can now mean anything on a continuum from intuitively false to not intuitively true, depending on the strength of the conjecture we would have been predisposed to make against it, had we not seen, and been won over by, the proof; and indeed, to our surprise, we often find out, in times of paradox, how weak and defeatible our ordinary intuitions are. Thompson claimed that the very idea that our intuitions should be both decisive and failsafe, derives historically from the maelstrom of senses which the term 'intuition' has acquired in a series of primitive epistemic theories in which some of these senses have been inherited from the large role introspection played in the indubitable bedrock of Cartesian-style philosophy, and some simply from the pervasiveness of out-moded theological convictions which seek to make certain modes of justification unassailable.

On the other hand, Hilbert's formal system fits the theory of recursive functions. Brouwer was very much opposed to these ideas, especially that of formalizing systems; he even opposed the formalization of logic; Brouwer had a very radical view of mathematics and languages relationship. According to Brouwer, in language, we can communicate the output of mathematical research, thus helping others recreate the mathematical experience; however, the proof itself is a pre-linguistic, purely conscious activity which is much more flexible than language. Brouwer thought formal systems could never be adequate to cover all the flexible options available to the creative mathematician; and thought that formalism was absurd. He thought that it was crazy to think that codified logic could capture the rules for correct mathematical thought. Brouwer showed particular rules of logic are inadequate with the most famous of the law of the excluded middle.

Brouwer beliefs that the research to applicability of traditional logic to mathematics was caused historically; he next stated that by the fact that, firstly, classical logic was abstracted from the mathematics of subsets of a definite finite set, that, secondly, an a priori existence independent of mathematics was ascribed to this logic, and finally, on the basis of this suppositious apriority, it was unjustifiably applied to the mathematics of infinite sets. Brouwer's hypothesis about the reason why philosophers and mathematicians included the law of the excluded middle; according him, logic was codified when the scientific community was concerned only with finite objects. Brouwer said that, considering only finite objects, the law of the excluded middle holds; however, a mistake was made when mathematics moved into the infinitary in which the rigid rules of logic were maintained without question.

became very important in the twentieth century that is a method of forming and arranging characters and signs to represent the relationships between mathematical thoughts.

Litlangs, 2004, further stipulated that Immanuel Kant perceived mathematical entities as a-priori synthetic propositions, which of course provide the necessary conditions for objective experience; time and space were matrices, the containers holding the changing material of perception. According to Kant, mathematics was the description of space and time; if restricted to thought, mathematical concepts required only self-consistency, but the construction of such concepts involves space having a certain structure, which in Kant's day was described by Euclidean geometry. Litlangs noted that for Kant, the distinction between the abstract "two" and "two pears" is about construction plus empirical matter; in his analysis of infinity, Kant accepted Aristotle's distinction between potential and complete infinity, but did not think the latter was logically impossible. Kant perceived that complete infinity was an idea of reason, internally consistent, though of course never encountered in our world of sense perceptions.

Frege and Russell and their followers research Leibniz's idea that mathematics was something logically undeniable; Frege used general laws of logic plus definitions, formulating a symbolic notation for the reasoning required. However, through the long chains of reasoning, these symbols became less intuitively obvious, the transition being mediated by definitions. Russell saw them as notational conveniences, mere steps in the argument; while Frege saw them as implying something worthy of careful thought, often presenting key mathematical concepts from new angles. Litlangs found that while in Russell's case the definitions had no objective existence, in Frege's case the matter was not so clear that is the definitions were logical objects which claim an existence equal to other mathematical entities. Russell researchs, resolving and side-stepping many logical paradoxes, to create with Whitehead the monumental system of description and notation of the Principia Mathematica.

Meanwhile, Thompson, P.,1993, exposed the critical movement of Cauchy and Weierstrass to have been a caution or reserve over the mathematical use of the infinite, except as a façon de parler in summing series or taking limits, where it really behaved as a convenient metaphor, or mode of abbreviation, for clumsier expressions only involving finite numbers. Thompson claimed that when Cantor came on the scene, the German mathematician Leopold Kronecker, who had already 'constructively' re-written the theory of algebraic number fields, objected violently to Cantor's belief that, so long as logic was respected, statements about the completed infinite were perfectly significant. According to Thompson, Cantor had further urged that we should be fully prepared to use familiar words in altogether new contexts, or with reference to situations not previously envisaged.

Kronecker, however, felt that Cantor was blindly cashing finite schemas in infinite domains, both by attributing a cardinal to any aggregate whatsoever, finite or infinite, and worse still, in his subsequent elaboration of transfinite arithmetic. Thompson insisted that although the interim strain on the intuition, at the time, was crucial to Euler's heuristic research, this particular infinite detour had been analyzed out of his subsequent proofs of the result, which appeared almost 10 years after its discovery. Thompson, P.,1993, clarified that Gödel's feeling is that our intuition can be suitably extended to a familiarity with very strongly axiomatic domains, such as extensions of ZF, or calculus on smooth space-time manifolds, thereby providing us with backgrounds for either accepting or rejecting hypotheses independently of our pre-theoretic prejudices or preconceptions about them.

C. Mathematical Research Insides the Blow of Epistemology

I. The Cartesian Doubt and Kant's Synthetic A Priori

Turan, H., 2004, elaborated that Descartes called mathematical propositions into doubt as he impugned all beliefs concerning common-sense ontology by assuming that all beliefs derive from perception seems to rest on the presupposition that the Cartesian problem of doubt concerning mathematics is an instance of the problem of doubt concerning existence of substances. Turan argued that the problem is not whether we are counting actual objects or empty images but whether we are counting what we count correctly; he argued that Descartes's works is possible to expose that the proposition '2+3=5' and the argument I think, therefore I am,' were equally evident. According to Turan, Descartes does not found his epistemological investigation upon the evidence of mathematician Leopold Kronecker, who had already 'constructively' re-written the theory of algebraic number fields, objected violently to Cantor's belief that, so long as logic was respected, statements about the completed infinite were perfectly significant. According to Thompson, Cantor had further urged that we should be fully prepared to use familiar words in altogether new contexts, or with reference to situations not previously envisaged.

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