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# Faded-Examples for Learning Contextual Mathematics Problem-Solving Skills

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**Abstract.** Some students may have learned the conceptual and procedural knowledge, however they might not be able to smoothly apply them simultaneously into problem solving in various situations. According to a CLT, faded-examples can assist students to develop more advanced problem solving skills. Faded-examples consist of completion problems followed by full problem solving. Completion problems are problem solving where the solution steps are partly shown, whereas full problem solving only consists of mathematics problem. There are two types of faded-examples, i.e. forward fading and backward fading types. In the forward fading, students are required to find the first step solution. In the second problem, students are asked to find the solution in the first and second steps, and so on, in such a way the last problem is to be fully solved by students. While the backward fading type is the reversed. Creating faded-examples should in accord with the principle of CLT, thus the faded-example instruction is efficient to study. Design of faded-examples is rarely found in mathematics textbooks. This paper attempts to describe possible design of faded-examples for learning year seven contextual problem related to algebra. Characteristics of faded-examples for learning mathematics contextual problem solving are discussed in this paper.

## 1. Introduction

Cognitive load can be viewed as a very important factor when designing effective instructions. When there is a high cognitive load, the working memory used becomes less effective and less relevant in the learning activities. To gain control over the cognitive load, the load needs to be measured. The ability to measure a cognitive load of a task forces learners to design effective instruction and can gain more knowledge into the underlying processes of cognitive load.

Cognitive load occurs when a person needs to process information in working memory. For example, during the task, test, or instruction given. Because of limited working memory, high cognitive loads may occur so that the acquisition and automation of schemes becomes impeded when one has to put too much mental effort into learning activities [1].

There are three types of cognitive load, namely intrinsic cognitive load, extraneous cognitive load, and germane cognitive load. [1]. The first is intrinsic cognitive load. That is the cognitive load that depends on the complexity of matter. The second is the extraneous cognitive load determined by the presentation technique of the material [2]. With the presentation of appropriate material will be able to



reduce extraneous cognitive load. However, if in the presentation of the material too much split attention and redundancy, then extraneous cognitive load will be high. The germane cognitive load occurs when there is a charge in the empty working memory due to the minimal intrinsic and extrinsic cognitive loads. This process can be influenced by students' motivation and attitudes toward the material being studied [3]. If the students' motivation and attitudes are not good to the learning process, although the material has been well conceived, it may result in less than optimal learning outcomes.

Learning is a cognitive process for constructing knowledge in working memory and stored permanently in long-term memory. The theory that develops instructional designs by considering the capacity of these two memories is Cognitive Load Theory (CLT) developed by John Sweller from Australia. Students with low initial ability can be facilitated with instructions in worked-examples. As for, students with high initial ability, can be facilitated with instruction in problem-solving. Many students have learned some knowledge but limited ability to transfer that knowledge into problem solving. CLT recommends faded-example as a learning design for such students. Faded-examples can be said to be a transition of cognitive skills, from low skills to high cognitive skills. With faded-examples, students with low or high initial skills can be facilitated in the learning process. By increasing demands and eliminating clues gradually, students will master problem-solving competencies better [1].

## **2. Problem-Solving**

Problem-solving in mathematics learning is thought to Polya. Polya proposed four troubleshooting steps, 1) understanding the problem, 2) planning the settlement, 3) solving the problem, and 4) re-checking [4]. As for, problem-solving is an important part in learning mathematics, so it should not be released from the learning of mathematics [5].

Problem-solving is to find the right way to achieve a goal [6]. A well-formulated problem is one that has a remedial solution using a particular method to find it, whereas a poorly formulated problem is to have more than one solution, ambiguous goals (unclear, and generally unapproved strategies for solutions) [7]. Attempts to solve mathematical problems, teachers can help students understand a problem-solving model that can be applied in different domains. Moreover, teachers can give students an understanding that math is widely applied in everyday life. Unlike most students think that math is something very abstract, for example in algebraic material.

With instructional formats that reduce guidance or support, such as problem-solving practices or exploration of learning environments, may be more efficient for relatively advanced learners [1]. For learners with higher abilities, problem-solving without the help of worked-examples may be superior during the next skill phase. According to the expertise reversal effect, worked-examples are less effective for learners with superior capabilities because detailed instructional guidance of the type is available and there is integration with existing knowledge structures in the long term memory of the learner, may require additional cognitive resources to impose unnecessary cognitive loads [1, 2]. To bridge the problem-solving practice and worked-examples, faded-examples can be used [1, 8].

## **3. Faded-Examples**

There are two types of faded examples: backward fading and forward fading [9]. In backward fading, students are required to try to find a solution in the last step on problem 1, the last two steps on problem 2, and so on. In other words, students are required to continue the steps given to solve the problem. As for forward fading has the steps as in the backward fading. The difference is, in backward fading, students are required to try to find the first step solution in problem 1, the first two steps on problem 2, and on the next problem, the students solve their own solution of the given problem. Steps in backward-fading can cause lower cognitive loads than forward fading. A grant of early troubleshooting steps is often an important step for the overall solution. However, further research shows that faded-examples result in fewer unproductive learning processes, resulting in better learning outcomes. Thus, there may be no specific recommendations regarding both types of faded-examples.

The selection of appropriate fading procedures depends on their use in the matter matter, to explain the type of workmanship solution to the structure and content of the specific material to be studied [1].

The following are given grid and sample questions with faded-examples instructions, both forward fading and backward fading type, about the application of algebraic forms and operations in solving problems related to rectangles in everyday life. This question is given to students who have studied algebraic forms and operations. The goal is that students can solve algebraic problems more fluently, especially on rectangular and algebraic material.

Also, students can learn more about forming algebraic equation and the operation. For students who are in junior high school in the early years, algebra is a new and fundamental material for learning the next material. More broadly, during learning this topic students can explore the role and application of mathematics in everyday life.

### 3.1. Forward fading type

The following table is a rectangular grid problem of daily life with forward fading type. There are five contexts used to develop the faded examples (see Table 1). The task is to understand how to find area of rectangular shape given the circumference.

**Table 1.** Grid Problem with Forward-Fading Type.

Number	Context	Activity
1	Pool	Given the circumference of pool. Asked the area of pool
2	Paperboard	Given the circumference of paperboard. Asked the area of paperboard
3	Oil Palm Plantation	Given the circumference of oil palm plantation. Asked the area of oil palm plantation
4	Field	Given the circumference of field area. Asked the area of field.

Below is the forward fading type that must be completed by the students. There are four problems where the first is a task accompanied by an answer with one initial settlement step omitted. Second is a task accompanied by an answer with three initials settlement step omitted. Third is task accompanied by an answer with five initials settlement step omitted. Forth is problem solving. The faded examples are presented in tables so students can match the steps and the description of the mathematics operation.

Task: learn how to solve the problem of applying algebra for rectangles by completing the following problem solving:

1. A pool has a length  $(x + 2)$  m and width  $(x - 2)$  m. If the circumference is 60 m, what is the area of the pool?

Answer:

Steps	Descriptions
Step 1 Make algebraic form of the problem	
Step 2 Specifies the value of a variable	$\Leftrightarrow x = \frac{60}{4}$ $\Leftrightarrow x = 15$
Step 3 Determine the necessary elements and find the solution to the problem	$\text{Length} = x + 2 = 15 + 2 = 17$ $\text{Width} = x - 2 = 15 - 2 = 13$ $\text{Area} = \text{length} \times \text{width}$ $= 17 \times 13 = 221$
Step 4	Thus, the area of the pool is 221 m <sup>2</sup> .

Conclude

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2. A paperboard has a length  $(3x + 10)$  cm and width  $(3x - 10)$  cm. The circumference of the paper is 120 cm. What is the area of the paperboard?

Answer:

Steps	Descriptions
Step 1	
Step 2	
Step 3	$\begin{aligned} \text{Length} &= 3x + 10 = 3 \times 10 + 1 = \\ &= 40 \end{aligned}$
Determine the necessary elements and find the solution to the problem	$\begin{aligned} \text{Width} &= 3x - 10 = 3 \times 10 - 10 \\ &= 20 \\ \text{Area} &= \text{length} \times \text{width} \\ &= 40 \times 20 \\ &= 800 \end{aligned}$
Step 4	Thus, the area of paperboard is 800 cm <sup>2</sup> .

Conclude

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3. The circumference of Albert's oil palm plantation is 50 m. The length and width of each garden are  $(4x + 1)$  m and width  $(x + 9)$  m. Determine the area of Albert's oil palm plantation.

Answer:

Steps	Descriptions
Step 1	
Step 2	
Step 3	
Step 4	Thus, the area of Albert's oil palm plantation is 52 m <sup>2</sup> .

4. A rectangular field has a length  $(2x + 5)$  m and width 9 m. If the circumference of the field is 40 m, then calculate the area of the field.

Answer:

Steps	Descriptions
Step 1	
Step 2	
Step 3	
<b>Step 4</b> Conclude	

- Conclusion: to solve a rectangular problem related to algebra, manipulate the given form by substitution, distribution, addition, subtraction, multiplication, and division of algebra operation [9].
- Create a rectangular problem related to algebra, similar to the above problems, and then solve it. If it is still to hard, try to solve it again and give an explanation of each step of the solution [9].

### 3.2. Backward Fading Type

Furthermore, a rectangular grid problem of daily life with backward fading type is given in the Table 2. There are five varied contexts can be used to develop this faded example based instruction. The task is to find area or circumference based on given situations, similar to the forward fading type discussed above.

The difference between forward fading and backward fading types lies in the algebraic form in the problem. For forward fading type, length and width of rectangular have been given in algebraic form. While for backward fading type, students must create algebraic form based on the given problem.

**Table 2.** Grid Problem with Backward-Fading Type

Number	Context	Activity
1	Corn field	Given the circumference of cord field. Asked the area of corn field
2	Land	Given the circumference of land. Asked the area of land
3	Workbench	Given the circumference of workbench. Asked the area of workbench
4	Rice field	Given the circumference of rice field. Asked the area of rice field.

Below is the backward fading type that must be completed by the students. There are four problems where the first is a task accompanied by an answer with one final settlement step omitted. Second is a task accompanied by an answer with three final settlement steps omitted. Third is task accompanied by an answer with five final settlement steps omitted. Forth is problem solving. The faded examples are presented in tables so students can match the steps and the description of the mathematics operation.

Task: learn how to solve the problem of applying algebra for rectangles by completing the following problem solving:

1. The corn field of Mr. Alvin has a circumference of 60 m and a length of 2 m more than its width. What is the area of the cornfield of Mr. Alvin?

Answer:

Steps	Descriptions
Step 1 Make algebraic form of the problem	Let the length of Mr Alvin's corn field is $p$ and the width is $l$ . Given $p = 2 + l$ . We get $60 = 2 \times (p + l)$ $\Leftrightarrow 60 = 2 \times ((2 + l) + l)$ $\Leftrightarrow 60 = 2 \times (2 + 2l)$ $\Leftrightarrow 60 = 4 + 4l$
Step 2 Specifies the value of a variable	$\Leftrightarrow 60 - 4 = 4l$ $\Leftrightarrow 56 = 4l$ $\Leftrightarrow l = \frac{56}{4}$ $\Leftrightarrow l = 14$
Step 3 Determine the necessary elements and find the solution to the problem	Length = $p = 2 + l = 2 + 14 = 16$ Area = $p \times l$ $= 16 \times 14$ $= 224$
Step 4 Conclude	

2. The circumference of a land is 70 m and the width of 5 m is less than the length. What is the area of the land?

Answer:

Steps	Descriptions
Step 1 Make algebraic form of the problem	Let the length of land is $p$ and the width is $l$ . Given $l = p - 5$ . We get $70 = 2 \times (p + l)$ $\Leftrightarrow 70 = 2 \times (p + (p - 5))$ $\Leftrightarrow 70 = 2 \times (2p - 5)$ $\Leftrightarrow 70 = 4p - 10$
Step 2 Specifies the value of a variable	$\Leftrightarrow 70 + 10 = 4l$ $\Leftrightarrow 80 = 4l$ $\Leftrightarrow l = \frac{80}{4}$ $\Leftrightarrow l = 20$
Step 3 Determine the necessary elements and find the solution to the problem	

Step 4

- 
3. A workbench has a length of twice the width. The circumference of the workbench is 12 m. What is the area of the workbench?

Answer:

Steps	Descriptions
Step 1	Let the length of land is $p$ and the width is $l$ . Given $p = 2l$ .
Make algebraic form of the problem	We get $12 = 2 \times (2l + l)$ $\Leftrightarrow 12 = 2 \times 3l$ $\Leftrightarrow 12 = 6l$ $\Leftrightarrow l = \frac{12}{6}$

Step 2

Specifies the value of a variable

Step 3

Step 4

- 
4. The rice field of Mr. Paul has a circumference of 24 m with its width is a half the length. Calculate the area of Mr. Paul's rice fields.

Answer:

Steps	Descriptions
Step 1	Make algebraic form of the problem

Step 2

Step 3

Step 4

- 
5. Conclusion: to solve a rectangular problem related to algebra, manipulate the given form by substitution, distribution, addition, subtraction, multiplication, and division of algebra operation [9].



6. Create a rectangular problem related to algebra, similar to the above problems, and then solve it. If it is still too hard, try to solve it again and give an explanation of each step of the solution [9].

The forward fading or backward fading solution steps and descriptions are omitted until at stage 4 where only 1 completion step is provided. In the final stage of learning, students are expected to conclude on their own the steps undertaken to solve problems of the same type and apply them to solve other problems. It is designed that students can also provide examples of similar problems that can be solved using the same steps and descriptions.

If necessary, a complete example of problems and solutions can be provided at the beginning of the problem, then in the second part, the problem is provided as faded examples. This might be more effective for students who have not developed sufficient knowledge. By eliminating clues in stages and increasing the completion tasks, students are expected to master the problem-solving competence. By this way, the designed faded examples may assist students to be more fluent in solving such contextual problems.

#### 4. Conclusion

When students solve new or complex problems, working memory will gain a high demand of cognitive processes because insufficient prior knowledge is shared from a long term memory. This student can be facilitated with instruction based on worked-examples for learning a specific problem solving. After they develop sufficient knowledge, teachers can facilitate them with an implicit instruction such as problem solving based instruction. When students have gained some prior knowledge but in limited use, an alternative is an instruction based on faded-examples, either using backward fading or forward fading strategies. To create faded-examples, instructor should find problem solving that has similar conceptual and procedural knowledge base. Therefore, the problem solution is similar and efficiently faded through the sequence of the problem solving. By faded-examples, students are expected to improve the ability to solve problem with various domain.

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