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Emergent Modelling: From Traditional Indonesian Games to a Standard Unit of Measurement

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In this paper, we describe the way in which traditional Indonesian games can support the learning of linear measurement. Previous research has revealed that young children tend to perform measurement as an instrumental procedure. This tendency may be due to the way in which linear measurement has been taught as an isolated concept, which is separate from children's daily experiences. Consequently, a set of experience-based activities was designed to help grade 2 students to connect their learning of linear measurement to their daily experiences. Design research was chosen as a means of designing and analysing the hypothetical learning trajectory. Theoretical development is driven by a cyclical process of designing instructional activities, teaching experiments and retrospective analysis. The main design heuristic that was used is the principle of emergent modelling. The analysis of the teaching experiments showed that conflicts of fairness while playing the game could be used to help students to acquire the concept of a standard unit of measurement.

***Key words**:* Linear measurement; Traditional Indonesian games; Design research; Emergent modelling

Introduction

In many countries, measurement is taught to young children as an isolated concept at a formal level (Castle & Needham, 2007; Kamii & Clark, 1997; Van de Walle & Folk, 2005). The teaching and learning of linear measurement focuses mostly on the use of a ruler as an instrumental procedure. One possible effect of this approach is that students tend to perform measurement as an instrumental procedure, without a consistent conceptual basis. The lack of a conceptual basis also plays a role in the inability of most students in grades 2 to 4 to measure correctly the length of an object that is not aligned with the first stripe of the ruler (Kamii & Clark, 1997; Kenney & Kouba, cited in Van de Walle, 2005; Lehrer, Jaslow & Curtis, 2003).

Buys and de Moor (2005), Castle and Needham (2007) and Van Nes and Doorman (2011) have suggested that the teaching and learning of linear measurement in kindergarten and elementary school should begin with meaningful measuring experiences. Consequently, it is important to involve young children in experience-based activities that embody some basic concepts of linear measurement. In these experience-based activities, a connection is made between the children's informal knowledge of measurement and the use of standard measuring instruments. One experience-based activity that suits young children is game-playing.

In Indonesia, traditional games are played that, without a doubt, are related to measurement. Games like "*gundu*" (marbles) and "*benthik*" embody the concepts of linear measurement including comparing, estimating and measuring distances. It is conjectured that game-playing provides a natural context for experience-based activities in which the concepts and skills associated with measurement can become meaningful and support further teaching and learning. Consequently, the main objective of this research is to contribute to a local instruction theory for the teaching and learning of linear measurement for Indonesian grade 2 students. The students' situational reasoning within the game can be used as a source from which the teaching and learning process can elicit the concepts of linear measurement. In addition, teachers can foster the learning of linear measurement by building on students' reasoning by referring to situations within the game and generalising these ideas to other measuring practices and tools. This trajectory, from the situational level towards the referential, general and finally the formal level, is precisely what is being targeted in the design heuristic of emergent modelling. Emergent modelling emphasises the role of didactical models in learning trajectories that can emerge from students' reasoning and have the potential to support formal mathematical concepts and skills. The research question of "*how can the modelling that emerges in playing traditional Indonesian games be used to elicit the issues and the basic concepts of linear measurement?*" will guide this investigation of the role of traditional Indonesian games in the learning of linear measurement.

Theoretical Framework

This research focuses on the role of experience-based activities in supporting second graders' learning of linear measurement. Experience-based activities also relate to Freudenthal's idea that stresses mathematics as a human activity, instead of as subject matter that has to be transmitted (Freudenthal, 1991). Freudenthal (*ibid.*) proposed the need to connect mathematics to reality through problem situations, because experience-based activities – such as game-playing – support students in contributing to emerging mathematical practices. Therefore, the instructional sequence used in this research was designed by considering and combining the theory of linear measurement with the theory of realistic mathematics education, as proposed by Freudenthal.

Linear Measurement

Van De Walle and Folk (2005) defined measurement as the understanding that a number indicates a comparison between the attribute of the object being measured and the same attribute of a given unit of measurement. Measurement as a procedure builds on a set of basic concepts. Barret, cited in Stephen and Clement (2003), mentioned two basic concepts of linear measurement, namely *unitisation* and *unit iteration*. *Unitisation* occurs when an attribute of a shorter object is compared to the same attribute of other objects. In the next stage, this shorter object becomes a unit of measurement. By establishing a unit of measurement, *unit iteration* emerges as the second basic concept of linear measurement. *Unit iteration* is the process of finding out how many units equate to the attribute of the measured object. When a unit is not enough to cover the measured object, then unit iteration is needed.

Lehrer et al. (2003) distinguished two conceptual accomplishments as the important ideas of linear measurement, namely conceptions of unit and conceptions of scale. The basic concepts included in these two accomplishments are described in Table 1.

Table 1
The Basic Concepts of Linear Measurement, as Formulated by Lehrer

	Basic concepts	Description
Conceptions of unit	• Iteration	A subdivision of a length is translated in order to obtain a measure
	• Identical unit	Each subdivision is identical
	• Tiling	Units fill the space
	• Partition	Units can be partitioned
	• Additivity	Measures are additive, so a measure of 10 units can be thought of as a composition of 8 and 2 units
Conceptions of the ruler	• Zero point	Any point can serve as the origin or zero point on a scale
	• Precision	The choice of units in relation to the object determines the relative precision of a measure. All measurement is inherently approximate

In Indonesia, the topic of measurement is introduced in grade 1, in which students start to learn terminologies which are related to the attributes of measurement, such as long-short, heavy-light and day-night, and those which indicate comparison, such as longer-shorter and heavier-lighter. In grade 2, students learn to measure using non-standard measuring instruments (such as body parts, sticks etc.) and standard units of measurement (rulers and weighing scales or balances). However, the focus is mainly on terminology and the procedural use of standard measuring instruments, without taking into account what Indonesian students can bring to the classroom and how their daily experiences relate to the concepts and skills of measurement.

Realistic Mathematics Education and Emergent Modelling

As mentioned in the introduction, this research is an attempt to connect the teaching and learning of the basic concepts of linear measurement to experience-based activities in game-playing. The process of designing the sequence of instructional activities that starts with experience-based activities was inspired by the theory of realistic mathematics education (RME; Treffers, 1987). The five tenets of RME are (Treffers, 1987; Van den Heuvel-Panhuizen, 2000):

Phenomenological exploration. This tenet focuses on the use of meaningful contexts as the basis for concept formation;

Using models and symbols for progressive mathematisation. The development of more formal mathematical concepts from informal, context-related notions is a gradual process. This process requires models and symbols which are meaningful for the students and have the potential for generalisation and abstraction;

Using students' own constructions and productions. Students' own constructions and productions are meaningful for them, and so should be used as an essential part of instruction;

Interactivity. Interaction is an important phase of learning in which students can reflect on their activities so that they can learn the developed models and symbols. The final goal of this reflection is to arrive at the intended formal mathematical concept;

Intertwinement. Mathematics is not split into distinctive strands of learning, and furthermore, solving contextual problems often requires the application of a broad range of mathematical tools and understandings.

In summary, RME stresses the importance of using meaningful contexts and modelling activities to build upon students' strategies. These activities and models foster a process of progressive mathematisation of students' strategies towards formal mathematical notions. This underpins the possibility of using children's game-playing strategies as a bridge between situational reasoning and formal reasoning about linear measurement, including the concepts of a standard unit of measurement, unit iteration, zero point and precision. Consequently, RME provides clues with which to design a sequence of instructional activities to create such a process of teaching and learning. Emergent modelling is a key design heuristic for RME that describes how *models of* a certain situation can become *models for* more formal reasoning. There are four levels of emergent modelling which need to be considered in designing a sequence of instructional activities, namely:

1. Situational level: The situational level is the basic level of emergent modelling at which domain-specific, situational knowledge and strategies are used within the context of the situation;

2. Referential level: The occupation of models and strategies in this level refers to the situation described in the problem. In other words, the referential level is the level of *models of*;
3. General level: At the general level, *models for* emerge, in which the mathematical focus on strategies dominates the reference to the contextual problem;
4. Formal level: At the formal level, reasoning with conventional symbolisations is no longer dependent on the support of *models for* mathematical activity.

The design heuristic of emergent modelling has proven useful in various topics in mathematics education (Doorman & Gravemeijer, 2009; Rasmussen & Blumefeld, 2007; Gravemeijer, 2004). The central issue in this research is to use the heuristics of emergent modelling in designing the instructional trajectory and investigating how emergent modelling fosters the learning process of students in eliciting the basic concepts of linear measurement.

Method

Participants

The participants were 46 second-grade students of a public elementary school in Yogyakarta, Indonesia. Twelve students participated in a pilot experiment which investigated students' pre-existing knowledge and resulted in some adjustments being made to the initial hypothetical learning trajectory (HLT). The other 34 students participated in the teaching experiment. The group of 12 students and the group of 34 students were both from the same school, but from different classes.

Research Design

We chose to use design research to investigate how traditional Indonesian games could be used to build upon students' reasoning and to teach the mathematical concept of linear measurement. According to Wang and Hannafin (cited in Simonson, 2006), design research is a systematic but flexible methodology aimed at improving educational practices through iterative analysis, (re)design and implementation, based on collaboration between researchers and practitioners in daily life, and leading to context-sensitive design principles and theories. Hence, in this research, a set of experience-based activities was designed as a flexible approach to understanding and improving educational practices in linear measurement for grade 2 students in elementary school.

There were four phases of this research, namely: (1) preliminary design; (2) pilot experiment; (3) teaching experiment and (4) retrospective analysis. In the preliminary design, which also included a literature review, the students' learning line was conjectured to formulate an initial HLT. The HLT consists of learning goals for students, planned instructional activities and a hypothetical learning process. The initial HLT was tried out in the pilot experiment in order to investigate students' pre-existing knowledge and to make any necessary adjustments to the initial HLT. The new HLT was implemented in the teaching experiments.

The teaching experiments were video-recorded in order to observe the strategies and tools used by students when performing measurement during their game-playing. Short discussions with students during the game and the class discussion afterwards were also recorded in order to investigate the students' reasoning. Written data, including students' work during the teaching experiments and observation notes, were collected to obtain more information about students' achievements in solving measurement problems.

Doorman (2005) mentioned that the result of design research is not a working design, but the underlying principles explaining how and why this design works. Hence, in the retrospective analysis, the HLT was compared with students' actual learning to investigate and explain how students' acquisition of the basic concepts of linear measurement was elicited by traditional Indonesian games. The videos of game-playing were annotated and described in order to analyse and discuss *how* students perform measurement during game-playing. Meanwhile, the reasoning of *why* students use a particular strategy in games was investigated from the students' arguments in the class discussion. Student-made measuring instruments (their own constructions) were also used as additional data to explain students' progress from experience-based knowledge to formal measurement using a ruler. Tests of the qualitative reliability for preserving the consistency of the data analysis were conducted in two ways, namely data triangulation and cross-interpretation. Data triangulation engages different data sources, namely video-taping, students' written work and notes from either the teacher or the observer. The data used in this research were also cross-interpreted with experts in order to reduce the subjectivity of the researcher's point of view. The validity of this research was ensured by using the HLT as a guideline and a point of reference to test conjectures about the students' learning process and to answer research questions. Extensive data were used to describe the situation and the findings in detail in order to give sufficient information for our reasoning.

The Instructional Design

Analysing possible learning lines for students in a particular domain was a crucial part of the process of designing instructional activities for students, because every stage of the instructional activities needed to be adjusted to the level of the students. Consequently, the HLT for linear measurement was analysed before a sequence of instructional activities was designed for learning measurement. According to Van de Walle and Folk (2005), there are three stages of learning measurement, namely: (1) comparison based on a specific attribute (i.e. length, weight etc); (2) using a physical unit to pace out, cover or match the measured attribute; and (3) using a measuring instrument. Hence, the analysis of the HLT resulted in a set of instructional activities which contained three main activities, namely experience-based activities (i.e. game-playing), bridging activities (i.e. "measuring using strings of beads" and "making our own ruler") and formal measurement activities (i.e. measuring using rulers).

The use of game-playing in the experience-based activities was in line with the first tenet of RME, namely *phenomenological exploration*. In the experience-based activities, we used two Indonesian games, *gundu* and *benthik*. These traditional Indonesian games provided a natural *situation* for linear measurement in which students compared and measured length in order to determine the winner of the games. The focus in the first traditional Indonesian game – *gundu* – was a comparison of lengths (the rules of the games can be found in the Appendix), because the winner of the game was the player whose marble was the nearest to a given circle. We conjectured that students would use three kinds of benchmark to compare the distances, namely: a mental benchmark (i.e. mental estimation); body parts and non-body parts such as a pencil, a stick etc. In the first game, a repeated unit did not emerge frequently, because the distances being compared were fairly short. In this game, the students focused on comparing the distances using the terms "shorter" or "longer" without using numbers to indicate the distances. This comparison of length was developed into a measuring activity in the second game, *benthik*. In the second game, the winner was the team that obtained the largest cumulative distance. This game changed the focus from the concept of direct comparison, as the first stage of measurement, to non-standard measurement, in which the students needed numbers to indicate the distance. This sequence of activities matched the aforementioned stages of learning measurement (see Van de Walle & Folk, 2005).

A class discussion was always conducted after each game-playing session to elicit issues with measurement and to support and develop students' acquisition of the basic concepts of linear measurement. The purpose of the class discussions was not merely to communicate some sensible idea or strategy, but also to encourage *all* students to share, discuss and develop their methods of reasoning. Therefore, these class discussions also aimed to develop *interactivity* as an accepted norm in the classroom.

The *experience-based measurement activities*, as the preliminaries to the instructional sequence, aimed to provide a *situational level* of emergent modelling in which the traditional Indonesian games would provide situational knowledge and strategies of linear measurement, such as iterating hand spans and pencils to cover the distances. The class discussion after the game aimed to develop the *situational level* – in which students used their own hand spans to determine the winner – to the *referential level*, at which students started to consider the need to use a "common unit of measurement" to ensure the fairness of the game. The shift from the *referential level* to the *general level* was facilitated by the *bridging activities*, whereby *models of*- the situation were developed into *models for* measurement (i.e. the use of an "*identical unit*" and unit iteration to measure distances). As the final activity, the *formal measurement activities* focused on the concept of a "*standard unit of measurement*" and using a "*standard measuring tool*" (i.e. a ruler). The "*identical unit*" at the general level differs from the "*standard unit of measurement*" at the formal level. An "*identical unit*" signifies that the unit used within a single measurement must be identical or constant in size. On the other hand, a "*standard unit of measurement*" emphasises the need to use an identical or standard unit for parallel measurements in order to give a fair result.

Although each of the three activities aimed to support a particular level of emergent modelling, some activities were also developed for more than one level of emergent modelling. In general, the relation between the students' learning line, the instructional activities and the basic concepts of linear measurement that need to be acquired is shown the scheme in Figure 1.

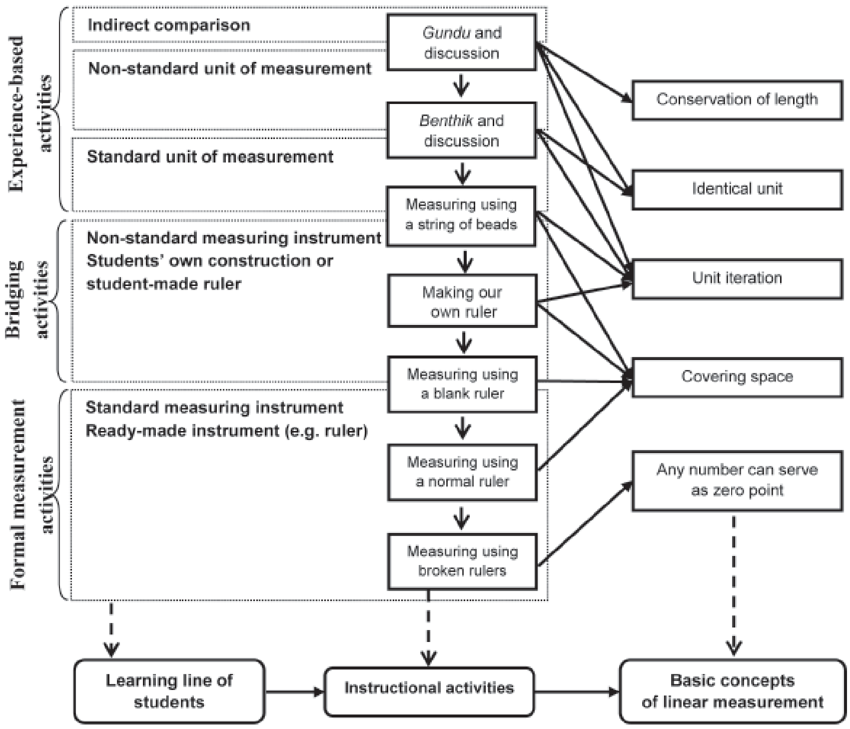


Figure 1. The main framework of experience-based activities for learning linear measurement.

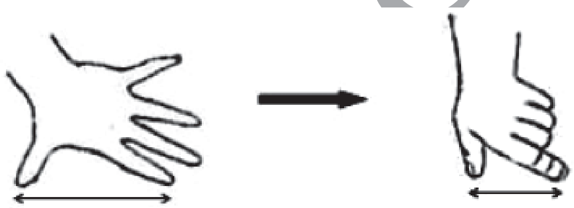
Note:

- A blank ruler is a ruler without any numbers on it
- A normal ruler is a ruler with numbers, starting from "0"
- A broken ruler is a ruler with numbers, starting from any non-zero number

As mentioned in the introduction, the focus of this paper is on the investigation of how traditional Indonesian games could be used to build upon students' reasoning and to reach the mathematical goal of linear measurement. Consequently, the focus of this paper is on game-playing in what we call *experience-based activities*.

Teaching Experiment

While game-playing, students started to use "third objects" as their *units of measurement* and also started to realise that they needed to *iterate the unit* when the measured distances were longer than the unit of measurement. At the beginning of playing *gundu*, students still used their body parts (i.e. hand spans and feet) and pencils to compare distances. However, in the last 15 minutes of the game, students started to think about other strategies when there was a conflict in which two marbles seemed to be the same distance from the circle, namely three spans in length. In fact, the distances between these marbles and the circle were different (i.e. approximately 2.50 and 2.33 spans), but both students adapted their last span in order to make a complete or integer span (see Figure 2).



A "*normal*" span

A bent span, counted as a "*normal*" span

Figure 2. Students bent their span to adjust the measure.

During the game-playing, all students experienced these aspects of measuring distance; i.e. the iteration of body parts and the need to be fair. Nevertheless, their strategies and understanding were still implicit and personal.

This conflict of *fairness* stimulated students to consider the need for *precision* (see Table 1 for the definition of *precision*), and furthermore, to generate the idea of a *standard unit of measurement*. This "need" or "motive" to improve their measurement practice emerged while they were playing. Finally, the students were able to determine the nearest marble when they used a piece of chalk to compare the distances from each marble to the circle. In this situation, the students started to consider that the choice of the unit size determines the precision of the measure (see Figure 3).

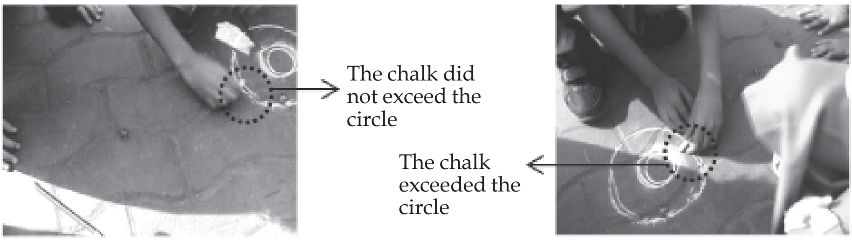


Figure 3. A piece of chalk used to compare distances.

The students' linear measurement experience from playing the game was combined with the teacher's story to elicit the basic concepts of linear measurement. The advantage of the *conflict of fairness* in determining the winner within the *situation* of game playing was also shown in the class discussion. The conflict of fairness could stimulate students to realise the need for *a standard unit of measurement* because of the shared game-playing experiences and the intrinsic motivation of being fair. The following vignette illustrates how the conflict of fairness plays this key role:

Haya: The game is not fair if there are many students measuring the distances because the different size of steps will give a different result [of the measurement].

Teacher: So… can we use different steps to measure the distances in our game?

Students: No, we cannot because it is not fair.

Teacher: What should we do?

Haya: In a game, we will have a fair game if there is only one person who measures the distances, because the different size of steps will give a different result [of the measurement].

Game-playing at the *situational level* provided an experience for the students that helped them to move to the *referential level*. Haya proposed a solution which would ensure a fair game by *referring* to a specific situation in the game. The phrase "*…the different size of steps will give a different result*" stated by Haya illustrates that Haya had come to understand the need for *precision* (i.e. the relation between the size of a unit of measurement and the result of the measurement). Next, the teacher posed the problem of "different steps" to elicit the concept of a standard unit of measurement from the students. The teacher's question and the word *fair* encouraged Haya to come up with a standardised unit of measurement, although she still used one person's body part as a unit. Haya's reasoning showed a *referential level* of emergent modelling because the need for a standard unit of measurement *referred* to the game-playing situation. Haya also showed that she had started to move from the *referential level* to the *general level* when she tried to generalise the solution using the phrase "*In a game…*"

In the second traditional game, *benthik*, most students started to use a stick instead of hand spans as the unit of measurement. In this activity, iterating sticks became the *model of* the activity of iterating hand spans. The students' understanding of a standard unit of measurement was further developed in the class discussion following the game of *benthik*. In the class discussion, the method of iterating sticks was also used to foster the idea of using a standard unit of measurement. Therefore, the stick became the *model for* reasoning about the characteristics of measurement and measuring instruments. The development from the *model of* to the *model for* was facilitated by a new conflict.

The following vignette illustrates how the teacher used a new conflict of *fairness* based on the students' results in the game to introduce and focus the class on the invention of a standard unit of measurement:

Teacher: Yesterday, Deva obtained 24 long sticks, and I still remember my brother told me that Agung, his best student in Kalimantan, got 50 short sticks. Who is the winner between Deva and Agung? Who threw the stick further?
[Students discuss this problem with their partners.]

Aira: We cannot determine the winner.

Teacher: Why did you say that we could not determine the winner? [The teacher asks for justification.]

Aira: Because one stick is longer than the other.

Teacher: Any other ideas? OK, Chia, share your opinion with your friends.

Chia: The length of the sticks is different, so if we measure the long stick with the short stick, we will know how many short sticks will match the long stick. So, 20 added to 20 is 40 and 4 added to 4 is equal to 8 and the sum of those is 48. Then the winner is Agung [because Agung got 50].

Teacher: Chia said that Agung is the winner.

The teacher created a new problem of indirect comparison that led to the development of a standard unit by connecting measurement to a game-playing situation. Chia – as well as Aira – tried to connect the short stick to the long stick. The sentence "*if we measure the long stick with the short stick, we will know how many short sticks will match the long stick*" showed that Chia tried to standardise the unit of measurement in terms of the short stick. Furthermore, Chia compared the length of Deva's stick to the length of Agung's stick. However, he did not really compare the short stick to the long stick, and therefore he only came to an assumption that the length of the long stick is twice that of the short stick.

The idea of Aira and Chia, that the winner could not be determined due to the different sticks used in the game, was used by the teacher to encourage the students to (re)invent a standard unit of measurement.

Teacher: What can we do to determine the winner?

Gilang: We can use marbles because marbles are always the same size.

Teacher: Yes, it is possible to use marbles, but the distances in benthik are too long to be compared using marbles. Are there any ideas?

Gilang: We cut the sticks and make them similar in length.

Shafa: Yes, we have to use units that have a similar length.

Gilang showed his achievement in understanding the need for a standard unit of measurement when he proposed the use of marbles and the notion of making the sticks a similar length by cutting one of them. Shafa proposed using units of measurement that have a similar length as a *general* solution to the conflict of fairness. This vignette illustrates how the social interaction among the students supported their contribution to a solution to the problem.

In summary, we can point out the importance of the conflict of fairness during these teaching experiments. This conflict seemed to help to stimulate the students to conceive the idea of a standard unit of measurement. However, student achievement at this stage was still at an informal level of knowledge. Consequently, the next important step in the instructional sequence was to provide "*bridging*" activities to develop students' informal knowledge into more formal knowledge of linear measurement. The use of strings of beads was proposed to shift their reasoning from iterating sticks to using a ruler. A string of beads represented an iteration of units (i.e. the beads as the units). In the next phase, the string of beads would be developed into a ruler in which *the beads* were replaced by *the spaces between every two stripes*. Therefore, measuring activities using a string of beads aimed to increase students' awareness that measuring by using a ruler did not involve counting the number of "stripes," but counting the number of "spaces."

Conclusion

The analysis of the instructional sequence in the classroom experiments resulted in empirical support for emergent modelling as a design heuristic for progressive mathematisation. This progressive mathematisation showed a gradual process from the situational level in game-playing to the formal reasoning of using a standard unit of measurement.

The process of the teaching and learning of linear measurement proceeded along the four levels of emergent modelling as follows:

1. Situational level: Game-playing provides informal knowledge of linear measurement, because there are some concepts of linear measurement that are elicited by traditional Indonesian games, such as indirect comparison and measurement. This game-playing needs to be part of the sequence, because it creates shared experiences to which the teacher can refer during the following activities;

2. Referential level: A class discussion seemed to successfully encourage students to shift from the situational level to the referential level. In the class discussion, students' arguments referred to specific situations in game-playing. One example is Haya's statement: "The game is not fair if there are many students measuring the distances, because the different size of steps will give different results";
3. General level: At the end of the class discussion, the students seemed to be able to generalise the specific solution – that referred to the game – into a general solution for other situations. The following is an example of a generalisation made by Shafa at the end of the class discussion: *"In a game, we will have a fair game if there is only one person who measures the distances, because the different size of steps will give different results [of measurement]."* This shows that the students had shifted from the *models of* situation towards *models for* performing a fair measurement;
4. Formal level: The formal level of the experience-based activities was shown when some students started to consider a specific characteristic of a standard unit of measurement. A student (i.e. Shafa) considered similarity of length to be the requirement for a standard unit of measurement. This is implied in her statement: *"Yes, we have to use measuring units that have a similar length."*

The general scheme of the students' process in eliciting the basic concepts of linear measurement from their game-playing is shown in Figure 4.

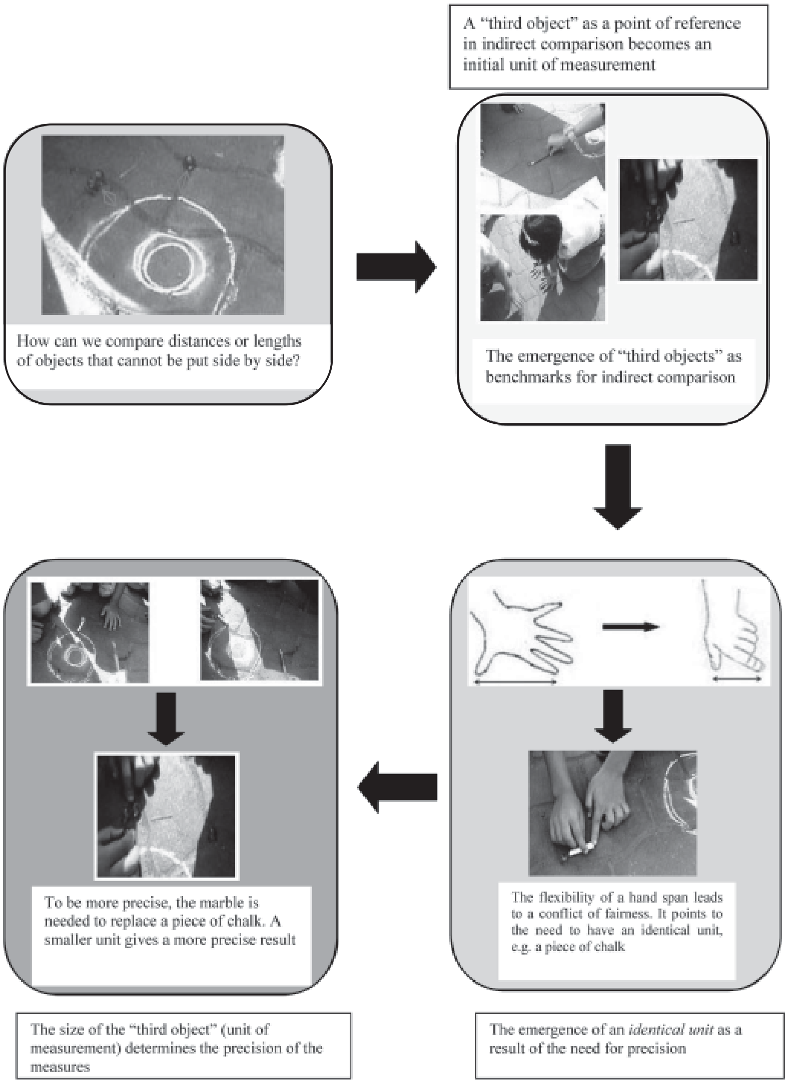


Figure 4. The scheme of the students' process of eliciting the basic concepts of linear measurement in traditional Indonesian games.

As an addition to emergent modelling, game-playing also supports social aspects of the learning process. Interaction between students emphasises that learning processes are not merely an individual process, but that they also have social characteristics, and that these processes run simultaneously. According to Vygotsky in Zack and Graves (2001), social interaction is at the core of learning processes, because learners first construct knowledge in their interactions with other people in the activity context. Consequently, the teacher plays an essential role in facilitating an interactive process among students. The roles of the teacher that were observed in the class discussion can be categorised in five ways, namely: (1) providing students with an opportunity to present their ideas; (2) stimulating social interaction; (3) connecting activities; (4) eliciting mathematical concepts and (5) asking for clarification.

Discussion

This research set out to contribute to the formulation and development of a local instruction theory for the teaching and learning of linear measurement in grade 2 of Indonesian elementary schools. The local instruction theory with regard to the sequence of experience-based activities and the intended concept development for the teaching and learning of linear measurement is summarised in Table 2. This table shows the interaction between the development of the tools that were used and the acquisition of mathematical concepts (Doorman & Gravemeijer, 2009; Gravemeijer, Bowers & Stephan, 2003).

Table 2
The Overview of the Proposed Role of Tools in the Instructional Sequence

<table>
<tr><th>Activity</th><th>Tools</th><th>Imagery</th><th>Practice</th><th>Concept</th></tr>
<tr><td rowspan="2">Traditional Indonesian games: playing gundu</td><td rowspan="2">Hand span, feet, marble</td><td rowspan="2"></td><td>Indirect comparison</td><td>Conservation of length
Emergence of a benchmark for indirect comparison</td></tr>
<tr><td colspan="2">The activity of iterating the benchmarks of comparison should become the focus for the introduction of measurement</td></tr>
<tr><td rowspan="3">Traditional Indonesian games: playing benthik</td><td rowspan="3">Hand span, feet, stick</td><td rowspan="3">Signifies that the “third object” in the comparison becomes the unit of measurement</td><td colspan="2">Playing benthik provides an opportunity to develop the use of “a third object” as a benchmark for indirect comparison, which becomes a unit of measurement</td></tr>
<tr><td>Measurement as the development of indirect comparison</td><td>Identical units and unit iteration</td></tr>
<tr><td colspan="2">The conflict of fairness in the game could lead to the need for a standard unit of measurement</td></tr>
<tr><td rowspan="2">Measuring using strings of beads</td><td rowspan="2">Strings of beads</td><td rowspan="2">Signifies the iteration of a unit of measurement, such as a hand span, feet and marbles</td><td>Measuring and reasoning about the activity of iterating and counting a unit of measurement
Beginning to use a standard unit of measurement</td><td>Standard unit of measurement for the fairness and precision of measurement</td></tr>
<tr><td colspan="2">The use of strings of beads should shift the focus of the learning process from units of measurement to measuring instruments</td></tr>
</table>

The role of the teacher in experience-based activities appears to be essential; the teacher should be able to find ways to structure class discussions in order to support both the process and the content of classroom discourses (Sherin, 2002). Consequently, the professionalisation of teachers is required in order to implement this kind of local instructional sequence. Further research should focus on how to professionalise teachers in conducting experience-based learning. Moreover, further research should also reveal how systematic analysis can support the teacher in performing experience-based learning.

The bent hand span (see Figure 2) shows that the result of measurement is not always an integer number. In games, it is natural and commonplace for the unit of measurement to exceed the measured object or distance. Considering the findings of this research and also that of Keijzer (2003), the suggestion for subsequent research is to use traditional Indonesian games to intertwine the topic of linear measurement with the concept of early fractions.

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Appendix

Two traditional Indonesian games – *gundu* and *benthik* – were used as the starting point of the instructional sequence. These games embody linear measurement activities, namely estimating, comparing and measuring distances. The students' linear measurement experience was combined with the teacher's story to elicit the basic concepts of linear measurement.

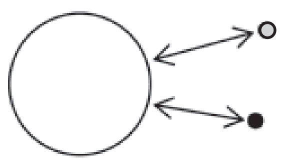
A. Playing *gundu* (marbles)

This game was intended to help the students to consider the need for a "*third object*" to compare distances or lengths. This "*third object*" will be developed into a unit of measurement in the next activity.

Rules of the game:

1. Each player has to throw a marble at a circle on the ground (the distance between the circle and the throwing point is approximately 2-3 metres);
2. The distances from the marbles to the circle on the ground are compared and the player whose marble is closest to the circle becomes the winner.

In this game, the students do not measure the distance or length along a line, but measure the distance from a certain point (i.e. the position of a marble) to the circle. This was actually more difficult than measuring the distance along a line, because determining the correct point on the circle was not an easy task for young children. However, the students were already familiar with this game, so they could compare the distance (although they did not choose the mathematically correct point on the circle).



The mathematical idea embodied in this game is:

- Measurement (students carry out a comparison (as a part of the measurement process) when they compare the distance from the marbles to the circle to decide which player has won).

B. Playing *benthik*

This game aims to develop the students' notion of a "third object" for comparing length into a "unit of measurement" for measuring a distance. A conflict of fairness that may potentially occur in the game will be used to raise the issue of a standard unit of measurement.

Materials: Two wooden sticks (long and short).

Players: The game is played by two groups (e.g. five students in each group).

Rules of the game:

1. The game is played by two groups; one group as "batter" and the other group as "guard." These groups will switch roles after a certain period;
2. A member of the batting group throws the short stick.
   - If the guard team can catch the short stick, the guard team gets 10 points and the game continues to rule/step number 3.
   - If the guard team cannot catch the short stick, the game continues to rule/step number 3;
3. The distance that the short stick has fallen from the hit point is measured, and the number indicating the distance is the number of points earned by the batting group;
4. Steps 1 to 3 are repeated until each member of the batting team has thrown the short stick. When each member of the batting group has thrown the short stick, then the batting and the guard teams will take turns;
5. The winner is the group that gains more points.

The mathematical ideas addressed in *benthik* are:

- Non-standard linear measurement (students measure distances using long sticks);
- Addition (students carry out addition when they add up all the points to find the accumulative distance).