System Response with Additional Poles

- Note that the formulas for T_p, T_s, and %OS that was derived before are only valid for 2^{nd} order systems with no zeros.
- However, under a certain conditions, a system having more than two poles or zeros can be approximated as a 2^{nd} order system having two complex *dominant poles*.
- Once the approximation is justified, the formulas that were derived before can be used without any changes.
- Consider a three-pole system:

$$p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

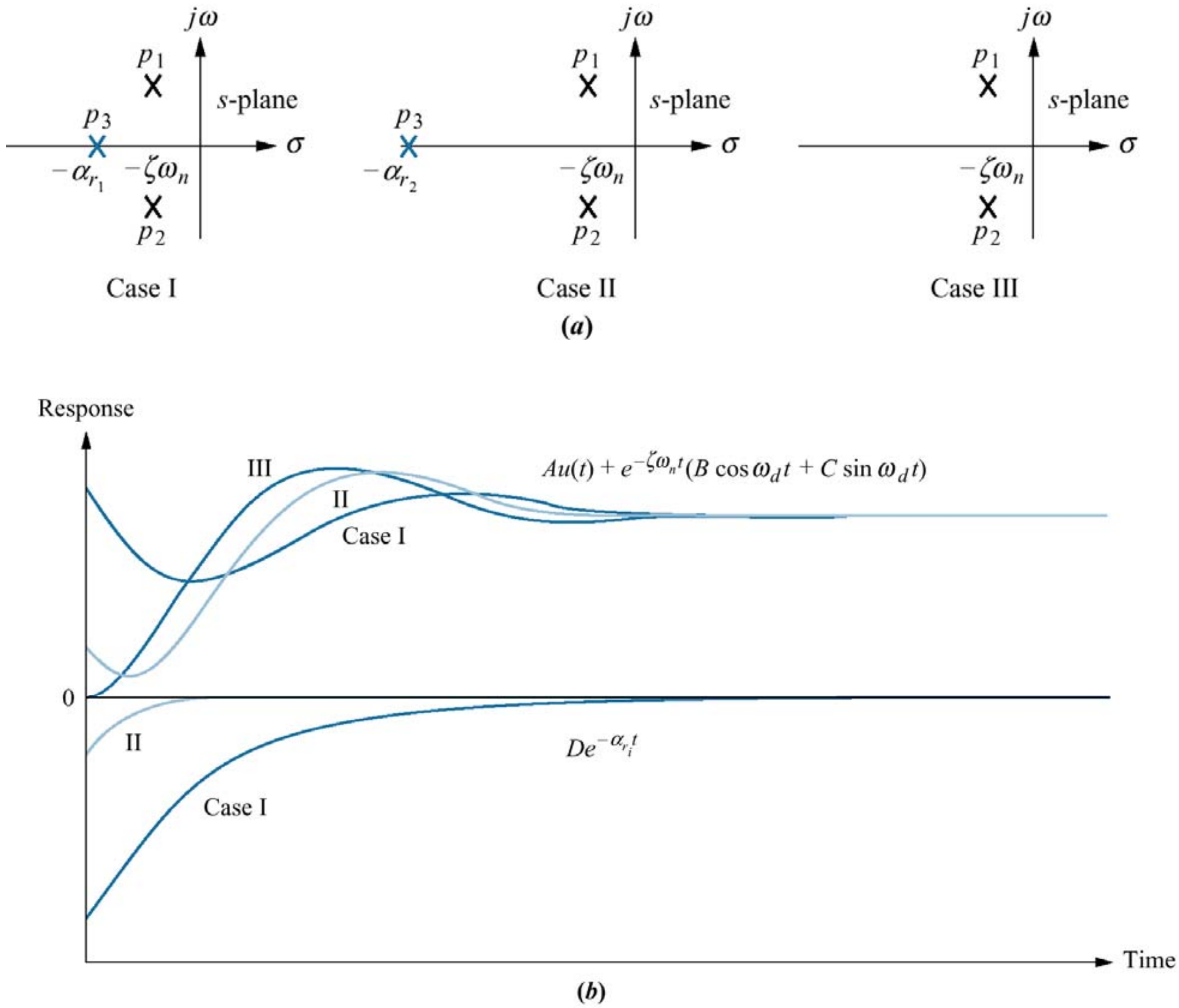
$$p_3 = -\alpha_r$$

- The step response of the system will then be:

$$C(s) = \frac{A}{s} + \frac{B(s+\zeta\omega_n) + C\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^{\,2}} + \frac{D}{s+\alpha_r}$$

$$c(t) = Au(t) + e^{-\zeta\omega_n t}\left(B\cos\omega_d t + C\sin\omega_d t\right) + De^{-\alpha_r t}$$

- Consider 3 cases. Case I: $p_3 > p_{1,2}$; Case II: $p_3 >> p_{1,2}$; case III: $p_3 >>>$.



- Case III behaves like a second order response.

- if the third pole, $p_3 = -\alpha_r$, is 'far' enough, then we can make 2nd order system approximation.
- Consider 'far' as more than 5 times farther to the left than the dominant poles.

Example:

Determine the validity of a second-order approximation for each transfer function shown below.

a. $G(s) = \dfrac{700}{(s+15)(s^2+4s+100)}$

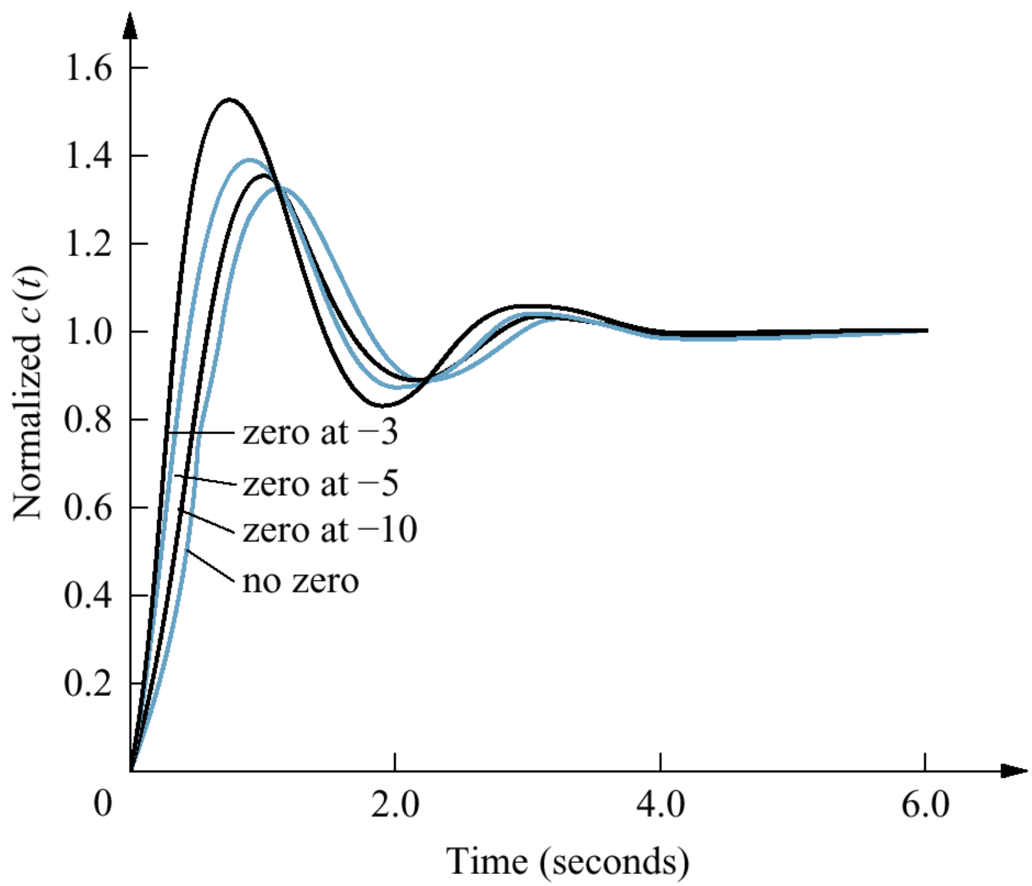
b. $G(s) = \dfrac{360}{(s+4)(s^2+2s+90)}$

System Response with Zeros

- From the previous section, the zeros affect the amplitude of response but do not affect the nature of the response.

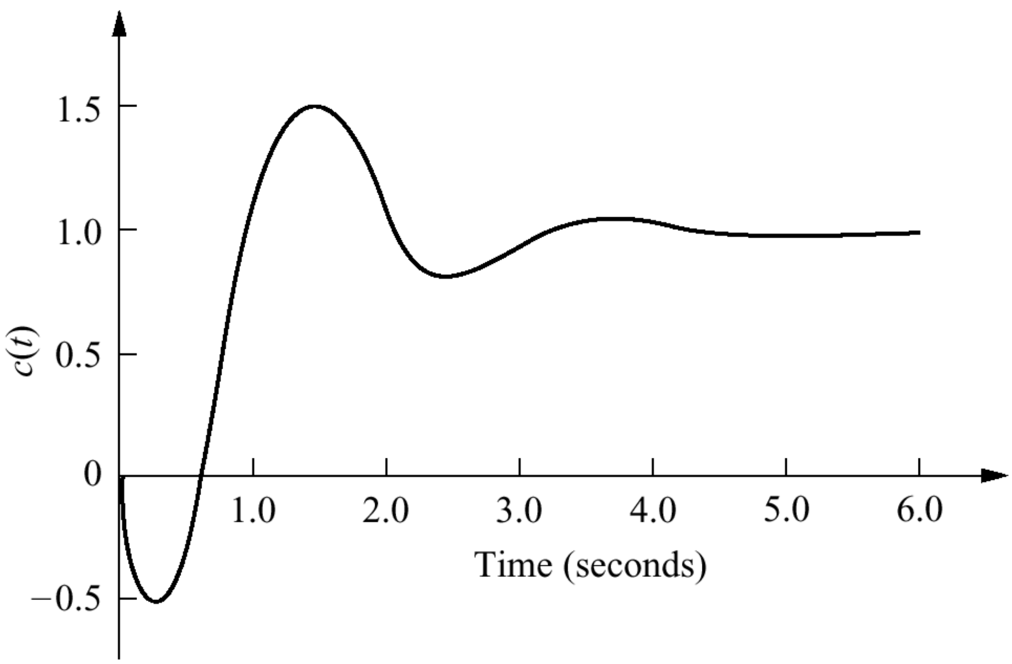
- This section studies the effect of adding a zero to the second order system.

- Consider a second order system with poles, $s_{1,2} = -1 \pm j2.828$. Adding zeros at -3, -5 and -10:



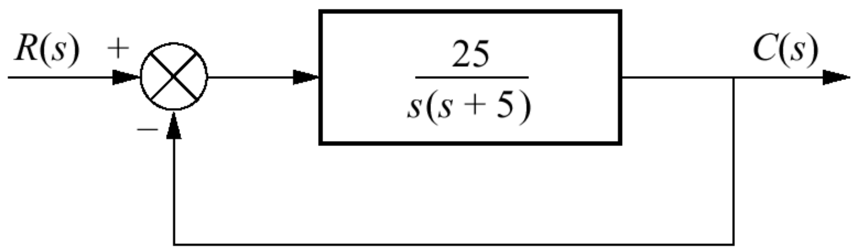
- The closer the zero to the dominant poles, the greater the effect on the transient response.

- Adding a positive zero yields a nonminimum-phase system. Undershoot occurs at the initial stage.



Example:

For the system shown below, find the peak time, percent overshoot and settling time.



Example:

Design the value of gain, K for the feedback control system so that the system will respond with a 10% overshoot.