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A GOAL PROGRAMMING APPROACH TO SOLVE VEHICLE ROUTING PROBLEM USING LINGO

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Abstract. Vehicle routing problem (VRP) which discusses a set of routes for some vehicles, starting and ending at a depot, serving a set of customers such that each customer must be visited once by exactly one vehicle, and having a time constrain is called vehicle routing problem with time window (VRPTW). This paper presents a goal programming approach to solve VRP, especially VRPTW. We have considered an objective function with four main goals: to maximize utilization of vehicle capacity, to minimize the total waiting time, to minimize the total cost to serve the customers and to maximize the number of served customers. The proposed model was implemented and has been solved numerically using LINGO software and the optimal solution is presented.

Key words and Phrases: goal programming, vehicle routing problem, LINGO.

1. Introduction

Vehicle Routing Problem (VRP) is a set of routes which formed to serve a set of customers using vehicles, starting and ending at a depot. In VRP, each customer must be visited once by exactly one vehicle. The route has to be designed such that the total demands of all customers must not exceed the capacity of the vehicle.

Based on Jolai and Aghdaghi [1], Sousa *et al* [2], Azi *et al* [3], if VRP has a time constrains on the periods of the day in which each customer must be visited, it is called Vehicle Routing Problem with Time Window (VRPTW). VRPTW is one of an important problem occurring in distribution systems. So, it has received many attentions not only on the development of the theory, but also on its application. For example, postal deliveries, delivery service of food business, Liquefied Petroleum Gas (LPG) deliveries. Routes which designed should be in a short duration and must be satisfy time constrain. Larsen [4], Cook and Rich [5], Cordeau *et al* [6] proposed an exact method for the VRPTW. As development of research, there are many other methods to solve VRPTW.

On the other side, because of its wide application to real-life situations,

3. Computational Result

Now we present some tests and results to a real problem of the Liquefied Petroleum Gas (LPG) agent. The LPG will be distributed to the customers. In this paper we only take the first 5 customers as an example. We denote each customer by N1, N2,…, N5. For depot and copy of depot will be denoted by N0 and N6, respectively. The travel time, travel cost and the demand of the customers are presented in Table 1, Table 2, and Table 3.

Table 1. Travel time between nodes (minutes)

	Depot	N1	N2	N3	N4	N5
Depot	0	2	3	3	5	3
N1	2	0	1	4	5	4
N2	3	1	0	5	6	5
N3	5	4	5	0	5	5
N4	5	6	7	3	0	5
N5	3	7	8	6	7	0

Table 2. Travel cost between nodes (x Rp 1000,00)

	Depot	N1	N2	N3	N4	N5
Depot	0	0.27	0.3	0.41	0.54	0.38
N1	0.25	0	0.06	0.48	0.67	0.48
N2	0.28	0.06	0	0.45	0.70	0.51
N3	0.45	0.41	0.45	0	0.54	0.41
N4	0.54	0.67	0.70	0.35	0	0.51
N5	0.67	0.77	0.80	0.64	0.57	0

Table 3. Demand and service time for each customer

Customer	N1	N2	N3	N4	N5
Demand (units)	70	420	80	120	240
Service time (minutes)	23	140	27	40	80

Complete script of the problem solved in LINGO 11.0 shown bellow

```
MODEL:
       SETS:
N/N0,N1,N2,N3,N4,N5,N6/:q,s,v,h,d2;
R/R1..R2/:B,TR,d4;
E(N,R):Y,W;
D(N,N,R):X;
A(N,N):c,t;
ENDSETS
DATA:
U = 56;
UC = 3;
UT = 6;
q = 0 7 42 8 12 24 0;
s = 0 23 140 27 40 80 0;
               !0;     !1;     !2;     !3;     !4;     !5;     !10;
c =     !0;    0.      0.27    0.30    0.41    0.54    0.38    0.
        !1;    0.25    0.      0.06    0.48    0.67    0.48    0.27
        !2;    0.28    0.06    0.      0.45    0.70    0.51    0.30
        !3;    0.45    0.41    0.45    0.      0.54    0.41    0.41
        !4;    0.54    0.67    0.70    0.35    0.      0.51    0.54
        !5;    0.67    0.77    0.80    0.64    0.57    0.      0.38
        !10;   0.      0.27    0.30    0.41    0.54    0.38    0.    ;

               !0;     !1;     !2;     !3;     !4;     !5;     !10;
t =     !0;    0       2       3       3       5       3       0
        !1;    2       0       1       4       5       4       2
        !2;    3       1       0       5       6       5       3
        !3;    5       4       5       0       5       5       5
        !4;    5       6       7       3       0       5       5
        !5;    3       7       8       6       7       0       3
        !10;   0       2       3       3       5       3       0     ;
v = 0 1 1 2 1 1 0;
h = 0 1 1 2 1 1 0;

ENDDATA
min = d1 + @sum(N(I)|I#NE#1 #AND# I#NE#7:d2) + d3 + @SUM(R(K):d4);
@FOR(R(K):@SUM(N(I):q(I)*Y(I,K))+d4(K)=U);
@FOR(R(K):(@SUM(A(I,J):t(I,J)*X(I,J,K))+@SUM(N(I):s(I)*Y(I,K)))/60=TR);
@SUM(R(K):TR)<=UT;
@SUM(R(K):TR)-d3=0;
@FOR (R(K):B=@SUM(A(I,J):c(I,J)*X(I,J,K)));
@SUM(R(K):B)<=UC;
@SUM(R(K):B)-d1=0;
@FOR(R(K):@FOR(N(I):@SUM(N(J):X(I,J,K))=Y(I,K)));
@FOR(N(I)|I#NE#1       #AND#   I#NE#8:@SUM(R(K):Y(I,K))+d2=1);
@FOR(R(K):@SUM(A(I,J)|J#EQ#7:X(I,J,K))=1);
@FOR(R(K):@SUM(A(I,J)|I#EQ#1:X(I,J,K))=1);
@FOR(R(K):@FOR(N(I):@SUM(N(J)|I#NE#1 #AND# I#NE#7:X(I,J,K))-
@SUM(N(J)|I#NE#7 #AND# I#NE#1:X(J,I,K))=0));
@FOR(R(K):@FOR(N(I):@FOR(N(J):(W(I,K)+S(I)+t(I,J))-100000*(1-
X(I,J,K))<=W(J,K))));
@FOR(N(I):@FOR(R(K): v*Y(I,K)  <= W(I,K)));
@FOR(N(I):@FOR(R(K): W(I,K)  <= h*Y(I,K)));
@FOR(D(I,J,K):@BIN(X));
@FOR(R(K):@BIN(Y));
@FOR(N(I):@BIN(d2));
END
```

Fig. 2. Complete Script Using LINGO 11.0

By inputting vehicle capacity 560 units, maximum distribution time 6 hours a week, and maximum distribution cost Rp 3.000,00 a week, we get two route which summarize in Table 4.

Table 4. Summarize output of LINGO

Route	Total Distribution Cost	Total distribution time (hour)	Total unit LPG
N0-N2-N1-N6	Rp 610,00	2.82	490
N0-N4-N3-N5-N6	Rp 1.970,00	2.77	440
Sum	Rp 2.580,00	5.59	930

From Table 4, all customers can be served by the agent. If we decrease the maximum distribution time to 5 hours a week, then only 4 customers can be served. It is also logic, if we decrease to 4 hours a week, then only 2 customers can be served. Decreasing the number of customers which can be served also happened whenever we decrease maximum distribution cost to Rp 2.000,00. It is only 4 customers can be served.

4. Conclusion

In this paper, we proposed a goal programming approach to solve Vehicle Routing Problem with Time Windows (VRPTW). Eq. (3) subject to (4) – (17) is called goal programming model of the VRPTW. The model was implemented and the results had been obtained using LINGO. From the simulation, by increasing the value of $T_{\max}$, it is followed the increasing customers can be served. As a future research, we suggest improving our model to solve larger nodes.

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