

7.4 FUNGSI EKSPONEN UMUM &

LOGARITMA UMUM

- Jika r bil. rasional ($r = p/q$), maka

$$a^r = \exp(\ln a^r) = \exp(r \cdot \ln a) = e^{r \ln a}$$

- Jika x bil. real, maka

$$a^x = \exp(\ln a^x) = \exp(x \ln a) = e^{x \ln a}$$

$$\boxed{a^x = e^{x \ln a}}$$

Contoh: $3^2 = e^{2 \ln 3}$

- $\ln(a^x) = \ln(e^{x \ln a}) = x \ln a \cdot \ln(e)$

$$\boxed{\ln(a^x) = x \ln a}$$

Contoh: $\ln 3^{\sqrt{5}} = \sqrt{5} \ln 3$

- Sifat

Misal $a > 0, b > 0$ dan x, y bil real:

1. $a^x \cdot a^y = a^{x+y}$

$$2. \frac{a^x}{a^y} = a^{x-y}$$

$$3. (a^x)^y = a^{xy}$$

$$4. (ab)^x = a^x \cdot b^x$$

$$5. \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

- Turunan Fungsi Eksponen Umum ($y = a^x$)

$$D_x(a^x) = a^x \cdot \ln a$$

Bukti:

$$\begin{aligned} D_x(a^x) &= D_x(e^{x \ln a}) \\ &= e^{x \ln a} \cdot D_x(x \ln a) = e^{x \ln a} \cdot \ln a = a^x \cdot \ln a \end{aligned}$$

Contoh: $D_x(12^x) = ?$

- Misalkan $u = f(x)$, maka

$$D_x(a^u) = a^u \cdot \ln a \cdot D_x u$$

Contoh: $D_x(3^{x^3+2x^2}) = ?$

- Integral Fungsi Eksponen Umum ($y = a^x$)

$$\int D_x(a^x) dx = \int a^x \cdot \ln a dx$$

$$a^x + C = \ln a \int a^x dx$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

- Misalkan $u = f(x)$, maka

$$\int a^u dx = \frac{a^u}{\ln a} + C$$

Contoh:

$$1. \int 2^{x^3} \cdot x^2 dx = \dots \dots \dots$$