THE NATURE AND HISTORY OF MATHEMATICS

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The nature of mathematics can be traced from ancient history of mathematics to contemporary one. The word "*mathematics*"[1] comes from the Greek "*máthema*" which means *science, knowledge,* or *learning;* and "*mathematikós*" means *"fond of learning"*. The invention of printing has largely solved the problem of obtaining secure texts and has allowed historians of mathematics to concentrate their editorial efforts on the correspondence or the unpublished works of mathematicians. The achievements of prehistoric mathematics and the flowering of Pythagorean have significant evidences to trace the nature of mathematics. While the challenge of non-Euclidean geometry to Euclidean geometry has some impacts to the development of contemporary mathematics.

A. Ancient Mathematics

For over a thousand years[2]--*from the fifth century B.C.* to the *fifth century A.D.*--Greek mathematicians maintain a splendid tradition of work in the exact

[1] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia,* Encyclopaedia Britannica, Retrieved 2004 < *http://www.google.search* >
[2] Knorr, W.R., 2004, *Mathematics in medieval Islām,* Encyclopaedia Britannica, Retrieved 2004 < *http:/ /www.google.search* >

sciences: *mathematics, astronomy,* and *related fields.* However, the exponential growth of mathematics means that historians are able to treat only the major figures in any detail. In addition there is, as the period gets nearer the present, the problem of perspective. Mathematics, like any other human activity, has its fashions, and the nearer one is to a given period, the more likely these fashions are to look like the wave from the past to the future. For this reason, the writer needs to have relevant references to assess the essence of the history of ancient mathematics.

1. Prehistoric Mathematics

Sumerian civilization[3] flourished before 3500 BC, an advanced civilization building cities and supporting the people with *irrigation systems, a legal system, administration,* and even *a postal service*. Writing developed and counting was based on a *sexagesimal system,* that is to say *base 60.* Around 2300 BC, the Akkadians[4] invented the *abacus* as a tool for counting and they developed methods of arithmetic with *addition, subtraction, multiplication* and *division.* Around 2000 BC, Sumerians had developed an abstract form of writing based on cuneiform i.e. *wedge-shaped symbols*. Their symbols were written on wet clay tablets which were baked in the hot sun and many thousands of these tablets have survived to this day.

The Babylonians[5] appear to have developed *a placeholder symbol* that functioned as a zero by the 3rd century BC, but its precise meaning and use is still

[3] Reed, I., 1998, *An overview of Babylonian mathematics,* The Math Forum, Retrieved 2004 <*http:// mathforum.org/*>
[4] Ibid.
[5] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia*, Encyclopaedia Britannica, Retrieved 2004 < *http://www.google.search* >

uncertain. They had no mark to separate numbers into integral and fractional parts as with the modern decimal point. The *three-place numeral 3 7 30* could represent:

a. $3\,7\,30 = \frac{3}{60^0} + \frac{7}{60^1} + \frac{30}{60^2} = \frac{3}{1} + \frac{7}{60} + \frac{30}{360} = 3\frac{1}{8}$

b. $3\,7\,30 = 3x60^2 + 7x60^1 + 30x60^0 = 3x3600 + 420 + 30 = 10,800 + 420 + 30 = 11,250$

c. $3\,7\,30 = 3x60^1 + 7x60^0 + 30x60^{-1} = 180 + 7 + \frac{1}{2} = 187\frac{1}{2}$

Berggren, J.L., 2004, describes that the Greeks divided the field of mathematics into *arithmetic* i.e. the study of *multitude* or *discrete quantity* and *geometry* i.e. the study of *magnitude* or *continuous quantity* and considered both to have originated in practical activities. Proclus[6], in his *Commentary on Euclid*, observes that *geometry*, literally, *"measurement of land*," first arose in surveying practices among the ancient Egyptians, for the flooding of the Nile compelled them each year to redefine the boundaries of properties. Similarly, *arithmetic* started with the *commerce* and trade of Phoenician merchants. Some hints[7] about the nature of early Greek practical mathematics are confirmed in the *arithmetic problems* in *papyrus* texts from Ptolemaic Egypt. Greek tradition[8] was much like the earlier traditions in Egypt and Mesopotamia; however, its development as *a theoretical discipline* was a distinctive contribution to mathematics.

[6] Ibid.

[7] Posy, C., 1992, "*Philosophy of Mathematics",* Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

[8] Ibid.

While the Mesopotamians[9] had procedures for finding whole numbers a, b, and c for which $a^2 + b^2 = c^2$ *(e.g., 3, 4, 5; 5, 12, 13; or 119, 120, 169)*. Greeks came a proof of a *general rule* for finding all such sets of numbers called *Pythagorean triples* (Figure. 1)

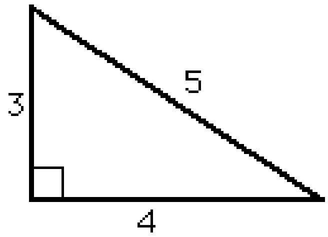

Figure 1: Pythagorean triples.

If one takes *any whole numbers p and q,* both being *even* or both *odd,* then *a = (p 2 - q 2)/2, b = pq*, and *c = (p 2 + q 2)/2*. For Mesopotamians it appears to be understood that the sets of such numbers *a, b, and c* form the *sides of right triangles*, but the Greeks proved this result by *Euclid* in *Element.* The transition from *practical* to *theoretical mathematics* initiated by Pythagoras by establishing that *all things are number* and a*ny geometric measure can be associated with some number.*

2. The Flowering of Pythagoreans

[9] Ibid.

Pythagoras of Samos[10] is often described as the *first pure mathematician*. He is an extremely important figure in the development of mathematics yet we know relatively little about his mathematical achievements. The society which he led, *half religious* and *half scientific,* followed *a code of secrecy* which certainly means that today *Pythagoras* is a mysterious figure. Pythagoras held that at its deepest level, *reality is mathematical in nature.* Pythagoreans[11] represents a coherent body of mathematical doctrines believed that *number rules the universe*. They made no distinction between mathematics and physics and concern with the study of properties of counting numbers. They believe all measurements could be expressed in terms of *natural numbers*, or *ratios of natural numbers*. They develop geometric theorems and insist that *mathematical ideas* required proofs. They think numbers had concrete representations as figures of point e.g. *square numbers, triangular numbers, etc.* Posy[12] points out three important *Pythagorean beliefs:* (1) they agree with Babylonian assumption of *commensurability* that any geometric measurement will be some rational multiple of the standard unit; (1) they think that *space* is ultimately *discrete or separable* that there is nothing between *1* and *2* and everything had to have *atomic parts*; and (3) they believe that *continuity* implied *infinite divisibility.*

O'Connor, J.J and Robertson, E.F., 1999, note that in the British museum, it was found one of four Babylonian tablets, which flourished in Mesopotamia between 1900 BC and 1600 BC, which has a connection with *Pythagoras's*

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[10] O'Connor, J.J and Robertson, E.F., 1999, Pythagoras of Samos, Retrieved 2004 <*http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Pythagoras.html*>

[11]Posy, C., 1992, "*Philosophy of Mathematics"*, Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

[12] Ibid.

theorem. The document elaborates the finding of the breath of a rectangle in which its length and diagonal hold, as the following:

> 4 is the length and 5 the diagonal. What is the breadth ?
> Its size is not known.
> 4 times 4 is 16.
> 5 times 5 is 25.
> You take 16 from 25 and there remains 9.
> What times what shall I take in order to get 9 ?
> 3 times 3 is 9.
> 3 is the breadth.

Jones R.B.(1997) exposes that for *Pythagoras* the square on the *hypotenuse* would certainly not be thought of as a number multiplied by itself, but rather as a geometrical square constructed on the side. To say that the sum of two squares is equal to a third square meant that the two squares could be cut up and reassembled to form a square identical to the third square (see Figure 2).

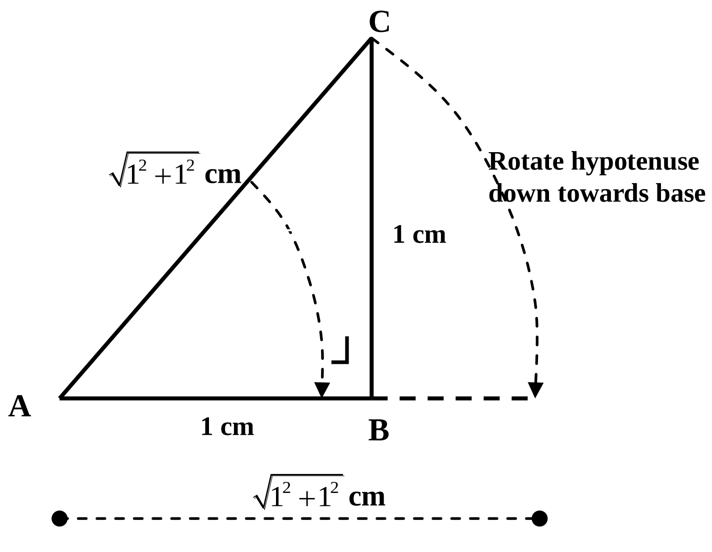

Figure 2: Commensurability of Pythagoras

Pythagoras[13] held that all things are number and any geometric measure can be associated with some number as the following examples:

a. the length of a given line is said to be so many feet plus a fractional part;
b. it breaks down for the lines that form the side and diagonal of the square;
c. if it is supposed that the ratio between the side and diagonal may be expressed as the ratio of two whole numbers, it can be shown that both of these numbers must be even and this is impossible,
d. there is no length that could serve as a unit of measure of both the side and diagonal;
e. the side and diagonal cannot each equal the same

3. Euclidean Geometry

Around 300 BC[14], Euclid was studying *geometry* in *Alexandria* and wrote a *thirteen-volume book* that compiled all the known and accepted rules of geometry called *The Elements.* Euclid [15] believes in absolute separation of discrete mathematics and magnitudes. Of the *Element*, for example, *Books 5* and *6* state the theory of proportion for magnitudes, while *Book 7* states the theory of proportion for numbers. In these *Elements*, Euclid attempted to define all *geometrical terms* and proposed *five undefined geometric term* that are the basis for defining all other geometric terms as follows: "*point", "line", " lie on", " between",* and

[13] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia,* Encyclopaedia Britannica, Retrieved 2004 < *http://www.google.search* >
[14] Posy, C., 1992, "*Philosophy of Mathematics",* Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>
[15] Ibid.

"congruent". Because mathematics is a science where every *theorem* is based on *accepted assumptions, Euclid* first had to establish some *axioms* with which to use as the basis of other *theorems*.

Euclid uses *five axioms* as the *5 assumptions,* which he needs to prove all other geometric ideas. The use and assumption of these *five axioms*[16] is what it called something to be categorized as *Euclidean geometry*. The first four of *Euclid's axioms* [17] are fairly straightforward and easy to accept, and no mathematician has ever seriously doubted them. The first four postulates[18] state about straight line that may be drawn from any two points; any terminated straight line that may be extended indefinitely; a circle that may be drawn with any given center and any given radius; and all right angles that are congruent. Explicitly, those postulates[19] are as follows:

> Postulat I
> For every point P and for every point Q not equal to P there exists a unique line l that passes through P and Q.
>
> Postulate II
> For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.
>
> Postulate III
> For every point O and every point A not equal to O there exist a circle with center O and radius OA.
>
> Postulate IV
> All right angles are congruent to each other.

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[16] -,2004, Non-EuclideanGeometry, Retrieved 2004 *<http://www.geocities.com/CapeCanaveral/7997/ index.html>*
[17] Ibid.
[18] Posy, C., 1992, "*Philosophy of Mathematics",* Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm>*
[19] Greenberg, M.J., 1973, "*Euclidean and Non-Euclidean Geometries: Development and History",* San Francisco: W.H. FREEMAN AND COMPANY

The *first postulate* is sometimes expressed informally by saying "*two points determine a unique line*". The *second postulate* is sometimes expressed informally by saying "*any segment AB can be extended by a segment BE congruent to a given segment CD".* In the *third postulate*, Euclid had in mind drawing the *circle with center A and radius r*, and this postulate tell us that such a drawing is allowed (Figure 3).

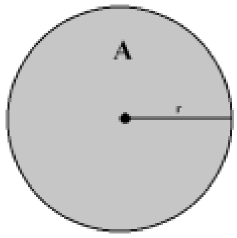

Figure 3: Euclid's circle

The fourth postulate expresses a sort of homogeneity and provides a natural standard of measurement for angles. The *fifth* or the *last postulate* listed by Euclid stands out a little bit. It is a bit less intuitive and a lot more convoluted. It looks like a condition of the geometry more than something fundamental about it. The *fifth postulate* is (see Figure 4):

> Postulate V
> If two straight lines lying in a plane are met by another line, and if the sum of he internal angles on one side is less than two right angles, then the straight lines will meet if the extended on the side on which the sum of the angles is less than two right angles.

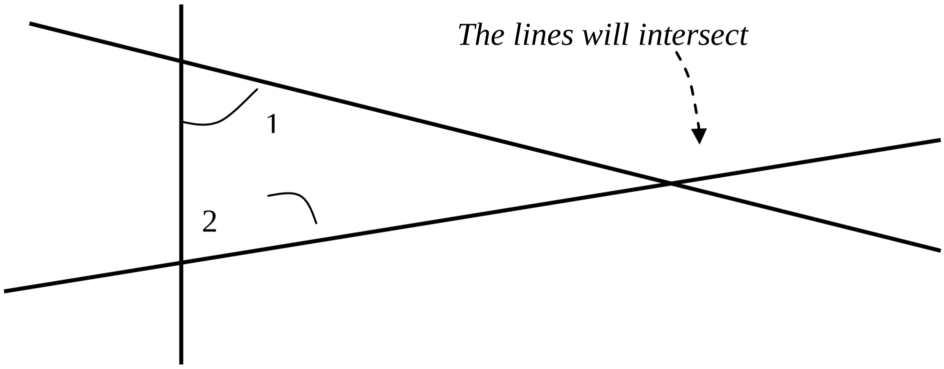

Figure 4: The Fifth Postulate of Euclides

The *Fifth Postulate of Geometry* (Parallel Postulate) means that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles and if the two straight lines are produced indefinitely, they will meet on that side on which the angles less than two right angles. From this postulate, we may have a question what is the *criterion* for line *l* to be parallel to line *m*?

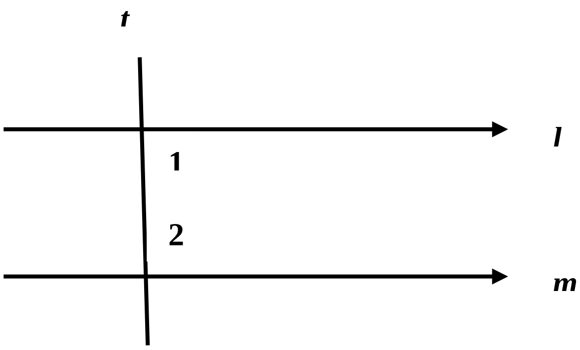

Figure 5: Criterion for parallel line

Euclid suggested drawing *a transversal* (Figure 5) , i.e., a line t that intersects both *l* and *m* in distinct points, and measuring the number of degrees in the *interior angles* 1) and 2) on one side of *t.*

4. Non-Euclidean Geometry

Pythagoras[20] was the first who inclined to regard number theory as more basic than geometry. The discovery of *in-commensurable* ratios presented them with a foundational crisis not fully resolved until the 19th century. Since *Greek*[21], *number theory*, which concerns only whole numbers, cannot adequately deal with the magnitudes found in geometry. Because of his belief that all things are numbers it would be a natural task to try to prove that the *hypotenuse* of an isosceles right angled triangle had a length corresponding to a number. For it found the irrational numbers, it can be proved that *commensurability* is false. To proof that *commensurability* is *false* we can use *reductio ad absurdum* procedure.

Posy, C., 1992, indicates that *Elements* is full of difficulties due to absolute separation resulted in a great deal of repetition and there are actual gaps and perceived gaps. *"If we continuously mark off segment AB in the direction of C, eventually we'll pass C".* This can not be proven from the other axioms. *(*See Figure 6*)*

[20] Jones, R.B.,1997, "*A Short History of Rigour in Mathematics*", Retrieved 2004 *<http://www.rbjones. com/rbjpub/rbj.htm>*
[21] Ibid.

Figure 6: Mark off segment AB in the direction of C

Euclid[22] may forget when it given a line, there must be two points on it, and there exists at least one point not on the line. He should have stated the "*self-evident*" assumptions. Modern mathematicians perceive that the *Fifth Axiom of Geometry* is too complex, and seemed derivable from the other postulates.

Wallis, J.[23] looks for simpler postulates to assume and try to proving the parallel postulate from those by simple postulate e.g. given a ABC and any line segment DE, there exists a triangle with DE as a side which is similar to ABC. In other words, *similarity* preserves shape. S*ize, shape*, and *location* are independent of each other and this simpler postulate allows derivation of *Euclid's fifth postulate.* Meanwhile, Saccheri and Lambert [24] independently tried proving the parallel postulate by the *reductio ad absurdum method*. Their proof starts with "*Neutral geometry*" that is *Euclidean geometry that excludes the parallel postulate.* In the following figure *(*Figure 7), Saccheri and *Lambert* prove that $\angle$ABC = $\angle$DCB with the following procedure. It shows that Δ ABD is congruent to Δ ADC. Hence AC = BD so Δ ABC is congruent to Δ BCD. Therefore $\angle$ABC = $\angle$DCB. There

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[22] Posy, C., 1992, "*Philosophy of Mathematics"*, Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

[23] in Posy, C., 1992, "*Philosophy of Mathematics"*, Retrieved 2004 < *http://www.cs.washington. edu/ homes/ gjb.doc/philmath.htm*>

[24] Ibid.

will three possibilities : 1) $\angle B = \angle C = 90$; 2) B, $\angle C > 90$ (obtuse angle hypothesis) ; and 3) $\angle B$, $\angle C < 90$ (acute angle hypothesis).

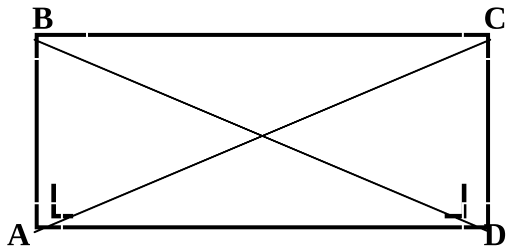


Figure 7: Neutral geometry

Possibility one is equivalent to the *parallel postulate*; possibility two can be shown to be contradictory to *Neutral geometry*; and possibility three couldn't be proven contradictory. Saccheri and Lambert[25] are repugnant to the nature of the straight line and space; and this was the discovery of *non-Euclidean geometry*.

In the late 18th and early 19th century, three men became interested in the acute angle hypothesis. Bolyai, Gauss, and Zobachevsky took the negation of the *parallel postulate* as a postulate and added it to *neutral geometry*; and resulting that nothing contradictory followed. *Negation*[26] *of the Parallel Postulate* states *that there is at least one line l and one point p outside of l such that through p there are at least two lines which do not intersect l.* This implies through any point outside of *l*, there are infinitely many lines parallel to *l*. This *new geometry* is called

[25] Posy, C., 1992, "*Philosophy of Mathematics*", Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

[26] Ibid.

hyperbolic geometry; and that some theorems that are derivable from *neutral geometry* and the negation of the *parallel postulate:*

> Angles in a triangles sum to less than 180 degrees.
> Angles in a quadrilateral sum to less than 360 degrees.
> Rectangles do not exist.
> If two triangles are similar, then they are congruent-- size and shape are not independent.

On the other hand, Riemann[27] develops *elliptical geometry* where there are no parallel lines. Klein and Belttrami independently prove that there is no hope of contradiction between neutral geometry and the negation of the *parallel postulate.* If *Euclidean geometry* is consistent, it must also be true that no contradiction can occur in *non-Euclidean geometry*. Accordingly, one could model *non-Euclidean geometry* inside *Euclidean geometry*.

B. The Road to Contemporary Mathematics

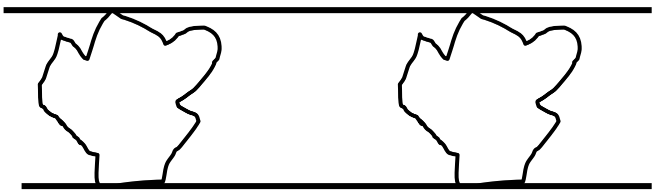
The 17th century[28], the period of the scientific revolution, witnesses the consolidation of *Copernican heliocentric astronomy* and the establishment of inertial physics in the work of Kepler, Galileo, Descartes, and Newton. This period is also one of intense activity and innovation in mathematics. Advances in *numerical calculation,* the development of *symbolic algebra* and *analytic geometry,* and the invention of the *differential* and *integral calculus* resulted in a major

[27] Ibid.
[28] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia,* Encyclo paedia Britannica, Retrieved 2004 < *http://www.google.search* >

expansion of the subject areas of mathematics. By the end of the 17th century a program of research based in *analysis* had replaced *classical Greek* geometry at the centre of advanced mathematics. In the next century this program would continue to develop in close association with *physics*, more particularly *mechanics* and *theoretical astronomy*. The extensive use of analytic methods, the incorporation of applied subjects, and the adoption of a pragmatic attitude to questions of logical rigor distinguished the new mathematics from traditional geometry.

1. The Invention of the Calculus

In his treatise *Geometria Indivisibilibus Continuorum* (1635) Cavalieri[29], formulates a systematic method for the determination of areas and volumes. Cavalieri[30] thinks *a plane strip* can be thought of as infinitely many parallel indivisibles, etc; and he then described principle that, called then *Cavalieri's principle*, one can move indivisibles composing a figure independently of each other and thus recreate the figure. Cavalieri showed that these collections could be interpreted as magnitudes obeying the *rules of Euclidean ratio theory* (See Figure 8)



[29] Ibid.

[30] Posy, C., 1992, "*Philosophy of Mathematics*", Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

Figure 8 : Area of figure

Cavalieri[31] produces two figures inside two parallel lines. *If all lines parallel to two containing lines intersecting the two figures cut chords of equal lengths, then the areas of the two figures are the same.* Cavalieri[32] admits that his methods clearly could not be rigorous. He thinks that rigor is for philosophers, and mathematics is for scientists. According to Cavalieri's, if two solids have equal altitudes and if sections made by planes parallel to the bases and of equal distance are always in a given ratio, then the solids' volumes are also in that ratio. The more effective instrument for scientific investigation to such problem that mathematics has ever produced, then is called *calculus.* As the mathematics of variability and change, *calculus* is the characteristic product of the scientific revolution.

Calculus[33] is properly the invention of two mathematicians, the German Gottfried Wilhelm Leibniz and the Englishman Isaac Newton. Both men published their researches in the 1680s, Leibniz in 1684 in the recently founded journal *Acta Eruditorum* and Newton in 1687 in his great treatise *Principia Mathematica.* The essential insight of Newton and Leibniz was to use *Cartesian algebra* to synthesize the earlier results and to develop algorithms that could be applied uniformly to a wide class of problems. The formative period of Newton's researches was from 1665 to 1670, while Leibniz worked a few years later, in the 1670s. Their

[31] In Posy, C., 1992, "*Philosophy of Mathematics*", Retrieved 2004 < *http://www.cs. washington.edu/ homes/ gjb.doc/philmath.htm*>
[32] Ibid.
[33] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia,* Encyclopaedia Britannica, Retrieved 2004 < *http://www.google.search* >

contributions differ in origin, development, and influence, and it is necessary to consider each man separately. Newton deals *calculus* with the *analysis of motion*.

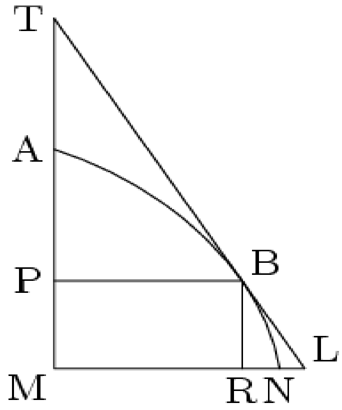


Figure 9: Locus of motion

He views curves as the locus of motion of a point and believed that notions of motion and flow must be used when analyzing continua (Figure 9). He calls his discovery the *method of fluxions* in which curve was a mapping between abscissa and ordinates. Newton called *fluents* for variables and *fluxions* for rates of change; the moment of a fluent was the delta of a variable. On the other hand, *Leibniz's* notation: $\frac{dy}{dx}$, dy and dx are both very small that they are insignificant, however, their ratio is a number; thus ratios were stressed, not the individual components.

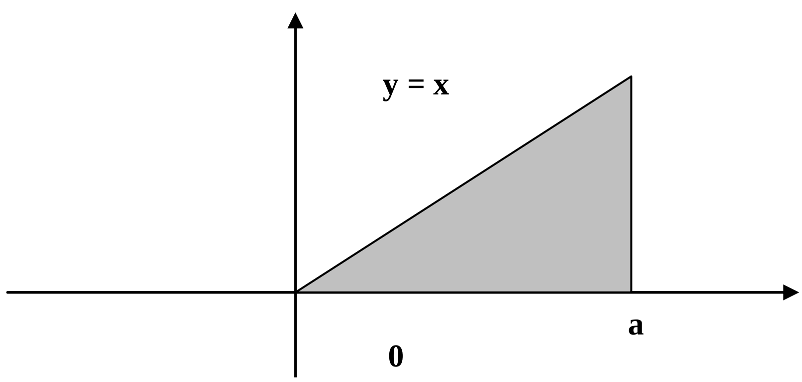


Figure 10: Area of geometrical shape

Calculation using modern calculus notation of the area of triangle in Fig. 10 resulting

$$Area\ \ of\ \ triangle = \int_0^a x\,dx = \frac{1}{2}a^2$$

In the late 18th century Bolzano and Cauchy[34], instead of talking about infinitely small quantities, think of a sequence of smaller and smaller quantities approaching a number and define the *Limit*. Cauchy defines the *Limit* as, when the successive values attributed to a variable approach indefinitely a fixed value so as to end by differing from it as little as one wishes. This last is called *the limit of all the others*. Bolzano and Cauchy take care of that; in term of the applications of converging to a limit, *Cauchy* used limits in describing the notion of a derivative. Cauchy introduces the notion $f^{'}(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$, to indicate the continuity of function.

2. Contemporary Mathematics

Contemporary[35], the major disciplines within mathematics arose out of the need to do *calculations in commerce*, *to measure land* and *to predict astronomical events.* These three needs can be roughly related to the broad subdivision of mathematics into the study of *structure, space* and *change.* The study of

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[34] Posy, C., 1992, "*Philosophy of Mathematics",* Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

[35] Ibid.

mathematical structure[36] starts with numbers, firstly the familiar natural numbers and integers and their arithmetical operations, which are recorded in elementary algebra.

The deeper properties of whole numbers[37] are studied in *number theory*. The investigation of methods to solve equations leads to the field of *abstract algebra*, which, among other things, studies *rings, fields and structures* that generalize the properties possessed by the familiar numbers. The physically important concept of *vector,* is generalized to *vector spaces* and studied in *linear algebra,* belongs to the two branches of *structure* and *space.* The study of *space*[38] originates with *geometry*, first the *Euclidean geometry* and *trigonometry* of familiar three-dimensional space, but later also generalized to *non-Euclidean geometries* which play a central role in general relativity.

The modern fields of *differential geometry* and *algebraic geometry*[39] generalize geometry in different directions. *Differential geometry* emphasizes the concepts of *coordinate system*, *smoothness* and *direction*, while in *algebraic geometry* geometrical objects are described as solution sets of *polynomial equations. Group theory* investigates the *concept of symmetry abstractly* and provides a link between the studies of *space* and *structure*. *Topology* connects the study of *space* and the study of change by focusing on the concept of *continuity*.

[36] Posy, C., 1992, *Philosophy of Mathematics,* Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>
[37] Ibid.
[38] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia,* Encyclopaedia Britannica, Retrieved 2004 < *http://www.google.search* >
[39] Ibid.

Understanding and describing change[40] in *measurable quantities* is the common theme of the natural sciences, and *calculus* was developed as a most useful tool for doing just that. The central concept used to describe a changing variable is that of *a function.* Many problems lead quite naturally to relations between a quantity and its rate of change, and the methods to solve these are studied in the field of *differential equations*. The numbers[41] used to represent *continuous quantities* are *the real numbers,* and the detailed study of their properties and the *properties of real-valued functions* is known as *real analysis.*

The invention of *analytic geometry*[42] is the most important mathematical development of the 17th century. Originating in the work of the French mathematicians Viète, Fermat, and Descartes, it had by the middle of the century established itself as a major program of mathematical research. Two tendencies in contemporary mathematics stimulate the rise of analytic geometry. The first is an increased interest in *curves,* resulting in part from the recovery and Latin translation of the classical treatises of Apollonius, Archimedes, and Pappus, and in part from the increasing importance of curves in such applied fields as *astronomy, mechanics, optics,* and *stereometry*. The second is the emergence a century earlier of an established *algebraic practice* in the work of the Italian and German algebraists and its subsequent shaping into a powerful mathematical tool.

The scientific revolution had affected to mathematics a major program of research in analysis and mechanics. The period from 1700 to 1800 is witnessed as

[40] Posy, C., 1992, *Philosophy of Mathematics*, Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>
[41] Ibid.
[42] Ibid.

the century of analysis. It covers *ordinary* and *partial differential equations, calculus of variations, infinite series,* and *differential geometry.* The applications of analysis are also varied, including the theory of the vibrating string, particle dynamics, the theory of rigid bodies, the mechanics of flexible and elastic media, and the theory of compressible and incompressible fluids. During the period 1600–1800 significant advances occur in the *theory of equations, foundations of Euclidean geometry, number theory, projective geometry,* and *probability theory.*

Most of the powerful *abstract mathematical theories* in use today originate in the 19th century. *Mathematics* grew so much during this period. This period comes together through the pioneering work of Georg Cantor on the concept of *a set.* He began to discover unexpected properties of sets. For example, he describes that the *set of all algebraic numbers* and *the set of all rational numbers* are *countable* in the sense that there is *a one-to-one correspondence* between the integers and the members of each of these sets. It means that any member of the set of rational numbers, no matter how large, there is always a unique integer it may be placed in correspondence with. But, more surprisingly, he could also show that the set of all real numbers is not countable. So, although the set of all integers and the set of all real numbers are both infinite, the set of all real numbers is a strictly larger infinity. This is incomplete contrast to the prevailing orthodoxy, which proclaims that infinite could only mean "*larger than any finite amount.*"

Frege's proposals goes in the direction of *a reduction of all mathematics to logic*. He hopes that every mathematical term could be defined precisely and manipulated according to agreed logical rules of inference. This, the *logicist* program, was dealt an unexpected blow by the English mathematician and philosopher Bertrand Russell in 1902, who pointed out unexpected complications with the *naive concept of a set.* Nothing seems to preclude the possibility that some sets are elements of themselves while others are not. In the 1920s *Hilbert* put forward his most detailed proposal for *establishing the validity of mathematics.* According to his theory of proofs, everything is to be put into *an axiomatic form,* allowing the rules of inference to be only those of *elementary logic,* and only those conclusions that could be reached from this *finite set of axioms* and rules of inference were to be admitted. He proposes that *a satisfactory system* would be one which was *consistent*, *complete,* and *decidable.* By *consistent Hilbert* meant that it should be impossible to derive both a statement and its negation; by *complete,* that every properly written statement should be such that either it or its negation was derivable from the axioms; by *decidable,* that one should have an algorithm which determines of any given statement whether it or its negation is provable.

In 1931 Kurt Gödel[43] shows that *there is no system of Hilbert's type* within which the integers could be defined and which is both *consistent* and *complete.* He[44] proves that no such decision procedure is possible for any system of logic made up of *axioms* and propositions sufficiently sophisticated to encompass the kinds of problems that mathematicians work on every day. Accordingly, *if we assume that the mathematical system is consistent, then we can show that it is incomplete.* Gödel and Alan Turing show that *decidability* was also *unattainable*.

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[43] Berggren, J.L., 2004, *The Foundation of Mathematics: Mathematics in ancient Mesopotamia,* Encyclopaedia Britannica, Retrieved 2004 < *http://www.google.search* >

[44] Posy, C., 1992, "*Philosophy of Mathematics*", Retrieved 2004 < *http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

In 1963, Paul Cohen exposes resolution of the *continuum hypothesis*, which was *Cantor's conjecture* that t*he set of all subsets of the rational numbers was of the same size as the set of all real numbers.* This turns out to be independent of the usual axioms for set theory, so there are set theories and therefore types of mathematics in which *it is true* and others in which *it is false*.

According to Hempel C.G. (2001), every concept of mathematics can be defined by means of *Peano's three primitives*, and every proposition of mathematics can be deduced from the five *postulate*s enriched by the definitions of the *non-primitive* terms. These deductions can be carried out, in most cases, by means of nothing more than the principles of *formal* logic. He perceived that the proof of some *theorems* concerning real numbers, however, requires one assumption which is not usually included among the latter; this is the so-called *axiom of choice* in which it asserts that given a class of mutually exclusive classes, none of which is empty, there exists at least one class which has exactly one element in common with each of the given classes. *Peano's* simple arithmetic including addition can be defined and many *theorems* proven by assuming :

1. a number called 0 exists
2. every number X has a successor called inc(X)
3. X+0 = X
4. inc(X) + Y = X + inc(Y)

Using these axioms, and defining the customary short names 1, 2, 3, and so on for inc(0), inc(inc(0)), inc(inc(inc(0))) respectively, we can show the following :

a. inc(X) = X + 1 and 1 + 2 = 1 + inc(1)   Expansion of abbreviation (2 = inc(1))
b. 1 + 2 = inc(1) + 1   Axiom 4
c. 1 + 2 = 2 + 1   Abbreviation (2 = inc(1))
d. 1 + 2 = 2 + inc(0)   Expansion of abbreviation (1 = inc(0))
e. 1 + 2 = inc(2) + 0   Axiom 4
f. 1 + 2 = 3   Axiom 3 and Use of abbreviation(inc(2=3)[45]

S*tructuralism*[46] provides a more holistic view of mathematics and science that can account for the interaction of these disciplines. Any structure[47] can be a mathematical structure if mathematicians, qua mathematicians, study it qua structure; the difference lies more in the way that structures are presented and studied. Accordingly, *mathematical structures* are described abstractly i.e. independent of what the structures may be structures of, and studied deductively. Bell [48] illustrates that the relationships between *mathematical structures* as embodied in the network of *morphisms* came to be seen as more significant than the objects which constitute the elements of the structures. The notion[49] of identity appropriate for structures is *not set-theoretic equality* but *isomorphism*.

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[45] *Ibid.*
[46] Shapiro in Stefanik, 1994, *"Structuralism, Category Theory and Philosophy of Mathematics*", Washington: MSG Press. Retrieved 2004 <*http://www.mmsygrp.com/mathstrc.htm*>
[47] *Ibid*.
[48] Bell's recognition in Stefanik, 1994, *"Structuralism, Category Theory and Philosophy of Mathematics*", Washington: MSG Press. Retrieved 2004 <*http://www.mmsygrp. com/mathstrc.htm*>
[49] Bell in Stefanik, 1994, *"Structuralism, Category Theory and Philosophy of Mathematics*", Washington: MSG Press. Retrieved 2004 <*http://www.mmsygrp.com/mathstrc.htm*>