

Identifying ways to improve student performance on context-based mathematics tasks

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Abstract

This paper reports the Context-based Mathematics Tasks Indonesia (CoMTI) project that was aimed at getting a better insight into Indonesian students' low performance on context-based tasks and identifying ways to improve it. The project addressed three main issues: (1) Indonesian students' difficulties when solving context-based tasks; (2) possible reasons for students' difficulties; and (3) offering students opportunity-to-learn and testing its effect on student performance. These issues were investigated in four consecutive studies. The studies revealed that the students' difficulties are related to students' opportunity-to-learn.

Keywords: Context-based mathematics tasks, modelling, low achievement, Indonesian students; opportunity-to-learn.

BACKGROUND OF THE STUDY

The ability to apply mathematics is considered as a core goal of mathematics education around the world (see, e.g., Eurydice, 2011; NCTM, 2000). This goal is similar to what in the Programme for International Student Assessment (PISA) is called mathematical literacy, which refers to students' ability "to identify, and understand, the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen" (OECD, 2003, p. 24). To develop students' ability to apply mathematics, it is recommended to offer students mathematics problems situated in real-world contexts (De Lange, 2003; NCTM, 2000). In PISA study problems with real-world contexts are used to assess mathematical literacy (OECD, 2003). In this paper such problems are called context-based tasks and defined as tasks that are situated in real-world settings and provide elements or information that need to be organized and modelled mathematically.

Similar to many other countries, Indonesia also places a premium on applying mathematics as a core goal of mathematics education and pays attention to the use of context-based tasks (Pusat Kurikulum, 2003). Nevertheless, there is an apparent discrepancy between this goal and student achievement. The PISA results showed that Indonesian students perform low on context-based tasks. More than three

quarters of Indonesian students did not reach the baseline Level 2, which means they could only answer tasks that have familiar contexts and present all relevant information (OECD, 2010). The low performance of Indonesian students on context-based tasks prompted an establishment of a project called “Context-based Mathematics Tasks Indonesia” (CoMTI), which was aimed at getting a better insight into Indonesian students’ low performance on context-based tasks and identifying ways to improve student performance.

KEY IDEAS FROM LITERATURE

Solving context-based mathematics tasks

Solving mathematics problem situated in real-world contexts, which in this paper are called context-based tasks, requires interplay between the real world and mathematics, which is often described as a modeling process. According to Blum and Leiss (2007) the process of modeling is considered to be carried out in seven steps. The first step is establishing a ‘situation model’ to understand the real-world problem. Second, developing the situation model into a ‘real model’ through the process of simplifying and structuring. Third, constructing a ‘mathematical model’ by mathematizing the real model. The fourth step is carrying out mathematical procedure to get a mathematical solution. In the fifth and sixth steps, the mathematical solution is interpreted and, then, validated in terms of the real-world problem. The final step is communicating the real-world solution. This modelling process is similar to what is called ‘mathematization’ in PISA studies (OECD, 2003). Mathematization involves: understanding the problem situated in reality; organizing the real-world problem according to mathematical concepts and identifying the relevant mathematics; transforming the real-world problem into a mathematical problem which represents the situation; solving the mathematical problem; and interpreting the mathematical solution in terms of the real situation.

Opportunity-to-learn

A so called ‘opportunity-to-learn’ is often used to find an explanation for students’ mathematics performance. In the First International Mathematics Study opportunity-to-learn was defined as “whether or not [...] students have had the opportunity to study a particular topic or learn how to solve a particular type of problem” (Husén, 1967, p. 162-163). This definition was further specified by Brewer and Stasz (1996) who distinguished three aspects for measuring opportunity-to-learn. First, *curriculum content*, which refers to the scope of the topics offered to students. Second, *teaching strategies* that are used by teachers to present the topics and to engage students. Third, *learning materials*, such as textbooks, which are used to teach the students.

THE CoMTI PROJECT

Although there are different ways used in different countries to improve educational achievement, improving Indonesian students’ performance cannot be simply done by applying an educational practice that is used in other countries because, according to

Pearson (2014), what works in one particular country will not necessarily give the same result in other countries. Careful thought about what is missing in current educational practices and what might be needed by students is necessary. Therefore, in order to identify possible ways to improve Indonesian students' performance the CoMTI project focused on three interrelated issues regarding context-based tasks in Indonesia. First, *what* difficulties are experienced by students when solving context-based tasks. Second, *why* students have difficulties, for which we investigated opportunity-to-learn to solve context-based tasks offered in Indonesian textbooks and in teachers' teaching practices. Lastly, to study *how* student performance can be improved, we offered students opportunity-to-learn to solve context-based tasks and test its effects on students' performance. These three issues were investigated in four studies, which are described in the following sub-sections.

Indonesian students' difficulties in solving context-based tasks [Study 1]

The first study of the CoMTI project was aimed at getting a better insight into the low performance of Indonesian students on context-based tasks. In this study the difficulties experienced by Indonesian students when solving context-based tasks were examined through an analysis of students' errors. This approach was chosen because students' errors provide access to students' reasoning and are considered as a powerful source to diagnose learning difficulties (Borasi, 1987). With respect to analysing students' difficulties in solving mathematical word problems, Newman (1977) developed a model that is known as Newman Error Analysis. Newman proposed five categories of errors, i.e. reading (error in simple recognition of words), comprehension (error in understanding the meaning of a problem), transformation (error in transforming a word problem into an appropriate mathematical problem), process skills (error in performing mathematical procedures), and encoding (error in representing the mathematical solution into written form).

Method

A total of 362 students from 11 schools in the Province of Yogyakarta, Indonesia participated in a so called CoMTI test. The test items were selected from the released PISA mathematics tasks. After the CoMTI test, an error analysis was carried out on the basis of students' incorrect responses to investigate the difficulties experienced by students. For this purpose, an analysis framework was developed based on Newman's error categories that were associated with the stages of modeling process and PISA mathematization. The analysis framework comprised four types of errors: comprehension, transformation, mathematical processing, and encoding. Newman's reading error was not used in our framework because this error category refers to a technical aspect and does not match to modeling process or PISA's mathematization.

Results and discussion

The error analysis revealed that of 1718 errors made by the students 38% were comprehension errors, 42% were transformation errors, 17% were mathematical processing errors, and 3% were encoding errors¹. A closer examination of the

comprehension and transformation errors disclosed that a half of the comprehension errors were errors in selecting relevant information. We also found that two thirds of the transformation errors were errors in selecting mathematical procedures required to solve the tasks.

The results of the error analysis indicate that Indonesian students mostly experienced difficulties in comprehending a context-based task and in transforming it into a mathematical problem. In addition to these specific results, this study showed how analyzing students' difficulties can be a crucial preliminary step in the process of improving student performance because it sheds light on key aspects of solving context-based tasks that need to be developed. The findings of this study suggest that improving the task comprehension of Indonesian students requires a focus not only on students' language competence, but also on the ability to select relevant information. Furthermore, the ability to identify the required procedure or concept was found to be another key competence that needs to be improved.

Opportunity-to-learn to solve context-based tasks offered in Indonesian mathematics textbooks [Study 2]

The next step in the CoMTI project was identifying possible explanations for students' difficulties. Several studies have shown that student performance is often influenced by textbooks. Tornroos (2005) found a relation between student achievement on a test and the amount of textbook content related to the test items. The method used in a textbook to help students understand the content is also an important aspect influencing student performance. As found by Xin (2007), students tend to solve word problems by using the solution strategies suggested in the textbooks. Another aspect of a textbook that has an influence on student performance is the cognitive demands of the tasks. What competences students will master depends on the cognitive demands of mathematics tasks they are engage in.

Considering the important influence of textbooks on student performance, in the second study of the CoMTI project we investigated the opportunity-to-learn context-based tasks offered in Indonesian textbooks. Three issues were addressed in this study: (1) the amount of exposure to context-based tasks in Indonesian textbooks, (2) the characteristics of the context-based tasks in the textbooks, and (3) the relation between the characteristics of textbook tasks and students' errors.

Method

Three mathematics textbooks that were used in the schools participating in the first study of the CoMTI project were analyzed. For this purpose, we developed an analysis framework that focused on three task characteristics. First, the type of context for which we used three types of context: relevant and essential context, camouflage context, and no context. Second, the type of information provided in a task: matching information, missing information, and superfluous information. Third, the cognitive demands of a task: reproduction, connection, and reflection tasks.

Results and discussion

The textbook analysis revealed insufficient number of context-based tasks in Indonesian mathematics textbooks². Only 10% of tasks in the textbooks were tasks that used either camouflage or relevant and essential context. Of these tasks, three quarters used camouflage context, i.e. the context can be ignored in the solving process, and explicitly implied the required mathematical procedures. This finding indicates that Indonesian textbooks do not offer students enough opportunity-to-learn to identify mathematical procedure that is required to solve a context-based task, which might explain the high number of transformation errors made by students. An in-depth analysis of the task characteristics revealed that of 276 context-based tasks in the three textbooks 88% provided only the information that is needed to solve the tasks (matching information). This result signifies a lack of opportunities for students to learn to select relevant information, which might contribute to students' comprehension errors, in particular errors in selecting information. Lastly, regarding the cognitive demands, of all context-based tasks in the textbooks almost no reflection tasks, i.e. tasks that require complex reasoning and a construction of original mathematical approaches.

Opportunity-to-learn to solve context-based tasks offered by Indonesian teachers' teaching practices [Study 3]

Several studies (e.g. Eurydice, 2011; Grouws & Cebulla, 2000) showed that student performance is affected by the teaching strategies used by teachers. How teachers teach mathematics and engage their students influences how well students learn. With respect to the teaching of context-based tasks, Antonius, Haines, Jensen, Niss, and Burkhardt (2007) argued that it requires more than an 'explanation-example-exercise' ritual because such directive approach does not offer students opportunity to develop strategic competences that are necessary to solve context-based tasks. Instead of using direct teaching, teachers should use a teaching approach in which they take a consultative role and give students opportunities to actively build new knowledge and reflect on their learning process (Antonius et al., 2007; Blum, 2011).

The purpose of this third study was to investigate the opportunity-to-learn (OTL) to solve context-based tasks offered in teachers' teaching practices. For this purpose, we investigated the teaching approach that was used by teachers to help students learn to solve context-based tasks. Teachers' beliefs were also investigated because they often influence teachers' teaching practices (see, e.g. Wilkins, 2008). Lastly, we investigate whether there was a relationship between the OTL to solve context-based tasks offered by teachers and the errors made by students when solving such tasks.

Method

A teacher survey and a series of classroom observations were used in this study. The survey was aimed at investigating teachers' beliefs and teachers' reported practices regarding the characteristics of context-based tasks offered to students. The

classroom observations were carried out to investigate the teaching approaches used by teachers to help their students learn to solve context-based tasks.

Twenty-seven teachers from the schools involved in the first study of the CoMTI project participated in the teacher survey and four of them participated in the classroom observations.

Results and discussion

The survey data showed that the teachers tended to perceive context-based tasks merely as plain word problems. They believed that the mathematical procedure required to solve a context-based task should be given explicitly. Furthermore, the teachers also did not consider missing and superfluous information as an important characteristic of a context-based task. In agreement with these beliefs, the teachers reported that they frequently offered their students context-based tasks that have explicit procedures, but rarely gave context-based tasks that provide superfluous or missing information. Such practice might explain students' difficulties in identifying the required procedures and in selecting relevant information.

The classroom observations revealed that the teachers mainly used directive teaching approach in which they tell the students what a context-based task is about, translate the task into a mathematical problem, and explain what mathematical procedure to be carried out. In such teaching students were not encouraged to actively carry out and reflect on the stages of solving context-based tasks. This directive teaching approach was mostly used in the comprehension and transformation stages. Consultative teaching in which students were actively engaged in the process of solving context-based tasks was barely used by the teachers. Moreover, this teaching approach was mostly observed in the mathematical processing stage; a stage in which students do not have to deal with the context of a task.

Correspondences were indicated between teachers' teaching practices and students' difficulties. A lack of opportunities for students to paraphrase a context-based task might be related to students' difficulty in comprehending the task because, as pointed out by Kletzien (2009), paraphrasing helps students understand the text of a task. Moreover, teachers' direct advice regarding the procedures to be carried out might correspond to students' transformation errors because it might discourage students from thinking about the mathematics concepts addressed in the task.

Effects of opportunity-to-learn on Indonesian students' performance in context-based tasks [Study 4]

For teaching context-based tasks it is recommended to use teaching practice that emphasizes on guiding students to construct new knowledge actively and independently by using their prior knowledge and experiences (Antonius et al., 2007; Blum, 2011), which in the CoMTI project is called 'consultative teaching'. Blum (2011) found that students who learned through such teaching approach made a better progress regarding their modeling competence in comparison to students who were

taught with directive teaching. In addition to teaching practices, it is also important to give students tasks that have superfluous and missing information and do not provide explicit suggestions about the required procedures (Maass, 2007).

The second and the third studies of the CoMTI project revealed a lack of opportunity-to-learn offered in Indonesian textbooks and in teachers' teaching practices. Therefore, in the final study of the project an intervention that offers students opportunity-to-learn to solve context-based tasks was developed. The effects of opportunity-to-learn on students' performance in solving context-based tasks were examined from the perspectives of students' score gains and students' errors.

Method

A field experiment with a pretest-posttest control-group design was used in this study, which involved a total of 299 students (144 students were in the experimental group and 155 students in the control group) from six schools.

An intervention program comprising a set of context-based tasks and a consultative teaching approach was used in the experimental group. The context-based tasks used in the intervention had three important characteristics: relevant and essential context, superfluous or missing information, and not explicit suggestion about the required mathematical procedures. The consultative teaching used metacognitive prompts, which included self-addressed questions and verbal prompts or instructions to help students focus attention on particular aspects of the solving process such as asking students to paraphrase a task and to underline relevant information.

Results³ and discussion

A univariate ANOVA with the gain score (posttest score minus pretest score) as dependent variable and intervention as a fixed factor was carried out to investigate the effect of the intervention. Contrary to our expectations, the difference in gain scores between the students in the experimental group ($M_{\text{experimental}} = 0.11$, $SD_{\text{experimental}} = 0.99$) and the students in the control group ($M_{\text{control}} = -0.09$, $SD_{\text{control}} = 0.95$) was only marginally significant and the effect of the intervention was small ($p = .068$; $\eta_p^2 = .011$). Nevertheless, a closer examination of the effect of the intervention on students' errors revealed a significant difference between the experimental group and the control group for the decrease in the total number of errors ($\chi^2(1, n = 4127) = 4.149$, $p = .042$). This finding reflects a positive influence of the opportunity-to-learn on reducing students' errors. Students who received the opportunity-to-learn could better understand the instruction for a context-based task and had improved performance in selecting relevant information. With respect to transforming a real-world problem into a mathematical problem in general no influence of the opportunity-to-learn was found. However, a positive influence was found for context-based tasks addressing graphs – i.e. the topic taught during the intervention period – in which students who got the opportunity-to-learn were better able to give a mathematical interpretation of a graph. Reflecting upon this finding and referring to Howson (2010), it can be learned that to improve students' ability to identify the required procedure it is

essential to provide not only context-based tasks that are related to the topic being taught, but also context-based tasks that address other topics.

FINAL REMARKS

In general, the results of the CoMTI study suggest two important ways to identify and to improve student performance; i.e. diagnosing students' difficulties and identifying opportunity-to-learn offered to students. By connecting students' difficulties with opportunity-to-learn, we could identify what was missing in the educational process. Our results show that textbooks and teaching practices are key aspects to improve students' performance on context-based tasks.

In the appendix we provide the summary of the findings of the four studies in the CoMTI project and show how the results of the four studies are interrelated.

NOTES

1. Examples of students' errors can be found in Wijaya et al. (2014) on pages 569–573.
2. Examples of tasks in Indonesian mathematics textbooks can be found in Wijaya et al. (2015) on pages 14–17.
3. We would like to thank Michiel Veldhuis for his contribution to the statistical analysis of the data.

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Appendix. The results of the CoMTI Project

STUDENTS' DIFFICULTIES (Study 1)

Analysis of Indonesian students' errors when solving context-based tasks

The most dominant error types:

- **comprehension** errors; in particular, errors in selecting relevant information.
- **transformation** errors; in particular, errors identifying the required mathematical procedures.

POSSIBLE REASONS FOR STUDENTS' DIFFICULTIES: OTLS AS A KEY CONCEPT (Study 2 and Study 3)

Analysis of Indonesian mathematics textbooks

Exposure of the context-based tasks:

- Only about 10% of all tasks were context-based.

Characteristics of the context-based tasks:

- most of the tasks used *camouflage contexts* and provide explicit indications about the required mathematical procedures.
- most of the tasks provide *matching information*, i.e. only the information that is needed to solve the tasks.
- almost no *reflection tasks*, i.e. tasks with highest cognitive demands which require constructing original mathematical approaches and communicating complex arguments and complex reasoning.

An investigation into Indonesian mathematics teachers' teaching practices

Teachers' report about the **characteristics of context-based tasks** offered to students:

- most of the teachers frequently give tasks with *explicit procedures*
- most of the teachers frequently give tasks with *matching information*
- a half of the teachers never or rarely give tasks with *superfluous information*
- a half of the teachers never or rarely tasks with *missing information*.

Teachers' **teaching approach**:

Over all stages of solving context-based tasks:

- *No instruction* was given in 42% of all questions discussed in the lessons.
- *Directive teaching* was applied in 47% of all questions discussed in the lessons.
- *Consultative teaching* was applied in only 12% of all questions discussed in the lessons.

Specified for the stages of solving context-based tasks:

- Directive teaching was most frequently applied in the comprehension and the transformation stages.
- Consultative teaching was mostly applied in the mathematical processing stage.
- Almost no attention was paid to the encoding stage.

OTL AND STUDENT PERFORMANCE (Study 4)

<u>Offering students opportunity-to-learn (OTL)</u>		<u>Effects of the OTL on students' performance</u>
Context-based tasks:	Consultative teaching approach with metacognitive prompts:	
<ul style="list-style-type: none"> - Context-based tasks with missing or superfluous information. 	<ul style="list-style-type: none"> - Paraphrasing: asking students to formulate a task in their own words. - Underlining all information and circling only the relevant information - Self- questioning; e.g. "<i>Do we have enough information to solve the task?</i>" 	<p>A positive effect of the OTL on students' task comprehension was found:</p> <ul style="list-style-type: none"> - Students could understand better the instruction of the task - Students' ability to select relevant information improved
<ul style="list-style-type: none"> - Context-based tasks with a relevant context that requires modeling - Context-based tasks with non-explicit procedure 	<ul style="list-style-type: none"> - Self-questioning; e.g. "<i>What are possible strategies to solve the task?</i>" 	<ul style="list-style-type: none"> - In general no effect of the OTL on students' ability to transform a real-world problem into a mathematical problem. However, a positive effect was found for tasks addressing an interpretation of a graph, which in fact was related to the topic taught during the intervention. - This finding leads to a recommendation to offer students 'mixed exercises', i.e. a set of context-based tasks that address various topics.