

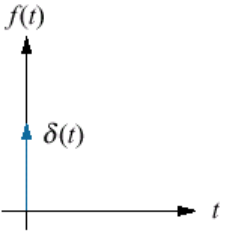
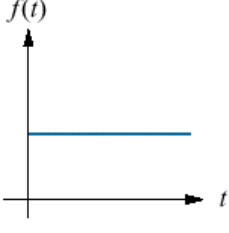
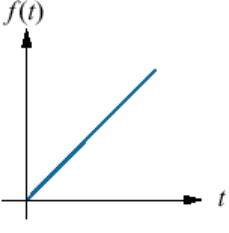
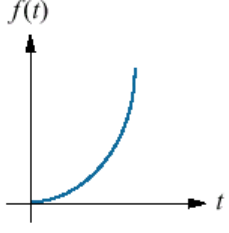
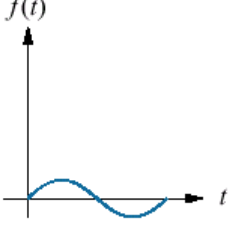
## TIME RESPONSE

- This topic involves analysis of a control system in terms of time response and steady-state error.
- At the end of this topic, the students should be able to:
  1. Recognize standard input test signal.
  2. Find and sketch the response of first, second and third order systems.
  3. Identify the poles and zeros of a transfer function.
  4. State the important specifications of a time domain response.
  5. Identify and state the order, type and steady state error coefficient given a transfer function.

## Standard Input Test Signal

- Test input signals are used, both analytically and during testing, to verify the design.
- Engineer usually select standard test inputs, as shown in Table 4.1: impulse, step, ramp, parabola and sinusoid.
- An *impulse* is infinite at  $t=0$  and zero elsewhere. The area under the unit impulse is  $1$ .
- A *step* input represents a *constant command*, such as position, velocity, or acceleration.
- Typically, the step input command is of the same form as the output. For example, if the system's output is position, the step input represents a desired position, and the output represents the actual position.
- The designer uses step inputs because both the transient response and the steady-state response are clearly visible and can be evaluated.
- The *ramp* input represents a *linearly increasing command*. For example, if the system's output is position, the input ramp represents a linearly increasing position.

Table 4.1

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t) dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

- The response to an input ramp test signal yields additional information about the steady-state error.
- The previous discussion can be extended to *parabolic* inputs, which are also used to evaluate a system's steady-state error.
- *Sinusoidal* inputs can also be used to test a physical system to arrive at a mathematical model (Frequency Response).

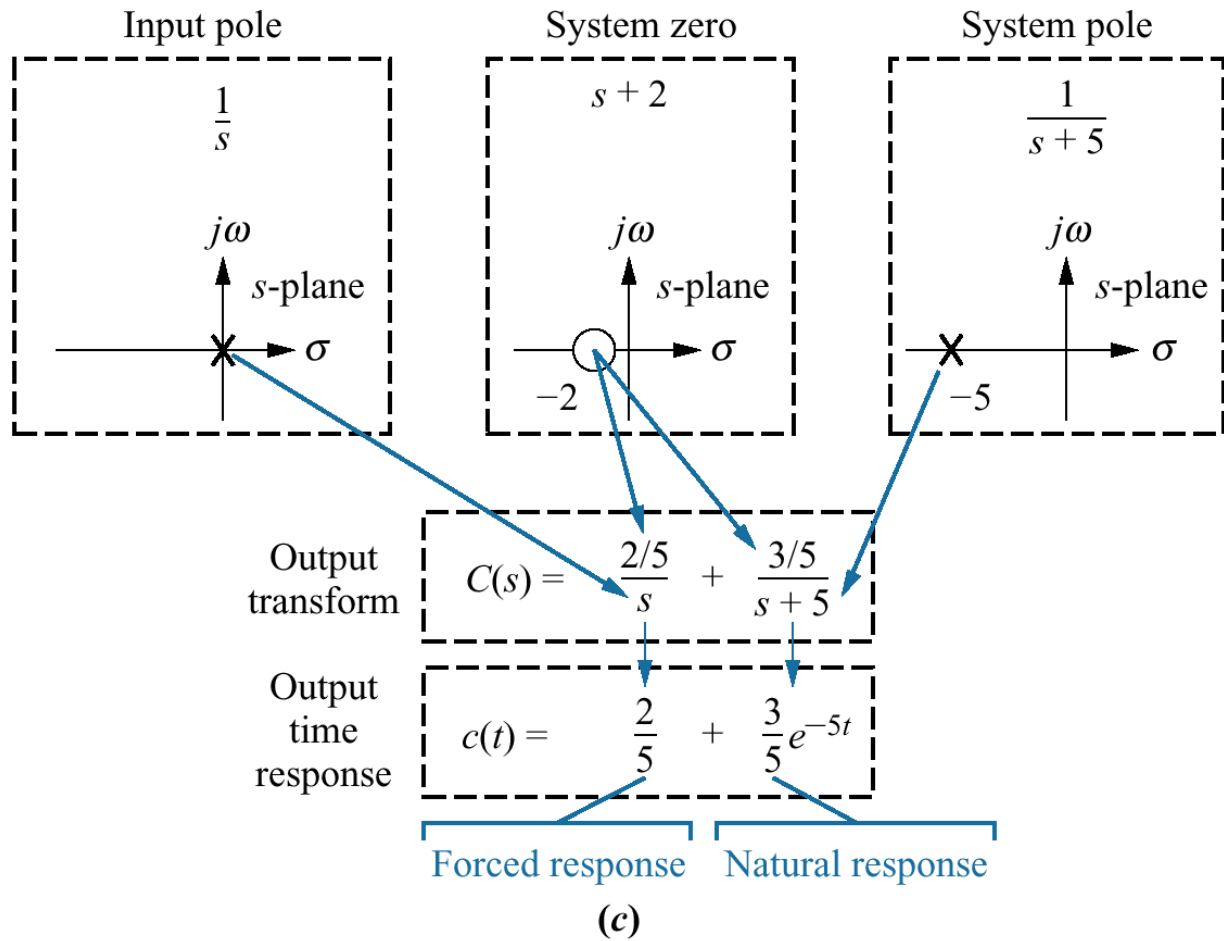
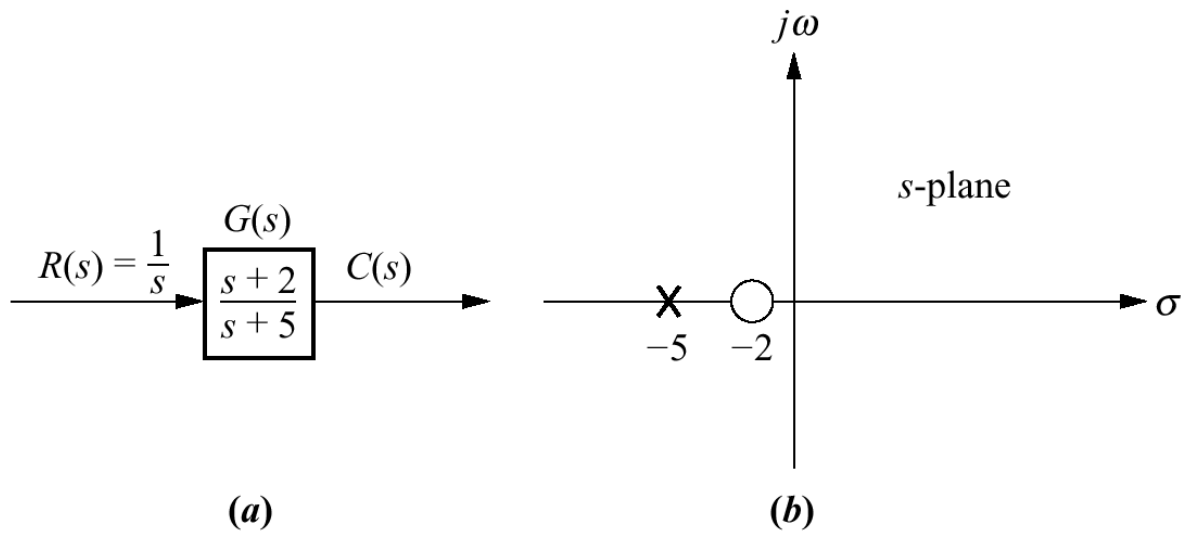
## **Poles, Zeros and System Response**

- The output response of a system is the sum of two responses: the *forced response* (*steady-state response*) and *the natural response*.
- It is possible to relate the poles and zeros of a system to its output response.
- The *poles* of a transfer function are:
  - The values of the Laplace transform variable,  $s$ , that cause the transfer function to become infinite.
  - Any roots of the denominator of the transfer function that are common to roots of the numerator.

- The *zeros* of a transfer function are:
  - The values of the Laplace transform variable,  $s$ , that cause the transfer function to become zero.
  - Any roots of the numerator of the transfer function that are common to roots of the denominator.

## **Poles and Zeros of a First-Order System**

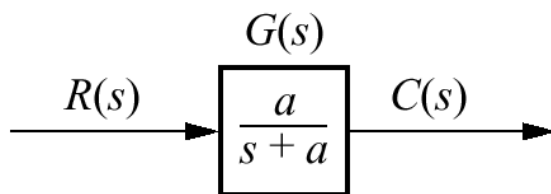
- Example: Given  $\frac{C(s)}{R(s)} = \frac{(s+2)}{(s+5)}$ . Find the pole and zero of the system and plot on the  $s$  plane. Find the unit step response of the system.



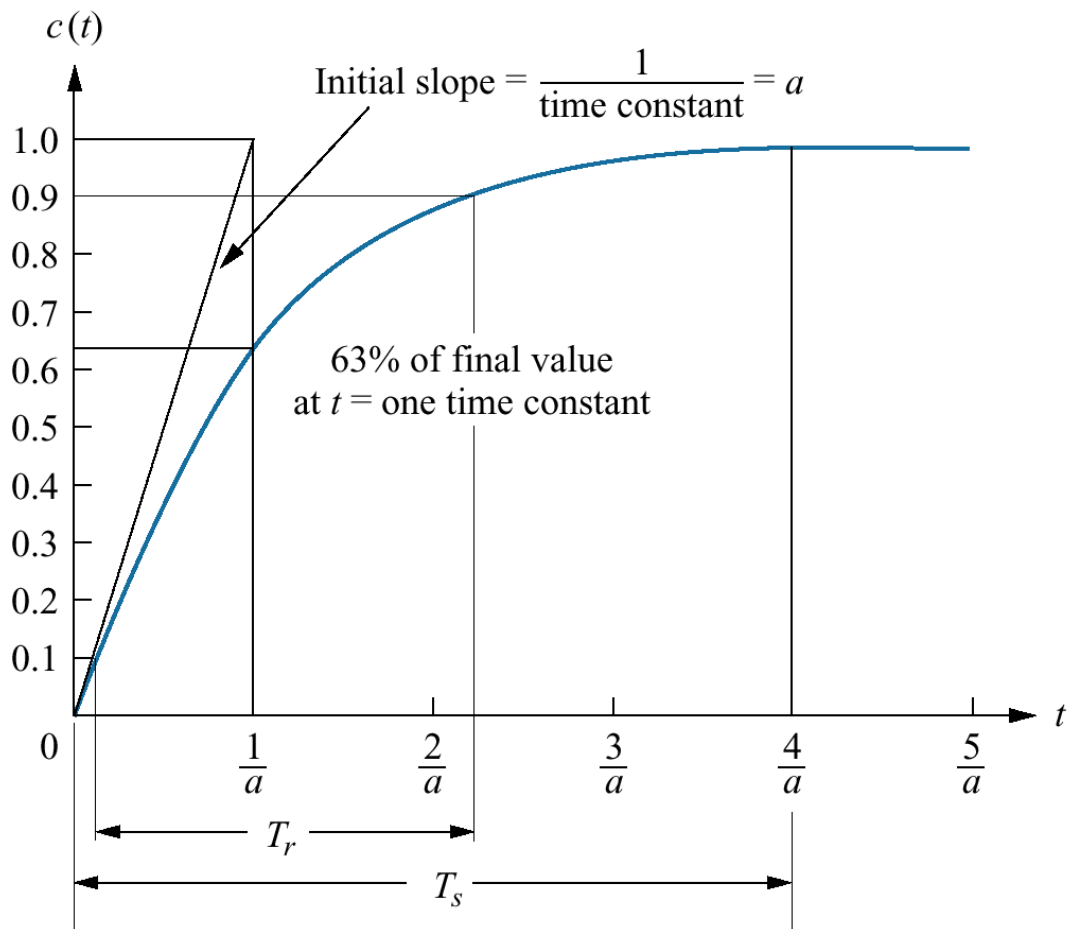
- Example conclusions:
  - A pole of the input function generates the form of the *forced response*.
  - A pole of the transfer function generates the form of the *natural response* (the pole at  $-5$  generated  $e^{-5t}$ ).
  - A pole on the real axis generates an *exponential* response of the form  $e^{-\alpha t}$ , where  $-\alpha$  is the pole location on the real axis.
  - The farther to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero.
  - The zeros and poles generate the *amplitudes* for both the forced and natural responses.

## First-Order Systems

- Consider a first order system,  $\frac{C(s)}{R(s)} = \frac{a}{s+a}$  with a unit step input



- Time response:





- At  $t=1/a$ ,

### **Specification: Time Constant**

- The Time constant,  $T_C$  is the time for the step response to rise to 63% of its final value.
- From the response,  $T_C = 1/a$ .
- The bigger  $a$  is, the farther it will be from the imaginary axis and the faster the transient response will be.

### **Specification: Rise Time**

- Rise time,  $T_r$  is the time it takes for the response to go from 0.1 to 0.9 of its final value.

## **Specifications: Settling Time**

- Settling time,  $T_S$  is the time for the response to reach and stay within, 2% of its final value.

## **First-Order Transfer Functions via Testing**

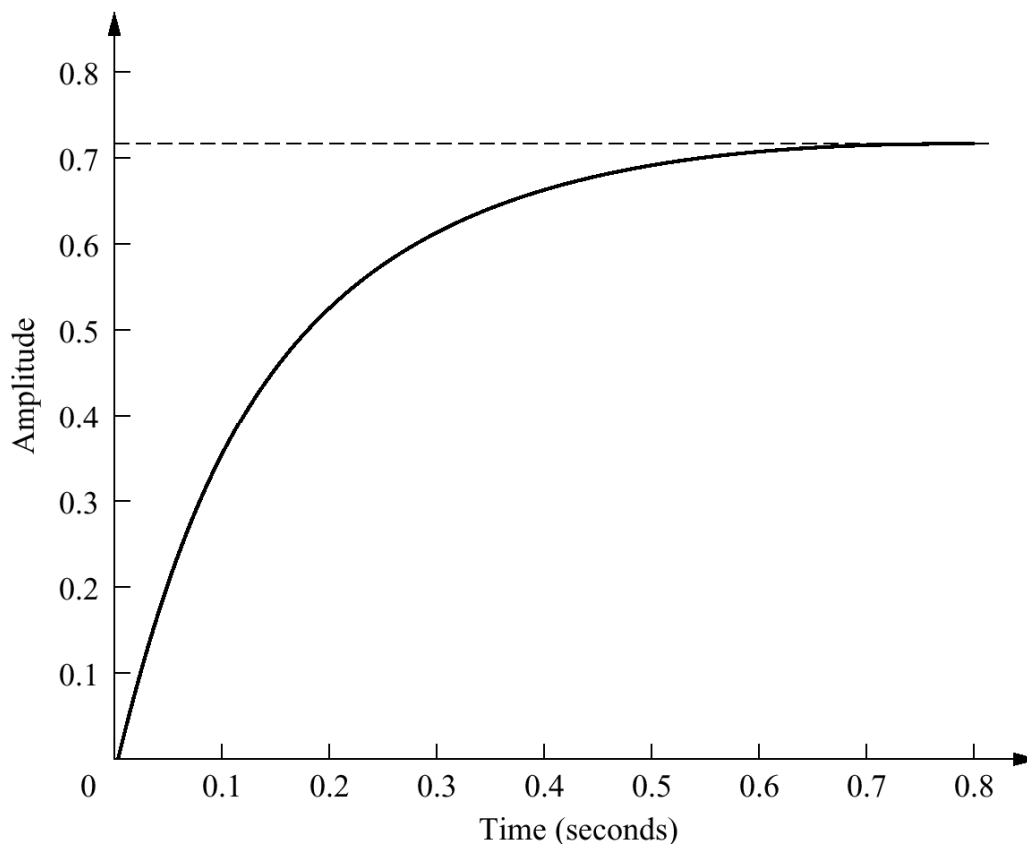
- Often it is not possible or practical to obtain a system's transfer function analytically (i.e. using techniques from last topic).
- We can have an approximation of a system's transfer function by looking at its step response.
- The actual system is fed with a step input, and its response recorded experimentally.
- Given that the step response is known, the transfer function can be found.
- Consider a simple first order system:

$$G(s) = \frac{K}{(s + a)}$$

$$\text{For step response, } C(s) = \frac{K}{s(s + a)} = \frac{K/a}{s} - \frac{K/a}{s + a}$$

$$\Rightarrow c(t) = \frac{K}{a} - \frac{K}{a} e^{-at}$$

- Hence, if we can identify  $K$  and  $a$  from laboratory testing, we can obtain the transfer function of the system.
- **Example**: Find the transfer function for the step response below:



- **Example**: A system has a transfer function,  
$$G(s) = \frac{50}{s + 50}$$
. Find the time constant, settling time and rise time. Sketch the unit step response of the system.

- **Example**: Sketch the unit step response for a system with transfer function,  $G(s) = \frac{100}{s + 50}$ .