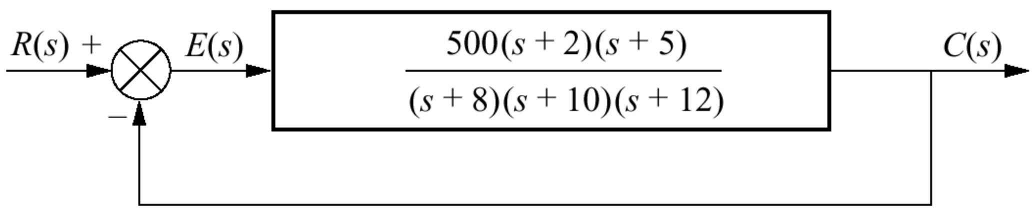
Static Error Constants and System Type

- Here we will define parameters that can be used as steady-state error performance specifications for unit negative feedback systems.
- These parameters are called *static error constants*.
- Recall previously that we have found the expressions for e_{ss} for the case of unity feedback systems, namely:
    - Step input:
    - Ramp input:
    - Parabolic input:
- The three terms in the denominator that are taken to the limit determine e_{ss}. These are the static error constants, i.e.:

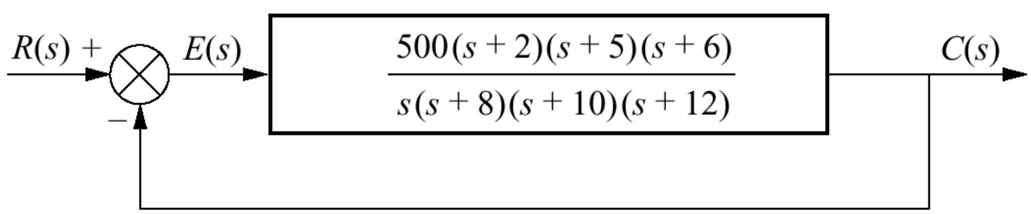
- Note that the values of e_{ss} decreases as the static error constants increases.

Example:

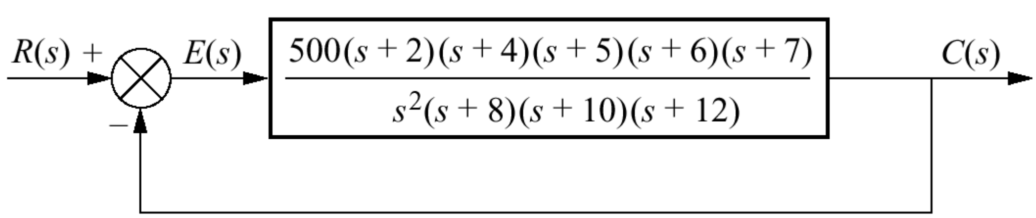
For each system shown, evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs.



(*a*)



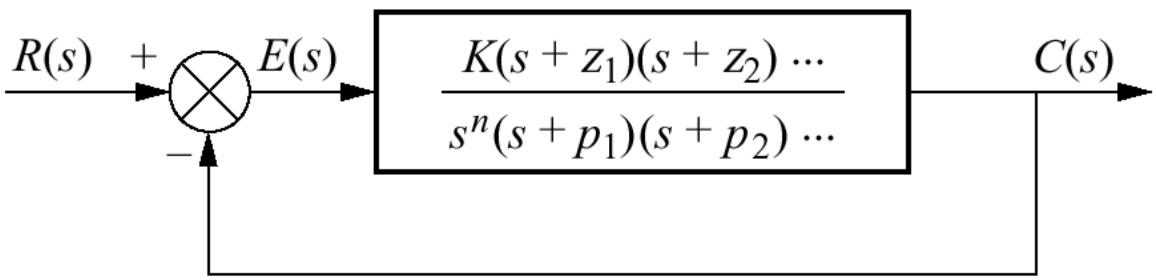
(*b*)



(*c*)

System type

- As seen in the previous example, the values of static error constants depend on the form of *G(s)* i.e. they depend on the number of integrations in existing in *G(s)*.
- Hence, define system type to be the value of n in the denominator of *G(s)*.



- The relationship between input, system type, static error constants, and steady state error is summarized in Table 7.2.

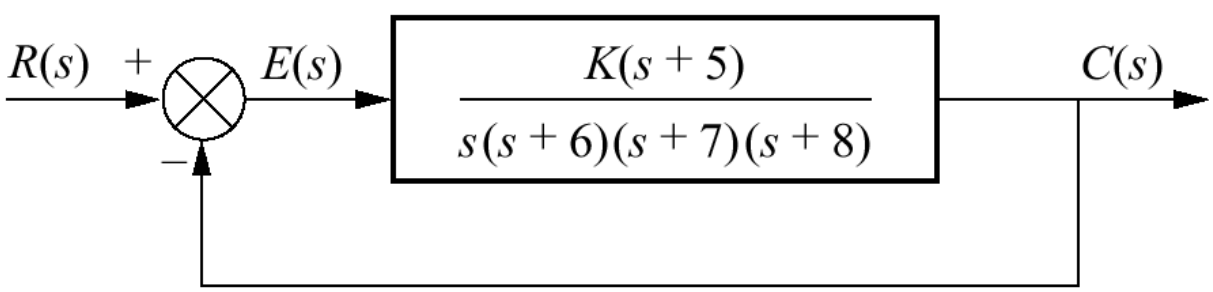
Input	Steady-state error formula	Type 0		Type 1		Type 2	
		Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Steady state error specifications

- A lot of information is contained within the specification of a static error constant.
- For example, if a control system has the specification K_v=1000, we can draw several conclusions:
    - The system is stable
    - The system is of Type 1, since only Type 1 systems have K_v's that are finite constants.
    - A ramp input is used as the test signal.
    - e_{ss} between input ramp and output ramp is $1/K_v$.

Example:

Find the value of K so that there is 10% error in the steady state.



Steady-state error for Non-unity Feedback Systems

- Example: Find the steady-state errors for the closed-loop system for the unit step, ramp and parabolic inputs where

$$G(s) = \frac{100}{s(s+10)};\ H(s) = \frac{1}{s+5}$$