Valid and Invalid Arguments

An argument is a sequence of statements such that

- all statements but the last are called *hypotheses*
- the final statement is called the *conclusion*.
- the symbol \therefore read "therefore" is usually placed just before the conclusion.

Example:

 $p \land \neg q \rightarrow r$ $p \lor q$ $q \rightarrow p$ $\therefore r$

An argument is said to be **valid** if - whenever all hypotheses are true, the conclusion must be true.

Example of a valid argument (form)

 $p \land (q \lor r)$ $\sim q$ $\therefore p \land r$

p q r	$p \land (q \lor r)$	~ q	$p \wedge r$
ТТТ	Т	F	Т
TTF	Т	F	F
ТГТ			
TFF	F	Т	F
FTT	F	F	F
FTF	F	F	F
FFT	F	Т	F
FFF	F	Т	F

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An invalid argument

$$p \to q \lor \sim r$$
$$q \to p \lor r$$
$$\therefore p \to r$$

	p q r	$p \rightarrow q \lor \sim r$	$q \rightarrow p \lor r$	$p \rightarrow r$
	ТТТ	Т	Т	Т
	TTF	Т	F	
	ТГТ	F	Т	
Invalid row	TFF	Т	Т	F
	FTT	Т	F	
	FTF	Т	F	
	FFT	Т		T
	FFF	Т	Т	Т
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Tautology

- is a statement (form) that is always true regardless of the truth values of the individual statement variables.

Examples:

- $p \lor \sim p$ (eg. the number *n* is either > 0 or ≤ 0)
- $p \land q \rightarrow p$
- $(p \rightarrow q \rightarrow r) \rightarrow (p \rightarrow r)$

We need to study tautologies because any valid argument is equivalent to a tautology. In particular, every theorem we have proved is a tautology.

Example: $p \land (q \lor r)$ $\sim q$ $\therefore p \land r$ is a valid argument,

 $[p \land (q \lor r)] \land [\sim q] \rightarrow [p \land r]$

is a tautology.

In other words, an argument H_1 H_2 \dots H_n \therefore Conclusion is valid if and only if $H_1 \wedge H_2 \wedge \dots \wedge H_n \rightarrow$ conclusion is a tautology.

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Two most important valid argument forms

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Modus Ponens : means method of affirming
       p \rightarrow q
       p
     ... q
     Example: If n \ge 5, then n! is divisible by 10.
                 n=7
              \therefore 7! is divisible by 10.
    Modus Tollens : means method of denying
       p \rightarrow q
       ~q
     ∴ ~p
    Example: If n is odd, then n^2 is odd.
                 n^2 is even.
               \therefore n is even.
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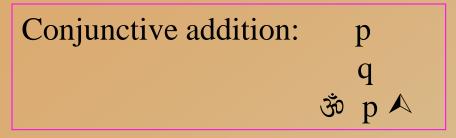
More valid forms

Disjunctive addition: p ॐ p ∀ q

Example: The function f is increasing. The function f is increasing or differentiable.

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More valid forms



Example: n is an integer, n is positive. \mathfrak{B} n is a positive integer.

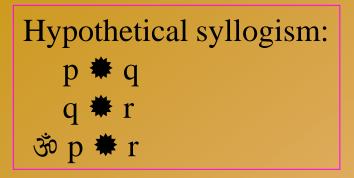
 Example:
 The graph of this equation may be a circle or an ellipse.

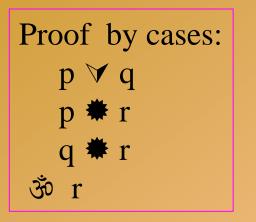
 The graph of this equation cannot be a circle.

 The graph of this equation cannot be a circle.

 The graph must be an (true) ellipse.

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Example: *n* is either odd or even. If *n* is odd, then n(n-1) is even. If *n* is even, then n(n-1) is even. Therefore n(n-1) is always even.

Rule of contradiction: "p ♣ c ॐ p Example: If $\sqrt{2}$ is not irrational, then there exists whole numbers *a*, *b* that are relatively prime and are both even.

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A valid argument with a false conclusion.

The following argument is valid by modus ponens, but since its hypothesis is false, so is its conclusion.

If p is prime, then $2^p - 1$ is also prime (False). 11 is prime (True). Therefore $2^{11} - 1$ is prime (False).

Actually, $2^{11} - 1 = 2047 = 23 \times 89$ is not prime.

Note: Any prime of the form $2^p - 1$ is called a Mersenne prime, the largest one up to date is $2^{6972593} - 1$ (discovered on 6-1-99)

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