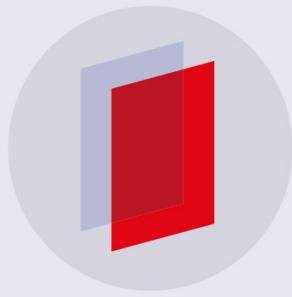


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# The influence of metacognition in mathematical problem solving

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**Abstract.** This paper is a review of ten papers about the relation of metacognition and mathematical problem solving. So, the aims of this paper is to analyze the influence of metacognition in mathematical problem solving at low, average, and high students' performance. Metacognition is an important factor of mathematical problem solving. Metacognition is the ability to monitor and control our own thoughts, how we approach the problem, how we choose the strategies to find a solution, or ask ourselves about the problem, in the other word, it can be defined as think about thinking. Solving mathematics problems requires analysis of the given problem, planning the strategy to be used to solve the problem, undertaking the planned strategy and checking whether the steps that have been done are correct. Therefore, metacognition is necessary for the successful solving of mathematical problems. This paper analyzes that the higher metacognition that students have, the better mathematical problem solving that students can do.

## 1. Introduction

Mathematics has an important role in various sciences and necessary to master mathematics since early stage because it underlies the development of modern technology and can advance the human mind. Therefore, the subjects of mathematics are subjects given at every level of education ranging from basic education to higher education. One of the goals of mathematical education was mathematical problems solving [1]. Problem solving skill is the most important cognitive activity and part of mathematics. [2,3,4]. In addition, problem solving skill is a cognitive process in finding solutions to a given problem [5,6]. Therefore, students need to improve their skills in solving problems. According to mathematicians, the problem of mathematics is a list of questions to be answered [7]. However, not all problems will be a problem. Even the problems we face today are not necessarily a problem either later or tomorrow. As said by [8] that something that becomes a problem is no longer a problem if we have found a solution. Problems are defined as a situation where the goals to be achieved and the steps to achieve those goals do not look as straightforward. This statement is in line with the opinion of [9] who said that the characteristics of the mathematical question (matter) that is said to be a problem is if the problem can not be solved directly. Problem solving is not enough just to mimic how to solve problems that students have known. Students should undertake additional efforts, such as modifying the problem in a familiar way of solving it, solving the problem into several known problems or, re-formulating the non-routine problem into a known (familiar) problem. In essence, mathematics is problem solving [10]. Problem solving is seen as a process to find ways that can be done to overcome problems that have never been faced. [11]. Understanding mathematics should be taught through problem solving [10]. Reference [10] also said that the curriculum wants



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tasks or activities based on the problem is a means to be developed in learning so that learning is the result of problem solving process. Problem solving skill is a key component of the curriculum and is also a basic skill (basic skill) mathematics [12].

The success of students in solving mathematics problems is not only supported by cognitive ability, but the students' self-control in solving mathematics problems process also has an effect [13]. The ability of students in self-control both emotionally and knowledge determine the success of students in the process of problem solving [14]. Monitor self-thinking and self-control on knowledge especially in problem solving with regard to metacognition meaning [15]. References [16] explain that metacognition emphasizes the importance of controlling cognitive mind during problem solving, so metacognition can help students understand concepts. References [17] states that in any cognitive transactions with human or non-human environments, various information processing activities will continue. Metacognition refers to active monitoring and regulation of cognitive processes to achieve a goal. In other words, metacognition is defined as thinking about thinking or cognition about cognition [18]. References [19] suggests that metacognition has two components, namely knowledge metacognition and metacognition skills. Knowledge of metacognition refers to knowledge of cognition such as knowledge of skills and a good work strategy for learning and is able to use skills and work strategies appropriately to achieve learning objectives [20]. Knowledge of metacognition has three aspects, namely 1) strategic knowledge, (2) knowledge of cognitive tasks, and 3) self-knowledge [21]. Reference [20] states that two components of metacognition are metacognition knowledge and metacognition regulation, each has sub-components as follows: 1) metacognition knowledge includes: a) declarative knowledge: knowing one's own abilities, strategies, skills and learning resources needed to achieve goals, b) procedural knowledge: knowing how to use prior knowledge in declarative knowledge, c) conditional knowledge: where a procedure, skill or strategy is used and where such things are not used, why a procedure proceeds and under what conditions it takes, and why a procedure is better than any other procedure, 2) metacognition regulation includes: a) planning: the ability to plan learning activities, b) information management strategy: the ability to organize information related to the learning process undertaken, c) comprehension monitoring: the ability to monitor the learning process and matters related to the process, d) debugging strategies: the ability of debugging strategies is a strategy used to correct wrong actions in learning, and e) evaluation: the ability to evaluate the effectiveness of its learning strategy, whether it will change its strategy, succumb to the situation, or end the activity.

## 2. Experimental Method

A search procedure was executed to find empirical studies of the influence of metacognition in mathematical problem solving. The search was performed with the keyword mathematics added with the specific keywords referring to the sub-domains in this study (metacognition and mathematical problem solving). In addition to mathematics, we used the following keywords in the abstract to define this search: mathematical problem solving, metacognition/metacognitive. Additionally, articles were added using the snowball procedure (using references in articles which had already been found). Studies were judged based on strict selection criteria. Studies were included in analysis when they required the following criteria: 1) the article is published in a scientific, 2) the study is executed in formal education, 3) the study explicitly aims to analyze the influence of metacognition in mathematical problem solving at low, average, and high students' performance, 4) the number of students in the study is appropriate for the analyses in the study. After the first selection, many studies were ruled out because they did not fit the criteria mentioned above. For instance: they were not about metacognition or mathematical problem solving or they were not executed in educational settings. After close reading, 10 studies were admitted for the analysis.

## 3. Result and Discussion

Some studies said that metacognitive give the influences to mathematical problems solving. The ten research results will be analyzed in this paper. Reference [22] illustrates the thinking skills, patterns of thinking skills, and the metacognitive function of learners in problem solving. Eighteen junior students take a Bachelor's degree in Secondary Education in Mathematics University of Bicol College is the

subject of research and is classified into two groups of abilities. When students are asked to reflect on their thinking as they think about the problem, their metacognitive activity is recorded and categorized into three metacognitive types namely metacognitive awareness (MA), metacognitive evaluation (ME) and metacognitive regulation (MR). There are six patterns of metacognitive action shown by the students. These are MA-ME, MA-ME-MR, MA-MR, MA-MR-ME, MR-MA-ME, ME-MR. All these metacognitive actions are employed by students in solving problems. Among the patterns, the most commonly practiced pattern is MA-ME exhibited 46 times, followed by MA-ME-MR occurring 14 times. Prior to the MA-operated thinking skills precede to assist the MA. In the same way cognitive activity occurs before or after ME or MR occurs. This supports the claim that metacognition and thinking ability can not be separated. One can never explain what is in his mind without referring to any thinking skill. Troubleshooting becomes an easy task if learners can know the content knowledge relative to the problem, which is being aware of the learning situation. But before solving the problem, they must use their thinking skills. Dealing with problems requires awareness of the learners. Such awareness involves what the problem is. Learners should involve their own knowledge of the tasks and interactions of their thinking skills. Reflection, or thinking about the thinking that takes place should continue to be used. That students should organize or monitor their own thoughts through the help of thinking skills may seem repetitive, but should be like that. The ultimate view of the model is to generate a learner's thinking capable of directing his thoughts and actions so as to be able to learn independently.

References [23] are carried out to identify and determine the impact of students' metacognitive skills on solving non-routine mathematical problems. The subject of this study was 304 students in Johor Bahru. The results show that students' ability to solve non-routine math problems is still very low. There are also significant differences in metacognitive skills in solving non-routine math problems between students in various categories. Students in the very high category provide excellent solutions and high metacognitive skills. The results of the statistical test analysis showed that there were significant differences between students in the very high and medium categories, very high and low categories, very high and very low categories, high and medium categories, high and low categories, categories. high and very low categories, medium and low categories and medium and very low categories. However, there were no significant differences between students in the very high and high categories and between students in the low and very low categories. It was found that problem solving abilities were directly proportional to metacognitive skills. This means that metacognitive skills affect their abilities. It means that if a students' metacognitive is high then students' problem solving abilities are also high.

References [24] have the aim of knowing the effect of using metacognitive strategies in mathematical problem solving abilities. Subjects in this study were 47 fifth grade students. The results showed that students in classes who used metacognitive strategies increased significantly both in math problem solving skills. The results showed that there were significant differences in mathematical problem solving abilities between students in the class using metacognition strategies and students in the control class. It can be concluded that metacognitive strategies can improve problem solving skills. Thus, metacognition can be used as a useful tool for developing problem solving abilities which is one of the objectives of the curriculum and plays an important role in students' academic development. Thus it is suggested that, all learning activities must include activities regarding metacognitive skills. This study supports that in the teaching of mathematical strategies in mathematics subjects improve the achievement of problem solving. This means that metacognitive strategies can improve mathematical problem solving abilities. So, metacognitive has a positive influence in solving mathematical problems.

Reference [16] aims to investigate the emergence of metacognition during solving mathematical problems. Subjects in this study were fifth grade students (ten years) of 20 students (10 men and 10 women). In open problems there are metacognitive activities that are often used, Meta of Procedural Knowledge. In addition, at the object level, student verbal reports are dominant in the Debugging strategy and Information Management Strategy. In authentic problems there are metacognitive activities that are often used, namely the Object Level in the Information Management Strategy. In complex problems there are metacognitive activities that are often used, namely the Meta-level of

Procedural Knowledge. At the object level, we cannot mention large differences in the appearance of metacognitive functions because their performance is very low. In general, fewer Object-level strategies appear than Meta-levels. It can be said that the Information Management Strategy, Understanding Understanding and Evaluation are more often used in authentic problems, Debugging Strategies are more often used in open problems, Planning strategies are almost the same for three types of mathematical problems, Strategies used by Procedural Knowledge are more often used in complex problems, The strategy used by Declarative Knowledge is almost the same for all three types of mathematical problems.

References [25] examine the relationship between metacognition and mathematical problem solving. The main component analysis on metacognition revealed that three metacognitive components, global metacognition, off-line metacognition, and effort attributions explained 66% to 67% of all major components in metacognition. The findings of this study support the use of off-line metacognition assessment (prediction and evaluation) to distinguish between students who have below-average mathematical problem-solving abilities, students who have the same mathematical problem-solving abilities, and students who have the ability solving math problems above average. So it can be said that metacognition influences students' mathematical problem-solving abilities.

Reference [26] is to see the role of metacognitive skills in solving mathematical problems. Another aim is to see the role of metacognition on routine and non-routine problems. Subjects in this study were a group of thirty-four students enrolled in discrete mathematics. In this study two problems were presented. Both problems were successfully solved by eleven students, fourteen students were only correct in solving one problem, nine students failed to solve both problems. There were significant differences between group 1 and group 2, group 2 and group 3 and group 3 and group 1 in solving the first problem. This means that the metacognitive activities of groups of students who can solve the first problem are higher than the group that fails to solve the first problem. There was a significant difference between group 1 and group 2, group 1 and group 3 in the second problem. This means that the metacognitive activity of the group of students who can solve the second problem is higher than the group that fails to solve the second problem. Therefore, high metacognitive activity causes students' mathematical problem-solving abilities are also high. Although the role of metacognition on routine and non-routine problems has conflicting results, metacognitive skills will be active in non-routine problems and can help in solving them. This means that if students' metacognitive is high, students' problem solving abilities are also high.

Reference [27] is to describe three components of metacognition, specifically knowledge, experience and skills, and see differences between students with learning disabilities (LD), low-achieving students (LA) and students with average achievement (AA) in solving mathematical problems (MWPS), as well as the relationship between these components and their influence on academic performance. The more metacognitive knowledge students possess, the more likely they will experience success. The more efficacious students' feel about their abilities to solve problems, the more likely they are to solve them correctly. Results of these analyses indicated that metacognitive knowledge (MK) was a significant predictor of MWPS performance for AA students. Examination of slope differences show that for AA students, the more metacognitive knowledge students possess, the more likely they will experience MWPS success. The slope difference between students with LD and AA students approached significance. This suggests that the relationship between MK and MWPS performance is different for students with LD. These students were lower in MK than AA students. Since the difference was not significant this may indicate that students with LD are able to report the declarative knowledge, but are unable to apply the procedural or conditional knowledge that allows them to effectively solve math word problems. Students with LD also had significantly lower metacognitive experience (ME) than AA students. For AA students there was positive but not significant relation between ME and MWPS performance. That is, the more efficacious students' feel about their abilities to solve problems, the more likely they are to solve them correctly. Interestingly, the relationship is the opposite for students with LD, that is, the more confident they are, the less likely they are to solve the problem correctly. This could be an indication of "illusions of understanding" whereby students with LD are overconfident in their abilities and therefore do not apply the

appropriate strategies or allocate enough resources to solve the problem correctly. It means that if a students' metacognitive is high then students' problem solving abilities are also high.

References [28] say that students are asked to tell about their metacognitive; use of metacognitive strategies in solving problems, problem solving processes and understanding of problems; and attitudes towards learning. Some of the findings that emerged were: (a) Normal flow students showed less metacognitive use compared to students from Express and special streams. (B) metacognitive strategies used by normal flow students tend to enter the "surface" type. (C) There is no significant difference between students from academic tracks that differ in the frequency of use of metacognitive strategies. (D) During the problem-solving process, students more often evaluate answers rather than monitor their understanding. (e) Students from various levels (Secondary 2, Secondary 4 and Pre-University) show that the use of metacognitive strategies in problem solving is the same. This means that if students 'metacognitive is high, students' problem solving abilities are also high.

References [29] assess metacognitive strategies on Algebraic problems to improve problem solving skills among students. Three groups of students have taken this course. Two groups of 86 students were randomly selected (two of three groups). The students involved were mathematics majors and mathematics education majors. The results show that there is a significant positive relationship between overall metacognitive strategies and mathematics achievement (ALGMA). However, the relationship between ALGMA and the three metacognition subscales (knowledge, planning and evaluation) is low, but still significant. And for algebraic problems, there is a significant positive relationship between overall metacognitive strategies and Algebra problem solving abilities (ALGPS). In addition, the correlation between ALGPS and the meta-cognition subscale (knowledge, planning, and evaluation) is weak, but significant. This means that if students 'metacognitive is high, students' problem solving abilities are also high.

Reference [30] is to examine the effects of both cognitive and metacognitive strategies on mathematical achievement and also to determine prediction models based on students' mathematical performance, algebra problem solving performance, meta-cognitive strategies, and cognitive strategies. The sample of this study is the first year students who take Algebra courses at one of the Malaysian state universities. Two groups of 86 students were randomly selected. The field of student study in this study is mathematics and mathematics education. The results show that there is a significant positive relationship between overall metacognitive strategies and mathematics achievement (ALGMA). However, the relationship between ALGMA and the three metacognition subscales (knowledge, planning and evaluation) is low, but still significant. In addition, the correlation between ALGPS and the meta-cognition subscale (knowledge, planning, and evaluation) is weak, but significant. This means that if students 'metacognitive is high, students' problem solving abilities are also high.

#### **4. Conclusion**

This can be seen from several studies which say that metacognitive strategies can improve problem solving skills, so students are accustomed to learning by using their metacognition. Some studies also say that students who have high metacognition also have high problem-solving abilities. This results in students who have or use their metacognitive optimally will have good problem solving abilities. The conclusion of this research is metacognition has positive influence to mathematical problem solving. The higher metacognition that students have, the better mathematical problem solving that students can do.

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