SYSTEM REPRESENTATION

In this topic, we will learn:

- Important definitions of block diagrams and signal flow graphs: signals, system, summing junctions, pickoff points, series, cascade and feedback forms.
- Techniques of simplifying block diagrams.
- Signal flow graphs.
- Changing block diagrams to signal flow graphs and vice versa.
- Mason's Rule and example questions.

INTRODUCTION

- A control system consists of the inter-connection of subsystems.
- A more complicated system will have many inter-connected subsystems.
- For the purpose of analysis, we want to represent the multiple subsystems as a single transfer function.
- A system with multiple subsystems can be represented in two ways:
  - Block diagrams
  - Signal flow graphs

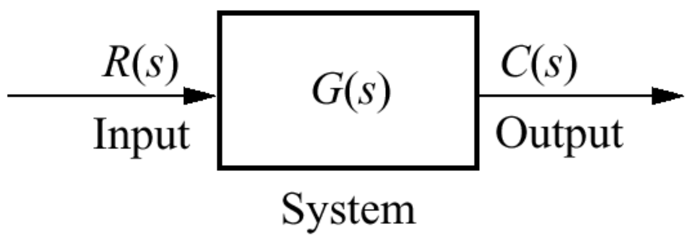
BLOCK DIAGRAMS

- The basic components in a block diagram are:

Signals

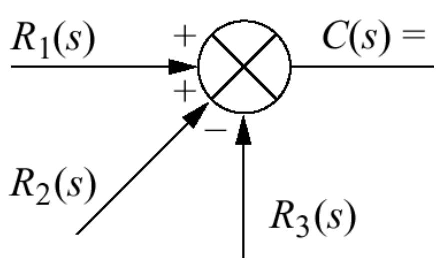
The direction of signal flow is shown by the arrow.

System blocks



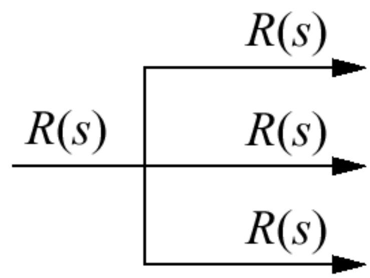
The system block represented by a transfer function.

Summing junctions



The signals are added/subtracted algebraically

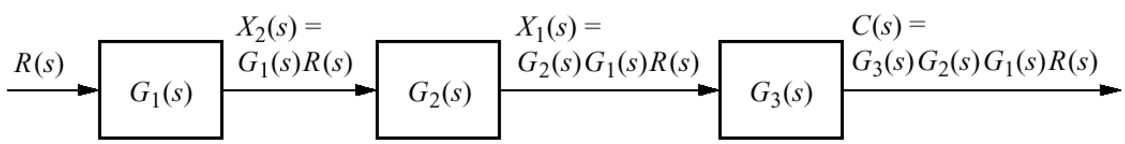
Pickoff points



The same signal is distributed to other subsystems

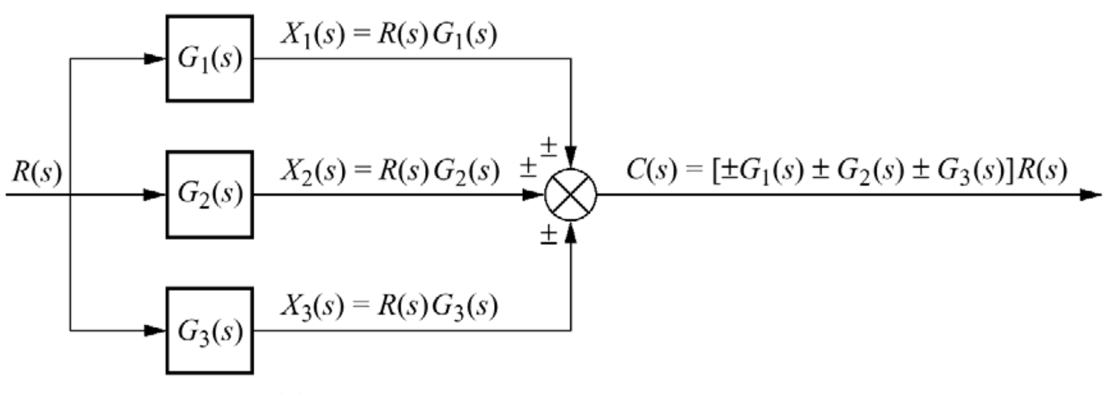
- The subsystems in a block diagram are normally connected in three forms:

Cascade form



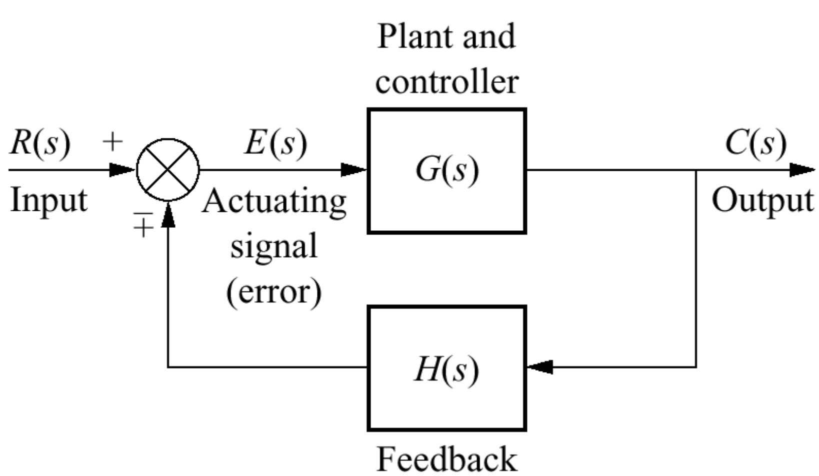
The block diagram can be reduced into a single block by multiplying every block to give:

Parallel form



The block diagram can be reduced into a single block by summing every block to give:

Feedback form



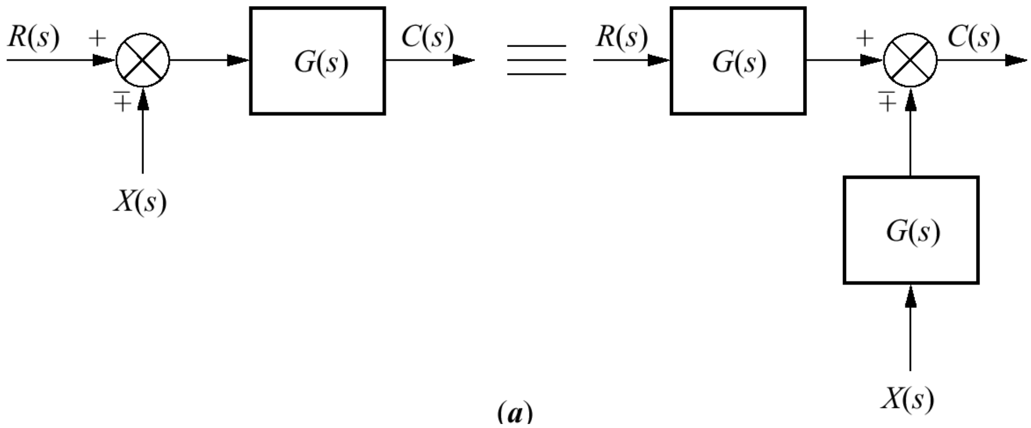
- closed-loop transfer function:

- open-loop transfer function:

Moving blocks to create familiar forms

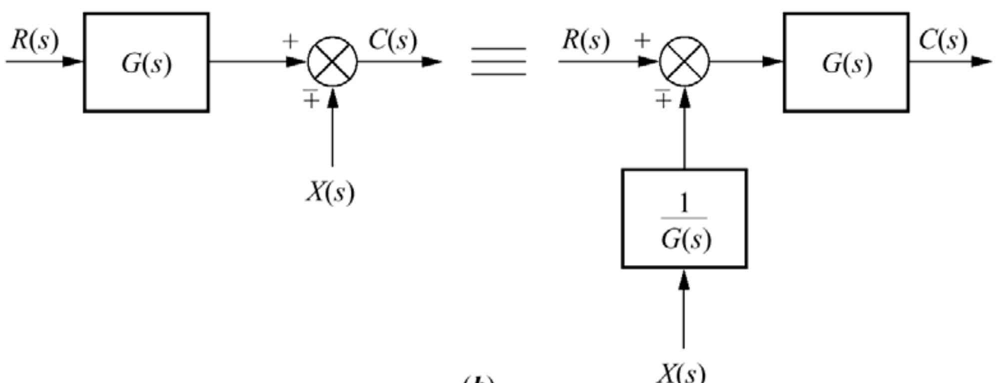
- It is not always apparent to get block diagrams in the familiar forms.
- We have to move blocks to get the familiar forms in order to be able to reduce the block diagram into single transfer function.

- Moving the summing junction to the front of a block



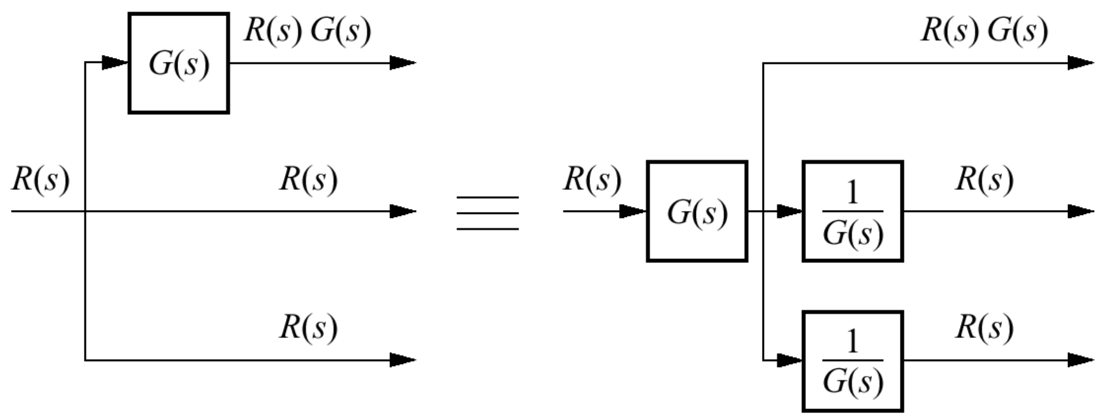
(*a*)

- Moving the summing junction to the back of a block

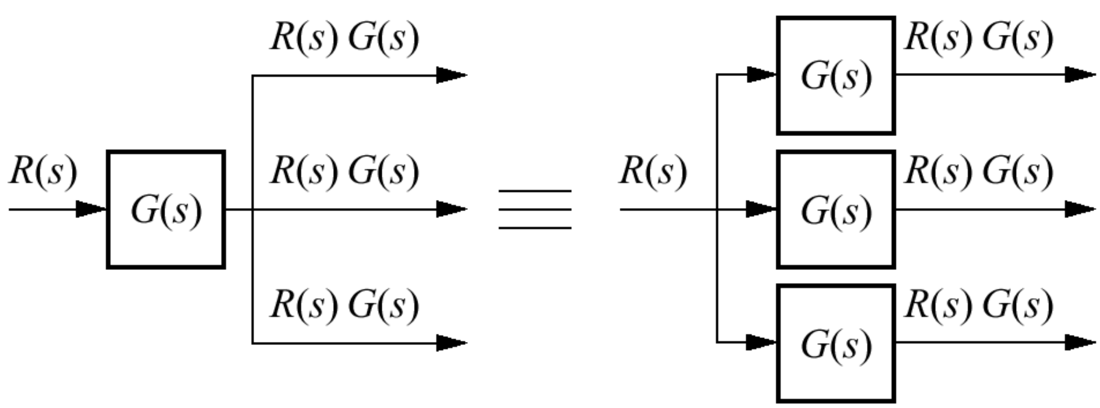


(*b*)

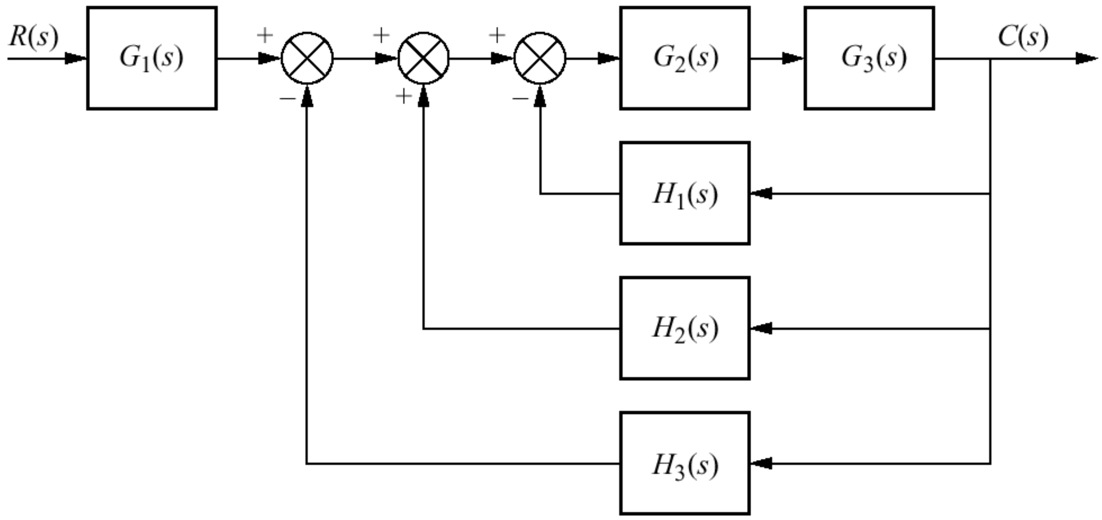
- Moving pick-off point to the front of a block



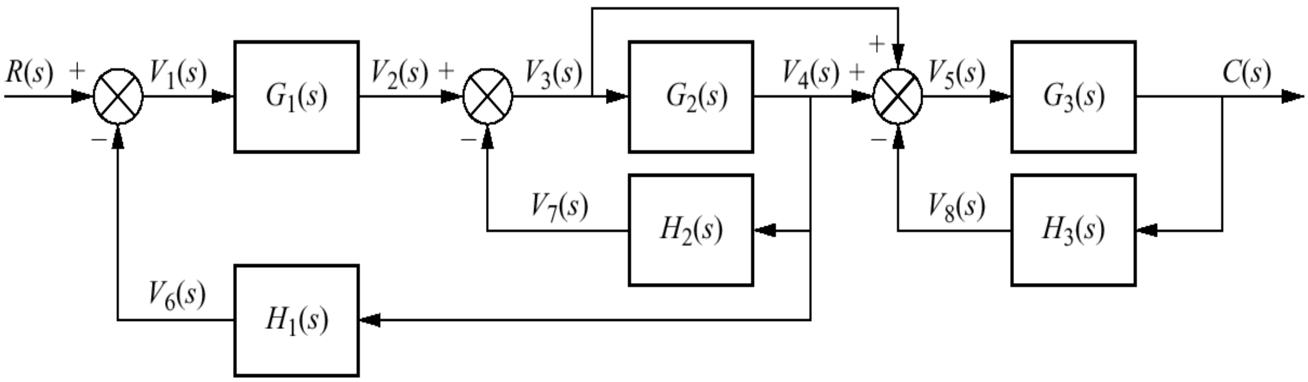
- Moving pick-off point to the back of a block



Example (5.1): Reduce the following block diagram into a single transfer function



Example (5.2): Reduce the following block diagram into a single transfer function.

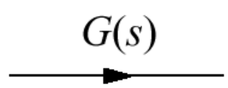


SIGNAL FLOW GRAPHS

- An alternative to block diagrams.
- It consists only of branches to represent systems and nodes to represents signals.

Branches

- Represented by a line with arrow showing the direction of signal flow through the system

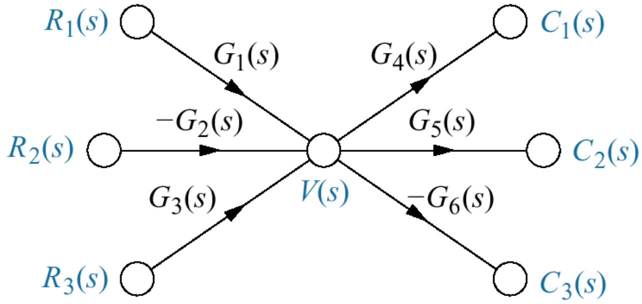


- The transfer function is written close to the line and arrow

Nodes

- Represented by a small circle with the signal's name is written adjacent to the node

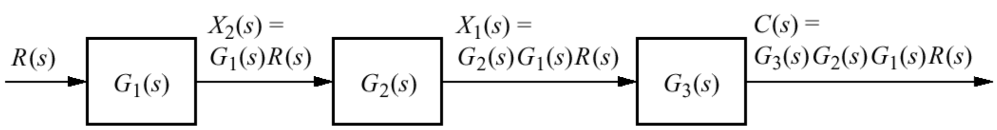
Example:



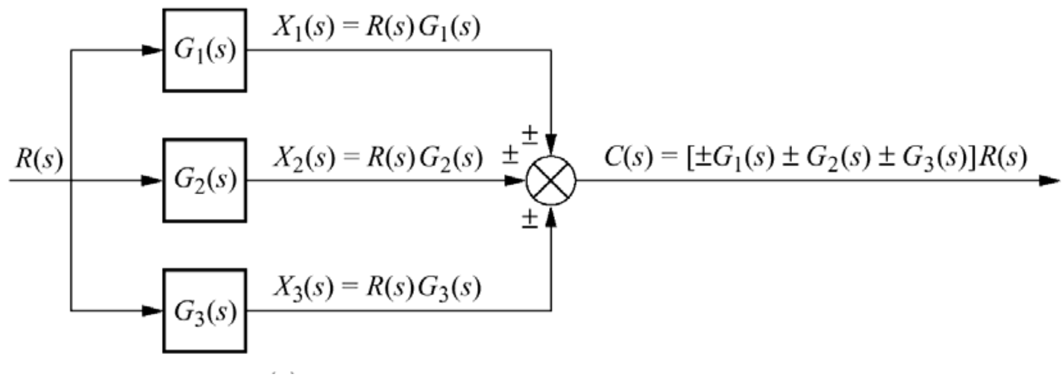
Converting block diagrams into signal-flow graphs

- The block diagrams in cascade, parallel and feedback forms can be converted into signal-flow diagrams.
- We can start with drawing the signal nodes, and then interconnect the signal nodes with system branches.

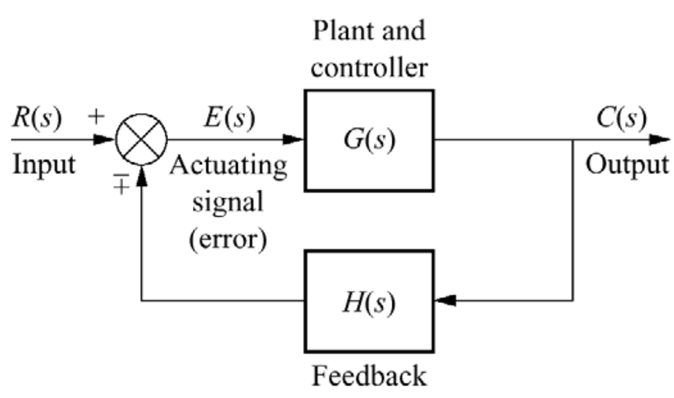
<u>Cascade form</u>



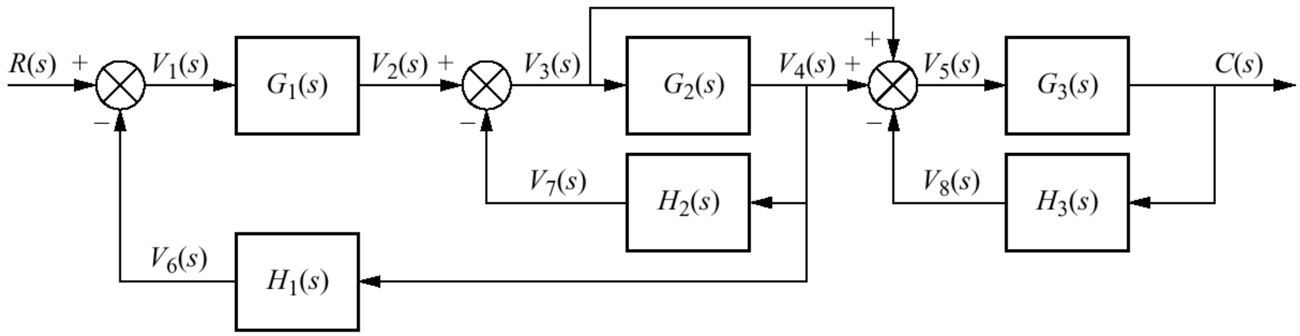
Parallel form



Feedback form

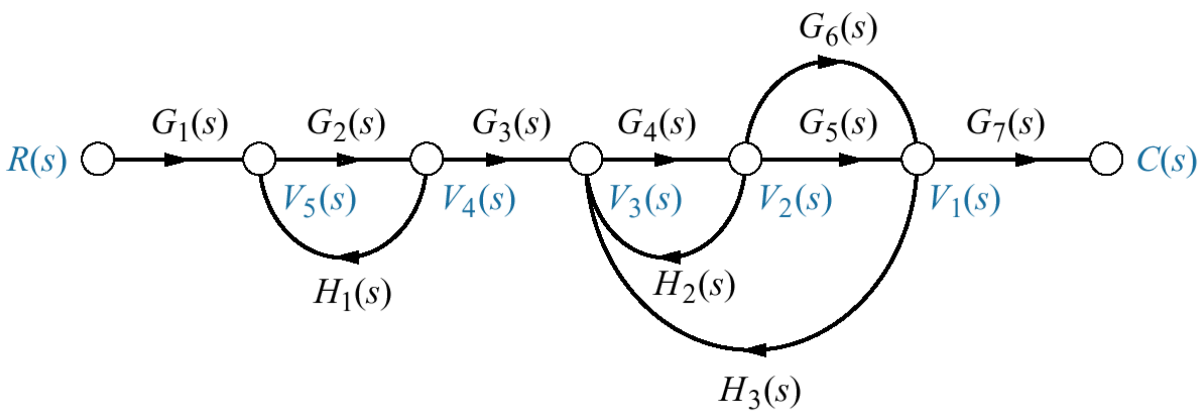


<u>Example (5.6)</u>: Convert the following block diagram into signal-flow graph



Mason’s Rule

- Mason’s rule is used to get the single transfer function in the signal-flow graph.
- We need to know some important definitions:



  - Loop – a closed path which starts and ends at the same node.
  - Loop gain – the product of branch gains found by traversing a loop.

  - Forward-path – a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

- Forward-path gain – the product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

- Non-touching loops – loops that do not have any nodes in common.

- Non-touching loops gain – the product of loop gains from non-touching loops taken two, three, four, or more at a time.

Mason's rule:

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

k = number of forward path

T_k = the k-th forward-path gain

Δ = 1 - Σ loop gains + Σ non-touching loops gain (taken 2 at a time) - Σ non-touching loops gain (taken 3 at a time) + Σ non-touching loops gain (taken 4 at a time) - …

Δ_k = Δ - Σ loop gain terms in Δ that touch the k-th forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the k-th forward path.

<u>Example (5.7)</u>: Find the transfer function, C(s)/R(s) of the following signal-flow graph,