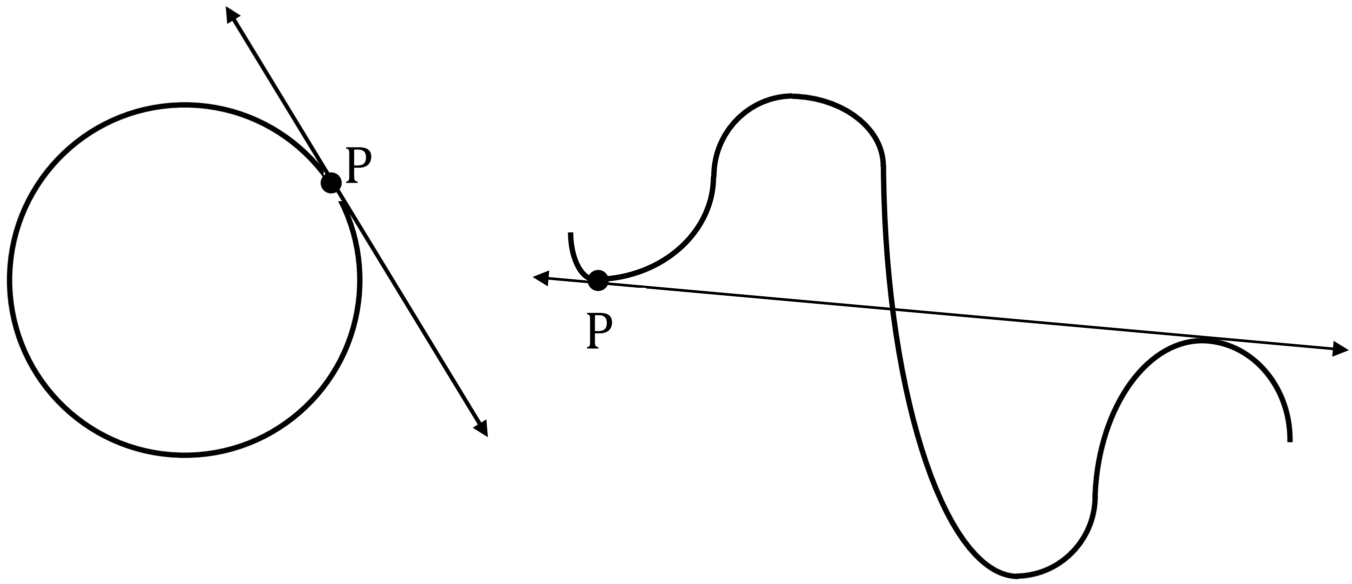
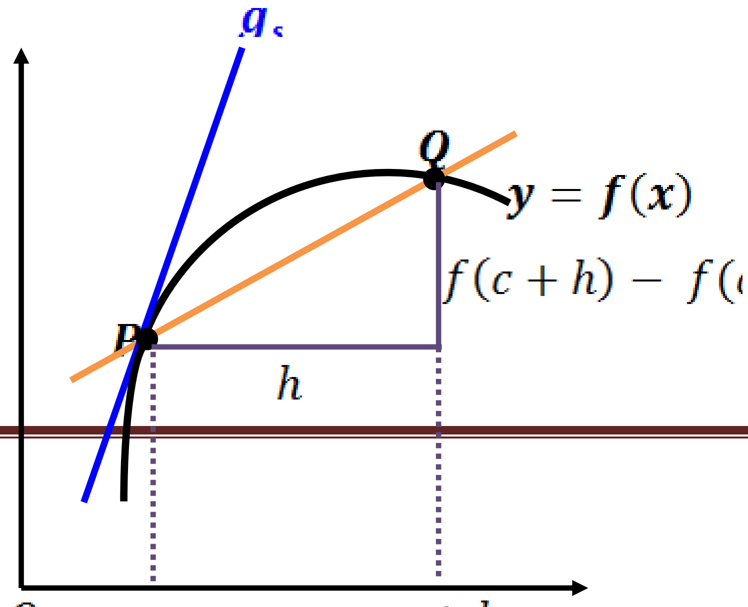
GARIS SINGGUNG

- Garis singgung?



What it means for a line to be tangent to a general curve is more difficult to describe precisely.

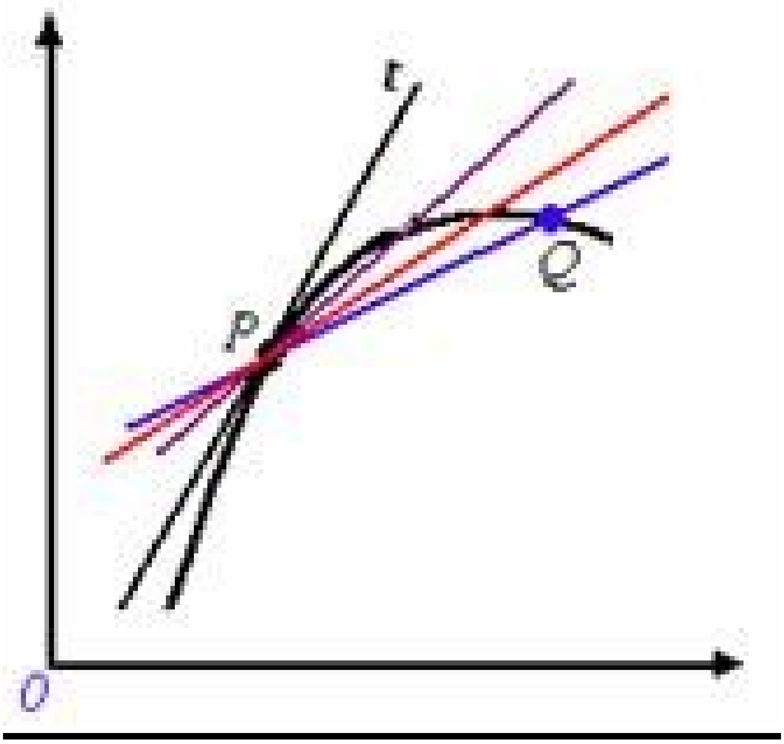
- Diketahui fungsi f dgn $y = f(x)$:



- Garis (tali busur) PQ mempunyai gradient:

$$m_{PQ} = \frac{f(c+h) - f(c)}{h}$$

- Apabila Q bergerak <u>mendekati</u> P sepanjang kurva hingga berimpit pd titik P ($h \longrightarrow 0$), maka terjadilah **garis singgung $\boldsymbol{y = f(x)}$ di titik $\boldsymbol{P(c, f(c))}$**,



dan kemiringan (gradien) grs singgung tsb adalah:

$$m_t = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Contoh 1:

Carilah kemiringan garis singgung pd kurva $y = x^2 - 3x + 4$ di titik (2,6).

Jawab:

$$m_t = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$m_t = \lim_{h \to 0} \frac{[(2+h)^2 - 3(2+h) + 4] - (2)}{h}$$

$$m_t = \lim_{h \to 0} \frac{h^2 + h}{h} = \lim_{h \to 0} \frac{h\,(h+1)}{h} = 0 + 1 = 1$$

Contoh 2:

Carilah kemiringan garis singgung pd kurva $y = x^2 - 1$ di titik $x = -1, 0, 1, 2$.

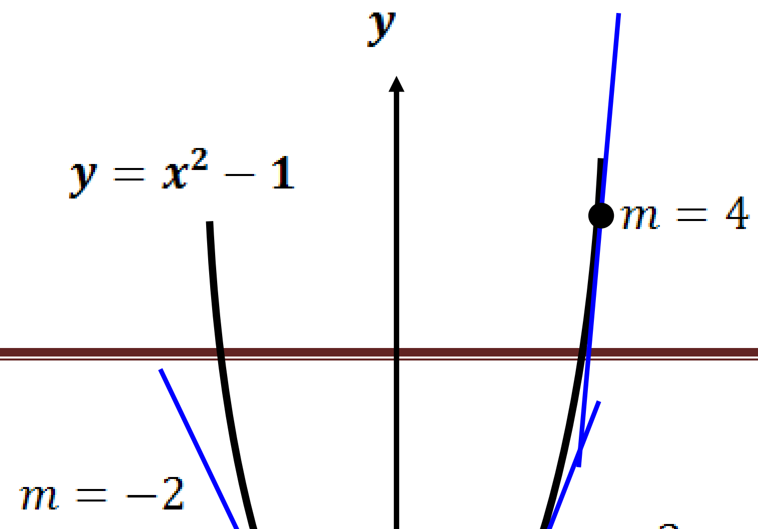
Jawab:

$$m_t = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$$= \lim_{h \to 0} \frac{[(c+h)^2 - 1] - (c^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{2ch + h^2}{h} = \lim_{h \to 0} \frac{h\,(2c + h)}{h} = 2c$$

Jadi, kemiringan grs singgung di titik $x = -1, 0, 1, 2$ berturut-turut adalah $-2, 0, 2, 4$.



TURUNAN

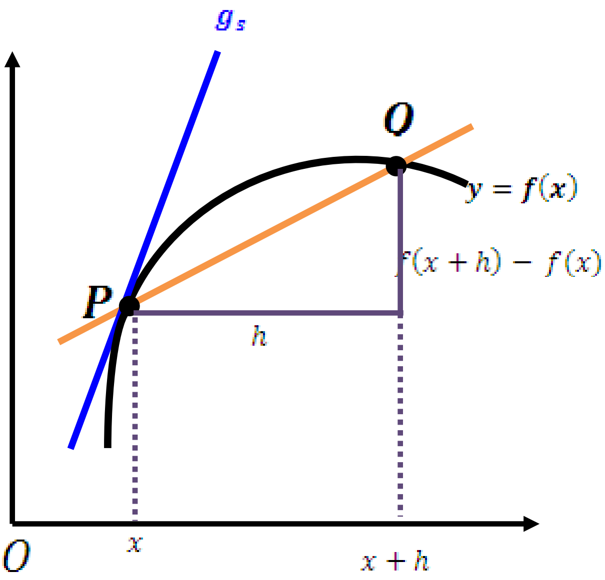
- Definisi

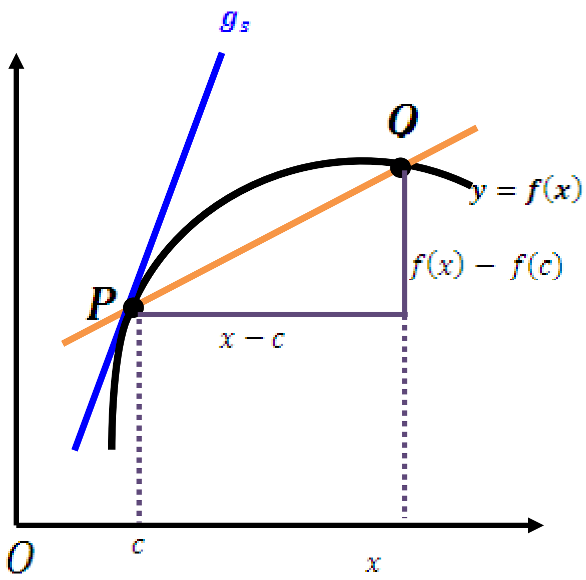
Turunan fungsi *f* adalah fungsi f' yang nilainya pd sebarang bilangan c adalah

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

asal nilai limit ada.

- Jika nilai limit ada, dikatakan ***f*** **terdifferensialkan (differensiable) di *c***.
- Sesuai dgn Bab 3.1, maka $f'(c) =$ kemiringan garis singgung kurva $f(x)$ di titik (c, f(c)).





$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

<u>Contoh 1</u>:

Tentukan $f'(c)$ utk $f(x) = 3x^2 - 4$

Jawab:

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$
$$= \lim_{h\to 0} \frac{[3(c+h)^2-4]-[3(c)^2-4]}{h}$$
$$= \lim_{h\to 0} \frac{[3(c+h)^2-4]-[3(c)^2-4]}{h}$$
$$= \lim_{h\to 0} \frac{6ch+h^2}{h} = \lim_{h\to 0} 6c+h = 6c$$

Contoh 2:

Tentukan $f'(x)$ utk $f(x) = \frac{1}{\sqrt{x}}$.

Jawab:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$
$$= \lim_{h\to 0} \frac{x-x-h}{h\sqrt{x}\sqrt{x+h}\left[\sqrt{x}+\sqrt{x+h}\right]}$$
$$= \lim_{h\to 0} \frac{-1}{x\sqrt{x+h}+\sqrt{x}(x+h)} = \frac{-1}{x\sqrt{x}+x\sqrt{x}} = \frac{-1}{2x\sqrt{x}}$$

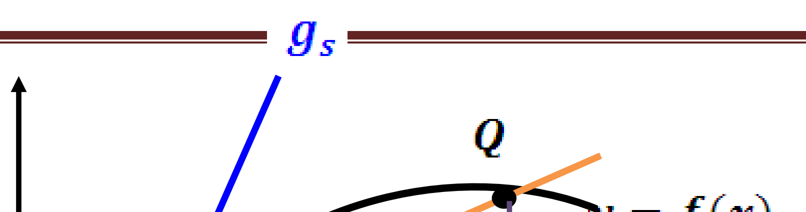
ATURAN PENENTUAN TURUNAN

Menghitung turunan menggunakan definisi limit sangatlah memakan waktu. Utk itu diperkenalkan aturan turunan.

ATURAN TURUNAN

1. $f(x) = k \rightarrow f'(x) = 0$, k suatu konstanta
2. $f(x) = x \rightarrow f'(x) = 1$
3. $f(x) = x^n \rightarrow f'(x) = nx^{n-1}$, n bil. bulat positif
4. $F(x) = f(x) \pm g(x) \rightarrow F'(x) = f'(x) \pm g'(x)$
5. $F(x) = f(x) \cdot g(x) \rightarrow F'(x) = f'(x) \cdot g'(x)$
6. $F(x) = \frac{f(x)}{g(x)} \rightarrow F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

NOTASI LEIBNIZ (Notasi $\frac{dy}{dx}$ untuk turunan)



$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = f'(x)$$

Note:

$$f'(x) = \frac{d(f)}{dx} = \frac{df(x)}{dx} = \frac{dy}{dx} = y'$$

TURUNAN FUNGSI TRIGONOMETRI

1. $f(x) = \sin x \rightarrow f'(x) = \cos x$
2. $f(x) = \cos x \rightarrow f'^{(x)} = -\sin x$
3. $f(x) = \tan x \rightarrow f'(x) = \sec^2 x$
4. $f(x) = \cot x \rightarrow f'(x) = -\mathrm{cosec}^2\, x$

Tentukan $f'(x)$ dari :

1. $f(x) = \sec x$
2. $f(x) = \operatorname{cosec} x$
3. $y = \frac{1}{\cos x}$
4. $y = \frac{\sin x + \cos x}{\tan x}$
5. $y = x^2 . \cos x$
6. Tentukan persamaan garis singgung pada kurva $y = \cos x$ di titik $x = 1$.