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# Designs of goal-free problems for trigonometry learning

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**Abstract.** This paper describes the designs of goal-free problems particularly for trigonometry, which may be considered a difficult topic for high school students. Goal-free problem is an instructional design developed based on a Cognitive load theory (CLT). Within the design, instead of asking students to solve a specific goal of a mathematics problem, the instruction is to solve as many Pythagoras as possible. It was assumed that for novice students, goal-free problems encourage students to pay attention more to the given information and the mathematical principles that can be applied to reveal the unknown variables. Hence, students develop more structured knowledge while solving the goal-free problems. The resulted design may be used in regular mathematics classroom with some adjustment on the difficulty level and the allocated lesson time.

## 1. Introduction

Cognitive Load Theory (CLT) is an instructional design theory based on our knowledge of human cognitive structure. CLT theory suggests that instructional designs must consider the characteristics of cognitive structure to facilitate efficient learning [1]. CLT is developed based on the assumption that when novices receive complex learning material, their working memory becomes limited in capacity and duration because they do not have sufficient knowledge base to organize the material [1-3]. However, long-term memory is unlimited storage of knowledge that might be retrieved into working memory [4]. The CLT focuses on learning techniques that assist learners to be able to use their cognitive resource in such a way learning does not burden working memory and schema are constructed and structured in long-term memory.

Goal-free problems are one of several instructional designs developed based on CLT. Ayres investigated that goal-free problems minimize extraneous cognitive load and improve transferability in problem-solving [4, 5]. The goal-free problems are characterized by replacing a specific question, such as in geometry: "evaluate the length of the segment a" by an open question: "evaluate the length of all unknown segments". It is found that using goal-free problems; students solve the problem from understanding the given information to the unknown variables, instead of using means-ends analysis or moving backward strategy. Solving a problem by understanding first the given information and using this to create understanding leads students to focus their cognitive resource to learn meaningfully. A student who learned by goal-free problems on angles formed by two parallel and transverse



lines have a better transfer ability scores compared to students who learned goal-given problems[6].

Cognitive load describes the amount of the cognitive process in the working memory should the learner takes into account when dealing with the to-be-learned material. Sweller, Ayres, and Kalyuga [1] explain that there are two types of cognitive load, firsts intrinsic cognitive load (the load imposes by the content of the information that learner needs to organize) and extraneous cognitive load (the load imposed by the presentation of the learning material). Moreover, Sweller [7] suggested that intrinsic cognitive load is determined by the level of interactivity of elements in the learning material. A difficult material although seems simple may have a high level of element interactivity and therefore may cause high intrinsic cognitive load. A new material presented to novices may impose heavy intrinsic cognitive load because they do not have sufficient prior knowledge to perceive and to organize the elements of the material. Also, when complex learning material has two sources, both must be presented integratively to lower extraneous cognitive load [1, 7]; because the cognitive process devoted to handling such separated information is unnecessary. Extraneous cognitive load also occurs when redundant information is presented. Eventually, both intrinsic and extraneous cognitive load must be considered to facilitate effective and efficient learning. In particular, the extraneous cognitive load should be minimised by designing instructions thoughtfully.

Mathematics in particular, problem-solving is an important competence in life [8, 9]. Jonassen [10] observed that problems solving are regarded as the most important cognitive activity in everyday and professional contexts. Problem-solving may be defined as an attempt to reach specific goals which cannot be performed automatically [11, 12]. In mathematics classroom, problem-solving is also referred as a learning strategy by which students develop their knowledge while solving given problems. According to the expertise reversal effect [1], nevertheless, this learning strategy is most efficient for students with sufficient knowledge base. When the relevant previously learned knowledge is available, these students will be able to use the knowledge to solve the given problem. Furthermore, with sufficient knowledge base when students have the opportunity to learn by challenging problem-solving, students will be able to improve their ability to transfer their knowledge into more advanced problem-solving [13].

Problem-solving activity may have a different cognitive load to the different level of learners. As was mentioned above, the expertise reversal effect [1] explains that students with insufficient knowledge base are unlikely to benefit from the problem-solving instruction. Based on cognitive load theory, students will use a random search method such as trial and error, means-ends analysis or working backward to find the problem solution. It has been argued that discovering the solution of the problem by this method is not efficient since it causes a heavy extraneous cognitive load [1, 5]. Students may find the solution to the given problem. However, their attention is not directed towards understanding the underlying knowledge and creating new knowledge structure.

Ayres and Sweller [5] have investigated the advantageous of goal-free problems for novice learners. They used geometry problems for applying the angle theorems for a triangle and parallel lines cut by a transversal line. The instruction was that students have to find as many as possible the measurement of the unknown angle. The goal-free problems do not provide a specific goal as the problem question. In their experiments, they found that students were able to work forward from the given information to the unknown angles. Instead, students who used conventional problem solving tended to create sub-goals from the given problems and

attempted to use means-ends analysis. The test results showed that students learned better from the goal-free problems compared to the conventional problem-solving.

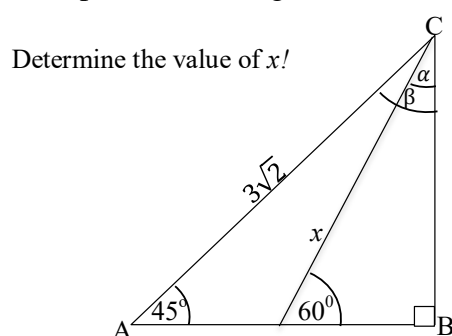
The current article aims to propose designs of goal-free problems for learning trigonometry. In mathematics, trigonometry is a topic that may not be easy to study. Trigonometry problem solving is mostly difficult to understand because it requires appropriate comprehension of the triangle and the trigonometric ratios. Using the perspective of cognitive load theory, the designs are described the effectiveness including how to implement the instruction in the trigonometry lesson.

## 2. Methods

### *Design of the Goal-free Problem*

The goal-free problem may be categorized as a problem solving based instruction. Sweller, Ayres, and Kalyuga [1] describe that goal-free problem is designed by creating a problem solving without a specific goal. Typical goal-free problems have several possible sub-goals to be created from the given information, which allow learners to apply their knowledge about the given information and to solve the possible unknown variables. An example is taken from Ayres and Sweller [4] in the angle problem. Instead of asking students to solve a specific angle measure, they asked students to calculate as many unknown angles as they can. This general instruction eventually lead students to focus on what can they do based on the provided information and the problem statement (solve the problem using a working forward strategy).

An example of a contextual trigonometry problem for high school is "A boy is looking up the top of a tree. His height is 1.2 meters and standing as far as 15 meters from the tree. If his sight makes an elevation angle of 45 degrees, how tall is the tree?". To answer such problem, a student usually draws a diagram to illustrate the context, which results in a right-angled triangle. If the student has acquired the knowledge base, that is the basic trigonometry ratios and the concept of angle of elevation, then most likely the problem can be solved. However, with the insufficient knowledge base, the student may end up by using guess and check method or using a scale to best estimate the height of the tree, without necessarily understanding or applying the trigonometry knowledge. A more complex problem-solving in this trigonometry topic may be represented in Figure 1 below.



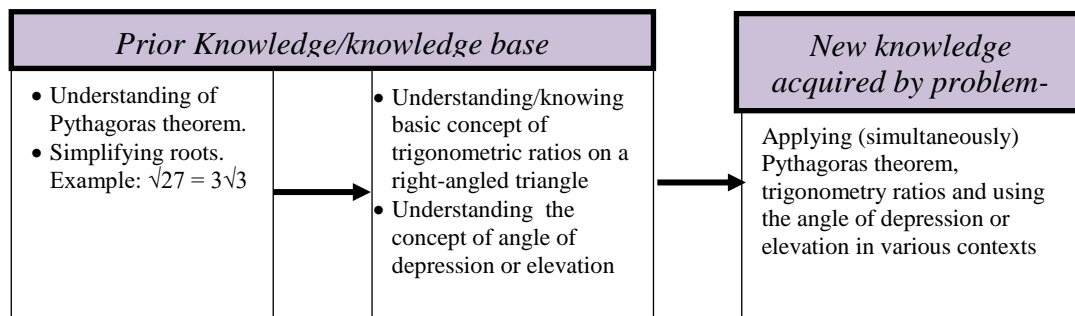
**Figure 1.** A goal-given problem in trigonometry

To solve the problem in Figure 1, students are required to have a knowledge base of trigonometry ratio and Pythagoras theorem. First, students have to find the length of  $AB$  using the cosine ratio on angle  $45^\circ$ . Second, using the Pythagoras theorem,  $BC$  can be calculated.

Finally, using the sine ratio on angle  $60^\circ$ , the value of  $x$  can be determined. Additionally, good knowledge of rational numbers and the operations benefits students because applying the Pythagoras theorem will yield in roots. The following Figure 2 describes the learning schema that shows the knowledge structure relevant to the above problem-solving.

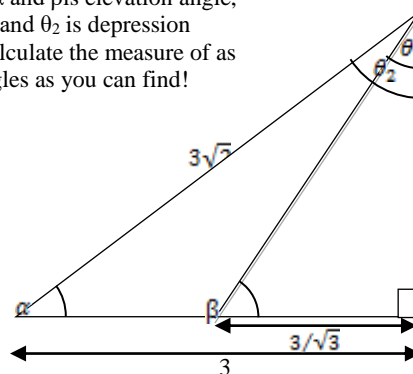
As discussed in the introduction section, solving a complex problem when learning imposes a high extraneous cognitive load and hence hinders the acquisition of new knowledge, particularly for novice learners [1, 2, 3]. As suggested, designing the problem-solving instruction using the format of a goal-free problem would assist novices more efficient learning [4, 5, 6]. Accordingly, if the trigonometry problem solving is to learn and solve by novice students, then presenting it in a goal-free problem is recommended. Specifically, novice students are those who do not have sufficient knowledge base (see Figure 2). These novices do not know much about either the Pythagoras theorem, basic trigonometry ratios (sine, cosine, tangent) or the concept of elevation or depression angles. Not only does the problem solving require each this knowledge but also how to apply them simultaneously. Therefore, solving a complex problem without prior knowledge is hardly possible.

The design of the goal-free problem proposed in this article can be seen in Figure 3. This problem is a modification of the problem shown in Figure 1. While in Figure 1, an  $x$  is determined as the given goal of the problem, in Figure 3, the  $x$  is removed. Instead, the problem question is “Calculate the measure of as many angles as you can find”. Students are directed to understand the given information (length of the triangle sides) and focus on how to understand the concept of elevation/depression angles and to apply the trigonometry ratios.



**Figure 2.** Schema or structured knowledge relevant to the trigonometry problem solving

See that  $\alpha$  and  $\beta$  is elevation angle, while  $\theta_1$  and  $\theta_2$  is depression angle. Calculate the measure of as many angles as you can find!



**Figure 3.** Goal-free problem solving on trigonometry

The problem solution of the above goal-free problem is the measures of angles  $\alpha$ ,  $\beta$ ,  $\theta_1$ , and  $\theta_2$ . From the given information, first of all, students can find the measure of angle  $\alpha$  using the cosine ratio. Students may try to find the vertical side first using the Pythagoras theorem and then apply the cosine ratio to calculate the measure of angle  $\beta$ . Then, using the sine ratios, the measures of angles  $\theta_1$  and  $\theta_2$  can be calculated. As noted that a goal-given problem asks a student to solve a particular variable (the final goal), consequently novice students tend to use a backward strategy to solve the problem by creating sub-goals from the final goal to close the gap with the provided information. The goal-free problems not only direct novices to develop their previously learned knowledge into more complex knowledge by avoiding working backward, but also enable students to reveal more variables in the problem from what is known.

### 3. Result and Discussion

The implementation of the goal-free problem-based instruction is similar to the conventional problem-based instruction. In the beginning, the teacher must make sure that students are novices because this instruction is most suitable for novices. Nevertheless, to ensure that the students have sufficient knowledge base, a phase to study the knowledge base or to strengthen the knowledge base is required. Because the knowledge base is rather simple, a direct instruction such as question-answer method may be implemented.

In the main acquisition phase, some goal-free problems in the topic are provided in a worksheet. Students are instructed to complete the worksheet in the allocated time. The instructor must be able to estimate the right time needed to complete all the goal-free problems. It should be underlined that the objective of the instruction is to assist students acquiring the complex knowledge of problem-solving in the topic. To support this learning, students must have appropriate time to do the problem solving and accomplish the learning goal. In this case, individual learning is suggested than collaborative learning [6]. By studying the goal-free problems individually, students are more focused on applying their understanding of the provided information to solve and learn the unknown variables by working forward [14-16].

It is commonly said that collaborative learning, where a small group of students studying together, is better than individual learning [17;18]. Collaborative learning may provide many benefits to students, such as to develop communication or interaction skills. Collaborative learning can encourage students to communicate with each other and hence creating a problem-solving environment that is more enjoyable [8]. Several research indicated how collaborative learning could be superior to individual learning [9, 18-21]. There is a strategy of allocating students in a small group which could minimize the extraneous cognitive load during group work by distributing the cognitive load to all members of the group [19, 20].

Moreover, collaborative learning is also assumed to encourage students to explain to others. Explaining to the others may develop understanding [18]. It should be noted. However, this takes effect when the learning materials are complex and suitable for discussion in the group [21]. Nonetheless, it has been suggested by the author the goal-free problems are suggested to be learned individually rather than collaboratively [5], because empirical evidence for the effectiveness of the collaborative learning, when the goal-free problems are used, is still needed.

#### 4. Conclusion

The design of goal-free problem for learning a trigonometry problem solving may be proposed by modifying the instruction of the problem solving removing the specific goal. Students are not asked to solve a particular side or angle using the trigonometry ratios, but instead to solve as many angles or side length as possible. The complexity of trigonometry problem solving is that it involves not only trigonometry ratios but also other knowledge bases, such as Pythagoras theorem or the concept of elevation and depression angles. How to illustrate the triangles would also increase the complexity. Therefore, to ensure the goal-free problems can be solved and learned effectively by novice students, they must have sufficient knowledge base. This can be facilitated by an introductory phase of learning to activate the knowledge base before the acquisition phase commences. It is suggested that individual learning for accomplishing goal-free problem is suggested. Eventually, the extraneous cognitive load may be imposed during learning should be considered by the instructor when deciding to facilitate students learning the goal-free problems individually or collaboratively.

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