Seameo QITEP

Training on Realistic Mathematics Education

LINEAR MEASUREMENT

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I. INTRODUCTION

Considering the importance of measurement in daily life, measurement has been taught since at elementary school in many countries. However, it is a common that measurement is directly taught at formal level of young children as an isolated concept (Castle & Needham, 2007; Kamii & Clark, 1997 and van de Walle & Folk, 2005). Teaching and learning of linear measurement mostly focuses on the use of ruler as an instrumental procedure and then, rapidly, followed by conversion of unit measurements. Van de Walle and Folk (2005) found that young children have difficulty in understanding the basic concepts of linear measurement in formal level. In higher grades, most students in grade 2 until grade 4 could not give the correct measure of an object that was not aligned with the first stripe of the ruler (Kamii & Clark, 1997; Kenney & Kouba in Van de Walle, 2005; and Lehrer et al, 2003). These students merely focus on the number that matches to the edge of the measured object. This fact shows that students tend to perform a measurement as an instrumental procedure, without a complete conceptual basis. Consequently, the teaching and learning of linear measurement need to focus on both how to use a measuring instrument and understand how this instrument works.

Hans Freudenthal stresses mathematics as a human activity, instead of subject matter that has to be transmitted (Freudenthal, 1991). Freudenthal (*ibid*) proposed the need to connect mathematics to reality through problem situation because experience-based activities could contribute to the emerging of mathematical practices. In line with Freudenthal's idea, the foundation of measurement education in kindergarten and elementary school needs to be laid on doing meaningful measuring experiences, through which a connection is made between informal measurement knowledge and the use of conventional and standard measuring instrument (Buys & de Moor, 2005 and Castle & Needham, 2007). Consequently, topic of linear measurement is given through series of activities to give participants measuring experience.

II. OBJECTIVES

The main goal of this topic is to give experience to the participants about the students' learning trajectory to understand the basic concepts of linear measurement. Participants are expected to be able to hypothesize students' learning trajectory in learning linear measurement and design series of activities for learning linear measurement.

III. MATHEMATICS CONCEPT

When we will measure an object, the first thing that we need to know is what attributes of the object that will be measured. Each object has more than one attributes that can be measured; for instance a box has weight, length, volume and area (of surface) that can be measured. Measuring width or length is categorized as linear measurement. Van De Walle and Folk (2005) mentioned that linear measurement consists of three stages, namely comparing length, estimating length, and measuring length. The sequences of a linear measurement procedure are described as follows:

a. Comparing length

Comparison as the simplest measurement can be done by "filling", "covering" or "matching" the unit with the attribute of the measured objects. The simple way to express the relation of attribute between the compared objects is given by words, such as "longer" and "shorter".

There are two kinds of comparison, namely:

- Direct comparison

  This comparison is used if the compared objects can be placed next to another.
- Indirect comparison

  When the compared objects cannot be placed next to another then we need to do indirect comparison. In an indirect comparison, a new benchmark is required as a reference point that is gradually developed into a measuring unit for measurement. The need of benchmark in indirect comparison supports the emergence of a unit measurement.

The concept of conservation of length is the main core of comparison because if students already perceive the idea of conservation of length, they will be able to do comparison of length (Kamii & Clark, 1997). Comparison itself serves as the base of measurement; therefore comparing activities embodied in Indonesian traditional games were used as preliminary for teaching and learning of linear measurement for grade 2.

b. Estimating length

Estimating length of an object is more as a mental comparison because it tries to relate the length of the object with the benchmarks in mind.

Benchmarks are needed as the points of reference in estimating the length of an object. According to Joram (2003), benchmarks can enhance the meaningfulness of standard units of measure and, therefore, benchmarks can be used as an important component of instruction on measurement and measurement estimation.

c. Using models of unit

At the beginning of measurement process, people are used to use non-standard measuring units. Therefore, the use of non-standard units at the beginning of measurement activities is crucial and beneficial at all grade levels. The first benefit is that non-standard units help students to focus directly on the attribute being measured. As the second benefit, the use of non-standard units at the beginning of measurement activities provides a good rationale for work with standard units. Using models of unit emerges when a benchmark is acquired to compare the length of objects which cannot be directly compared.

A discussion of the need for a standard unit will be more meaningful to students after they have measured objects using their own non-standard units. The different non-standard measuring units used by students could be used as a conflict to stimulate and support the emergence of standard measuring unit. The need to have a "fair" game was also expected to stimulate student to "standardize" the measuring units that were used in the game. Consequently, the emergence of a standard measuring unit was expected to be acquired in the class discussion. The agreement-based standard measuring unit as the result of standardization became the starting point of the emergence of a standard measuring unit in the formal measurement.

d. Measuring length

The need of measurement is initiated in indirect comparison when the objects cannot be directly compared by placing next to another. Each object is compared to a benchmark and the relation between those two objects is derived from the relations between each object to the benchmark. In this process the benchmark becomes a "unit" for measuring.

Measuring length requires the second basic concepts of linear measurement proposed by Barret (Stephen and Clement, 2003), namely *unit iteration*. There are two kinds of unit iteration, namely:

- Arranging a number of similar units to cover the attribute of the measured objects.
- Iterating a unit from one to another end of the measured object.

Measuring length is also built up by the concept of *covering space* and any number as *zero point* of measurement. A problem that frequently occurs when young children measure the length of objects using paper strips is counting the number of stripes, instead of the number of spaces between two stripes. This fact shows that many young children do not fully perceive the idea of measuring as *covering space*. Consequently, the concept of *covering space* became the focus in measuring activity using strings of beads, making our own ruler activity and measuring using blank ruler activity in this research.

Many prior researchers found that young children also have difficulty to give the correct measure of an object that is not aligned with number zero on the ruler (Kamii & Clark, 1997; Van de Walle, 2005; and; Lehrer et al, 2003). It indicates that many young children do not seemed to know that any number can serve as *zero point* of measurement. Hence, the use of broken ruler aimed to help students in understanding the concept that any number can serve as *zero point* of measurement.

Those measurement procedures are built upon a set of basic concepts of measurement. Barret (Stephen and Clement, 2003) mentioned two basic concepts of linear measurement, namely *unitization* and *unit iteration*. *Unitization* occurs when we bring in shorter object or mentally create a shorter object and compare its attribute to the attribute of other objects. In the next stage, this shorter object becomes a unit of measurement. By establishing a unit of measurement, we anticipate the second basic concept of linear measurement that is *unit iteration*. *Unit iteration* is the process of finding how many units would match the attribute of the measured object. When a unit is not enough to cover up the attribute of the measured object, then the unit iteration is needed. In addition to the idea of Stephen and Clement (2003) about linear measurement, Lehrer et al (2003) separated important ideas of linear measurement into two conceptual accomplishments, namely the conceptions of unit and the conceptions of scale.

The basic concepts included in these two accomplishments are described in the following table.

	Basic concepts	**Description**
Conceptions of unit	• Iteration	A subdivision of a length is translated to obtain a measure
	• Identical unit	Each subdivision is identical
	• Tiling	Units fill the space
	• Partition	Units can be partitioned
	• Additivity	

		Measures are additive, so that a measure of 10 units can be thought of as a composition of 8 and 2
Conceptions of ruler	• Zero – point • Precision	Any point can serve as the origin or zero point on the scale The choice of units in relation to the object determines the relative precision of a measure. All measurement is inherently approximate.

Table 1. The basic concepts of linear measurement that are formulated by Lehrer

The combination between the procedure and basic concepts of measurement directs to a formulation of instructional activities for linear measurement. Van de Walle and Folk (2005) formulate a set of general instructional activities for linear measurement that are described as follows:

Conceptual knowledge to be developed	**Type of activity to use**
1. Understand the attribute being measured	1. Make comparisons based on the attribute
2. Understand how filling, covering, matching, or making other comparisons of an attribute with units produces what is known as a measure	2. Use physical models of measurement units (such as hand spans, foot, etc) to fill, cover, match, or make the desired comparison of the attribute with the unit. At the next stage, measuring instruments signifying physical models of unit (e.g. hand spans and foot).
3. Understand the way measuring instruments work	3. Combining the measuring instruments (ruler) and the actual unit models (such as string of beads) to compare how each works.

Table 2. The set of general activities for linear measurement generated by Van de Walle

Analyzing students' learning line or learning trajectory for a particular domain is a crucial part in designing instructional activities for students. Every stage of instructional activities should be adjusted to the level of students.

The following is a general overview of student's learning line for linear measurement in grade 2:

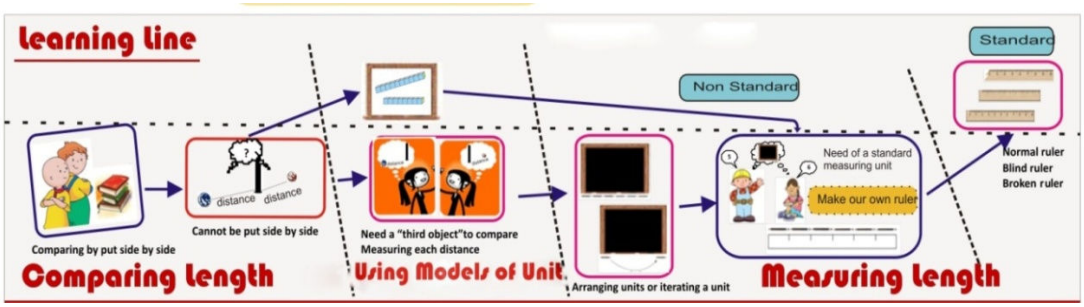

Figure 1. The learning line of students in learning linear measurement

In line with Realistic Mathematics Education, the learning process for linear measurement can be done in three levels of activities, namely:

1. Experience-based activities
   In this level, contextual problems are important as the starting point for learning linear measurement.
2. "Bridge" activities
   This level plays an important role in the learning process because in this level models are developed to "bridge" informal knowledge and concrete level into formal and abstract concept of mathematics.
3. Formal measurement activities
   This level is the goal of the learning process in which the mathematics concepts become the central point.

The following diagram shows the relationship among students' learning line, examples of activities and mathematics concepts in three different levels.

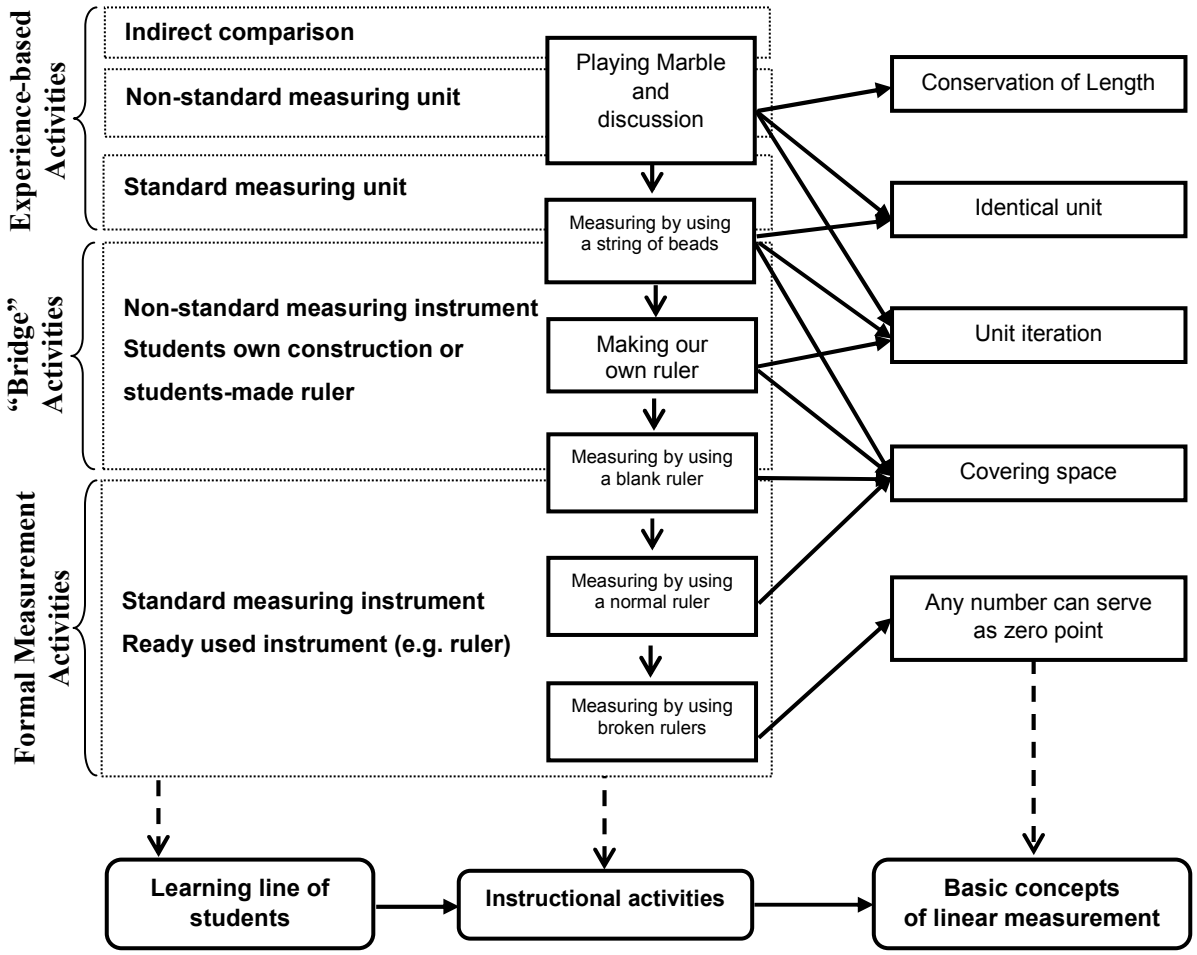

Figure 2. The main framework for learning linear measurement

IV. ACTIVITIES

According to Van De Walle and Folk (2005) , linear measurement consists of three stages, namely comparing length, estimating length, and measuring length. Consequently, the activities in this book are designed to fulfill the three stages of linear measurement.

There are five activities of linear measurement, namely:

1. Activity for comparing length
2. Activity for estimating length
3. Activity for measuring length (non standard measurement)
4. Activity of making our own ruler
5. Activity for measuring length (standard measurement)

The detail information about the activities can be found in the appendices.

V. REFERENCES

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