

# LQR Robust Control of a Flexible Link Robot Manipulator

M. Khairudin<sup>1</sup>, Z. Mohammed<sup>2</sup>, A.R. Husain<sup>2</sup>

<sup>1</sup>Department of Electrical Engineering, Universitas Negeri Yogyakarta  
Jl. Colombo 55281 Karangmalang-Yogyakarta, Indonesia, Telp/Fax +62274-548161

<sup>2</sup>Faculty of Electrical Engineering, Universiti Teknologi Malaysia,  
P08 Building, FKE-UTM, Skudai, 81310 Johor, Malaysia, Telp. +607-5535292  
email: moh\_khairudin@uny.ac.id<sup>1</sup>, zahar@fke.utm.my<sup>2</sup>, rashid@fke.utm.my<sup>2</sup>

## Abstrak

*Paper ini menyajikan pemanfaatan system kendali LQR robust pada robot lengan lentur. Model dinamik system dirancang menggunakan teknik element berhingga (FEM). Problematika yang terjadi dalam robot lengan lentur adalah adanya parameter yang tidak terbatas dan tidak linear. Penelitian sebelumnya banyak membahas model kendali untuk system ini tetapi sedikit membahas sistem kendali terhadap effect ketidakterbatasan dan ketidaknonlinearan parameter. Penelitian ini untuk menguji model dinamik, sistem response pada hub dan titik ujung lengan. Sebagai metode kendali, LQR dikembangkan untuk mengatasi masalah kekokohan dan kemampuan mencapai posisi yang akurat. Hasil LQR controller diverifikasi melalui perbandingan hasil PID kontroler, sebagai alternative solusi dalam control robot lengan lentur. Hasil menggunakan metode LQR menunjukkan settling time yang lebih cepat, overshoot yang lebih kecil, serta performa yang lebih baik dibandingkan dengan menggunakan PID controller.*

**Kata kunci:** Model, LQR, robot lengan fleksibel link.

## Abstract

*This paper presents investigations into the development of linear quadratic regulator (LQR) robust control of a flexible link manipulator. The dynamic model of the system is developed using a finite element methods (FEM). The problems of a flexible link manipulator are uncertainties and parametric nonlinearities. Previous researches have fathom around control methods for the system, few presented the effect of uncertainties and parametric nonlinearities. Simulation is performed to assess the dynamic model and system responses at the hub and end-point responses of links. For the control of the system, LQR is developed for solving flexible link robustness and achieve high-precision position input tracking capability. The results achieved by the proposed controller are compared with conventional PID, to substantiate and verify the advantages of the proposed scheme and its promising potential in control of a flexible link manipulator. It is shown that faster settling time and smaller overshoot response and tracking performance of the proposed controller than PID controllers.*

**Keywords:** Modeling, LQR, flexible link manipulator.

## 1. Introduction

Flexible manipulators have several advantages over rigid robots: they require less material, are lighter in weight, consume less power, require smaller actuators, are more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio [1]. These types of robots are used in a wide spectrum of applications starting from simple pick and place operations of an industrial robot to micro-surgery, maintenance of nuclear plants and space robotics [2].

Due to the flexible nature of the system, the dynamics are highly non-linear and complex. Problems arise due to lack of sensing, vibration due to system flexibility, imprecise

positional accuracy and the difficulty in obtaining accurate model for the system [3]. Moreover, the complexity of this problem increases when the flexible manipulator carries a payload. Practically, a robot is required to perform a single or sequential task such as to pick up a payload, move to a specified location or along a pre-planned trajectory and place the payload. Previous investigations have shown that the dynamic behaviour of the manipulator is significantly affected by payload variations [4].

The main goal on the modeling of flexible link manipulator is to achieve an accurate model representing the actual system behaviour. It is important to recognize the flexible nature and dynamic characteristics of the system and construct a suitable mathematical framework. The numerical analysis methods that are utilized include finite difference and finite element methods. Both approaches have been used in obtaining the dynamic characterization of single-link flexible manipulator systems incorporating damping, hub inertia and payload [5]. Performance investigations have shown that the finite element method can be used to obtain a good representation of the system [3]. De Luca and Siciliano [6] have utilised the AMM to derive a dynamic model of multilink flexible robot arms limiting to the case of planar manipulators with no torsional effects. M. Khairudin *et al.*, [7] also derived the models using combined Euler Lagrange and AMM for various payload profiles.

There is great interest in the development of control strategies for use with flexible link manipulators at present. The difficulties of flexible manipulator control are exacerbated by the fact that control inputs as well as external disturbances induce flexural vibrations in the manipulator structures. However, the complexity of the modelling process increases dramatically as compared to the case of a single-link flexible manipulator. Yang and Sadler [8] and Dogan and Istefanopulos [9] have developed the finite element models to describe the deflection of a planar two-link flexible robot manipulator. On the other ways, Han *et al.* [10] using model reference adaptive control with PD (MRAC-PD) gain to keep certain relationship which is defined as synchronization motion of the two-link mechanism. In order to guarantee a desired tracking performance, a time-variable and time-coefficient of the reference trajectory is used model predictive control [11].

This paper presents the dynamic modeling and control of flexible link robot manipulator. System responses namely angular position and modal displacement are evaluated. The work presented forms the basis of design of a suitable control method for flexible link manipulator systems using a LQR robust control. This controller is used to achieve of high-precision position tracking. The LQR robust controller design maximizes the control performance guaranteeing a good precision when regulating the tip position of the flexible link manipulator in the presence of parameter uncertainties [12] and [13].

## 2. Research Methodology

Figure 1 shows a structure of flexible link manipulator system considered in these investigations. The links are cascaded in a serial fashion and are actuated by rotors and hubs with individual motors. The link has length  $l$  with uniform mass density per unit length  $\rho$ . The first link is clamped at the rotor of the first motor.  $E$  and  $I$  represent Young modulus and area moment of inertia of both links respectively.  $X_0Y_0$  is the inertial co-ordinate frame,  $XY$  is the rigid body coordinate frame associated with the  $i$ th link.  $\theta$  is the angular positions and  $w(x, t)$  is the transverse component of the displacement vector.  $m_p$  is an inertial payload mass with inertia  $I_p$  at the end-point of link-2.

The physical parameters of the flexible link manipulator system  $m_h$  is the mass considered at the motor which is located in clamped with motor,  $I_h$  is the inertia of the motor and hub. The input torque,  $\tau(t)$  is applied at each motor and  $G$  is the gear ratio for the motor. Both links and motors are considered to have the same dimensions.

#### 4. Conclusion

The development of dynamic model and LQR robust control of flexible link manipulator has been presented. A set of linear model has been developed. A PID controller has, initially, been developed for control of a flexible link manipulator. A LQR robust controller has been implemented for input tracking control of the flexible link manipulator. Performances of the control schemes have been evaluated in terms of the input tracking capability of the system with compared PID controller. Simulations of the dynamic model and LQR robust control have been carried out in the time domain where the system responses including angular positions, modal displacement, end-point residual and hub velocity are studied. In term of input tracking, LQR robust controller has been shown to be more effective technique compared with PID controller. These results will be verified on the hardware experimental work for future work.

#### References

- [1] Tokhi M. O., Mohamed Z. and Shaheed M. H. Dynamic characterisation of a flexible manipulator system. *Robotica*. 2001; 19: 571-580.
- [2] Dwivedy, S. K. and Eberhard, P. Dynamic analysis of flexible manipulators, a literature review. *Mechanism and Machine Theory*. 2006; 41:749-777.
- [3] Martins J. M., Mohamed Z., Tokhi M. O., Sa da Costa J. and Botto M. A. Approaches for dynamic modeling of flexible manipulator systems. *IEE Proc-Control Theory and Application*. 2003; 150: 401-411.
- [4] Mohamed Z. and Tokhi M. O. Command shaping techniques for vibration control of a flexible manipulator system. *Mechatronics*. 2004; 14: 69-90.
- [5] Aoustin Y, Chevallereau C, Glumineau A and Moog C. H. Experimental results for the end-effector control of a single flexible robotic arm. *IEEE Transactions on Control Systems Technology*. 1994; 2: 371-381.
- [6] De Luca A. and Siciliano B. Closed-form dynamic model of planar multi-link lightweight robots. *IEEE Transactions on Systems, Man, and Cybernetics*. 1991; 21: 826-839.
- [7] M. Khairudin, Z. Mohamed, A.R. Husein, M.A. Ahmad. Dynamic Modelling and Characterization of A Two-Link Flexible Manipulator. *Journal of Low Frequency Noise and Vibration Active Control*. 2010; 29(3): 207-219.
- [8] Yang Z. and Sadler J. P. Large-displacement finite element analysis of flexible linkage. *ASME Journal of Mechanical Design*, 1990. Vol. 112, 175-182.
- [9] Dogan, M., and Istefanopulos, Y. Optimal nonlinear controller design for flexible robot manipulators with adaptive internal model, *IET Control Theory and Applications*, 2007, 1, (3), pp. 770-778.
- [10] Han, Q., Yang, X., Zhao, X., Wen, B. Controlled synchronous motions of a two-link mechanism based on improved MRAC-PD controller. *Int. Journal ICIC*, 2010. 6 (11): 4911-4922.
- [11] Henmi, T., Ohta, T., Deng, M., Inoue A. Tracking control of a two-link planar manipulator using nonlinear model predictive control. *Int. Journal ICIC*, 2010.6 (7): 2977-2984.
- [12] Araghi, L.F, Habibnejad M.K, Nikoobin A, Setoudeh F. Neural Network Controller Based on PID Controller for Two links- Robotic Manipulator Control. *Proceedings of the World Congress on Engineering and Computer Science WCECS 2008*. October 22 - 24, 2008; San Francisco. USA.
- [13] Tian L., and Collins C. A dynamic recurrent neural network based controller for a rigid-flexible manipulator system. *Mechatronics*. 2004; 14; 471-490.