

Generalized Space-Time Autoregressive Modeling

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Abstract. Generalized Space-Time Autoregressive (GSTAR) model is relatively new method for spatial time series data. It is a generalization of Space-Time Autoregressive (STAR) model that allows the autoregressive parameters vary in each location. In this paper we discuss the procedure to build the GSTAR model. The steps are similar to those of Box-Jenkins procedure, starting from order selection of spatial and temporal autoregressive model, parameters estimation, and diagnostic check of white noise errors. The Akaike's Information Criterion (AIC) is used for order selection. Particularly, there are two types estimation in GSTAR model. The first is determining space weights; here we propose normalization of cross-correlation between locations. The second is estimating the autoregressive parameters, which is performed by using least square method. We develop Wald test for evaluating the significance of the least square estimators. The Pormanteau multivariate test is adopted for detecting white noise error in multivariate case. Simulation study is performed to all the steps to illustrate the computation of the procedure.

Keywords:

GSTAR Modeling, Akaike's Information Criteria, normalization of cross-correlation, Wald test, Pormanteau multivariate test

1 Introduction

Spatial time series is addressed to the data that depend on time and site. It deals with single variable observed over time at a number of different locations. Daily air pollution measurements recorded from detector at many location, daily oil production at a number of wells, and traffic flow measurements taken from a set of loop detectors in a very frequent basis are examples of spatial time series data. Such kinds of data are also recognized as space time data.

Space time models are models that consider simultaneously spatial and time. So, this model is more appropriate to spatial data observed at ordered time than solely spatial or time series models. A classical multivariate time series model VARMA (Vector Autoregressive Moving Average) can be used to analyze space time data, but it requires so many parameters. Cliff and Ord [4] introduced space time model STARMA (space-time autoregressive moving average) and STAR (space-time autoregressive) that have less number of

parameters than VARMA model. Then, Pfeifer dan Deutsch [6,7] further studied those models and developed the procedure of their modeling

In STAR model, the autoregressive parameters are assumed to be the same for all locations. This assumption is impractical, since different locations usually lead different parameters. A more flexible model, i.e., Generalized STAR model was proposed by Borovkova et.al [2], allowing different autoregressive parameters.

This paper present the method of GSTAR modeling through the procedures adopted from Box and Jenkins [1]. The procedures start from stationer condition. Discussions about stationery GSTAR model can be read extensively in [8,9] and [12]. This paper focuses the discussion on the three stage modeling procedure. Those are model identification, parameters estimation, and diagnostic check to ensure that the model has white noise error vector.

2. GSTAR (*Generalized Space-Time Autoregressive*) Model

The GSTAR model is specific form of VAR (Vector Autoregressive) model. It reveals linear dependencies of space and time. The main difference is on the spatial dependent, that in GSTAR model, it is expressed by *weight matrix*. Let $\{Z(t) : t = 0, \pm 1, \pm 2, \dots\}$ be a multivariate time series of N components. In matrix notation, the GSTAR model of autoregressive order p and spatial orders $\lambda_1 \lambda_2, \dots, \lambda_p$, GSTAR ($p \lambda_1 \lambda_2, \dots, \lambda_p$) could be written as (see [2])

$$Z(t) = \sum_{s=1}^p \left[\Phi_{s0} + \sum_{k=1}^{\lambda_s} \Phi_{sk} W^{(k)} \right] Z(t-s) + e(t) \quad (1)$$

where

- $\Phi_{s0} = \text{diag}(\phi_{s0}^1, \dots, \phi_{s0}^N)$ and $\Phi_{sk} = \text{diag}(\phi_{sk}^1, \dots, \phi_{sk}^N)$,
- weights are choosen to satisfy $w_{ii}^{(k)} = 0$ and $\sum_{i \neq j} w_{ij}^{(k)} = 1$.

3. Model Identification

As in time series modeling, the first step is identifying a tentative model which is characterized by spatial and time order. Spatial order is generally restricted on order 1, since the higher order is difficult to be interpreted. Approving the method in VAR and VARMA models, the time (autoregressive) order is determined by using the Akaike Information Criterion (AIC) (see [13])

$$AIC(i) = \ln\left(\left|\hat{\Sigma}_i\right|\right) + \frac{2k^2i}{T} \quad (2)$$

where k is the number of parameters in the model and T is the number of observation. The autoregressive order of GSTAR model is p such that $AIC(p) = \min_{0 \leq i \leq p_0} AIC(i)$. This value p can be obtained by performing SAS program using PROCSTATSPACE.

4. Parameters Estimation

4.1 Determination of space weight

Determination of space weight by using the normalisation result of cross-correlation between locations at the appropriate time lag is firstly proposed by Suhartono and Atok (see [10,11]). They demonstrated by simulation study that

the method well performed on GSTAR(1₁) model. In general, cross-correlation between two variables or location i and j at the time lag k , $\text{corr}[Z_i(t), Z_j(t-k)]$, defined as (see [1, 14])

$$\rho_{ij}(k) = \frac{\gamma_{ij}(k)}{\sigma_i \sigma_j}, \quad k = 0, \pm 1, \pm 2, \dots \quad (3)$$

where $\gamma_{ij}(k)$ is cross-covarians between observation in location i and j at the time lag k , σ_i and σ_j is standard deviation of observation in location i and j .

The estimated of cross-correlation in sample data is

$$r_{ij}(k) = \frac{\sum_{t=k+1}^n [Z_i(t) - \bar{Z}_i][Z_j(t-k) - \bar{Z}_j]}{\sqrt{\left(\sum_{t=1}^n [Z_i(t) - \bar{Z}_i]^2 \right) \left(\sum_{t=1}^n [Z_j(t) - \bar{Z}_j]^2 \right)}}. \quad (4)$$

Then, determination of space weight could be done by normalisation of the the cross-correlation between locations at the appropriate time lag. This process generally yields space weight for GSTAR(p₁) model, i.e.

$$w_{ij} = \frac{r_{ij}(k)}{\sum_{k \neq i} |r_{ik}(k)|}, \quad \text{where } i \neq j, k = 1, \dots, p \quad (5)$$

and this weight satisfies $\sum_{j \neq i} |w_{ij}| = 1$

Space weights by using the normalisation of the cross-correlation between locations at the appropriate time lag give all form possibilities of the

relationship between locations. Hence, there is no strict constraint about the weight values that must depend on distance between locations. This weight also gives flexibility on the sign and size of the relationship between locations.

4.2. Autoregressive Parameter Estimation of GSTAR ($p\lambda_1\lambda_2,\dots,\lambda_p$) model

Recall GSTAR ($p\lambda_1\lambda_2,\dots,\lambda_p$) model

$$Z_i(t) = \sum_{s=1}^p \sum_{k=0}^{\lambda_s} \varphi_{sk}^{(i)} \left(w_{i1}^{(k)} Z_1(t-s) + w_{i2}^{(k)} Z_2(t-s) + \dots + w_{iN}^{(k)} Z_N(t-s) \right) + \mathbf{e}_i(t) \quad (6)$$

for $t = p, p+1, \dots, T$, $i = 1, 2, \dots, N$, where $w_{ij}^{(0)} = 1$ for $i = j$ and zero otherwise.

It is assumed that $\mathbf{e}(t) \sim \mathbf{White Noise}(\mathbf{0}, \Sigma)$, where $\mathbf{e}(t) = (e_1(t), e_2(t), \dots, e_N(t))$

Least square estimator of autoregressive parameter has been derived by Borovkova *et.al* [3]. They define new notations

$$V_i^{(k)}(t) = \sum_{j \neq i}^N w_{ij}^{(k)} Z_j(t) \text{ for } k \geq 1 \text{ and } V_i^{(0)}(t) = Z_i(t),$$

$$\mathbf{Y}_i = (Z_i(p), \dots, Z_i(T))', \quad \mathbf{u}_i = (e_i(p), \dots, e_i(T))'$$

$$\mathbf{X}_i = \begin{pmatrix} V_i^{(0)}(p-1) & \dots & V_i^{(\lambda_1)}(p-1) & \dots & V_i^{(0)}(0) & \dots & V_i^{(\lambda_p)}(0) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ V_i^{(0)}(T-1) & \dots & V_i^{(\lambda_1)}(T-1) & \dots & V_i^{(0)}(T-p) & \dots & V_i^{(\lambda_p)}(T-p) \end{pmatrix}$$

and $\boldsymbol{\beta}_i = (\phi_{10}^{(i)}, \dots, \phi_{1\lambda_1}^{(i)}, \phi_{20}^{(i)}, \dots, \phi_{2\lambda_2}^{(i)}, \dots, \phi_{p0}^{(i)}, \dots, \phi_{\lambda_p}^{(i)})'$

Equation (6) can be expressed for all location simultaneously as linear model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \quad (7)$$

where $\mathbf{Y} = (\mathbf{Y}'_1, \dots, \mathbf{Y}'_N)'$, $\mathbf{X} = (\mathbf{X}_1, \dots, \mathbf{X}_N)$, $\boldsymbol{\beta} = (\boldsymbol{\beta}'_1, \dots, \boldsymbol{\beta}'_N)'$, $\mathbf{u} = (\mathbf{u}'_1, \dots, \mathbf{u}'_N)'$

Thus, least square estimator $\hat{\boldsymbol{\beta}}_T$ is of the form

$$\hat{\boldsymbol{\beta}}_T = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u} \quad (8)$$

Asymptotic normality of the estimator $\hat{\boldsymbol{\beta}}_T$ will be explained below.

Write $\mathbf{M} = \text{diag}(\mathbf{M}_1, \dots, \mathbf{M}_N)$ where $\mathbf{M}_i = \text{diag}(\mathbf{M}_{i1}, \dots, \mathbf{M}_{ip})$, for $i = 1, 2, \dots$,

N and $s = 1, 2, \dots, p$

$$\mathbf{M}_{is} = \begin{pmatrix} 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ w_{i1}^{(1)} & \dots & w_{i,i-1}^{(1)} & 0 & w_{i,i+1}^{(1)} & \dots & w_{i,N}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{i1}^{(\lambda_s)} & \dots & w_{i,i-1}^{(\lambda_s)} & 0 & w_{i,i+1}^{(\lambda_s)} & \dots & w_{i,N}^{(\lambda_s)} \end{pmatrix} \quad (9)$$

Define covariance matrix

$$\boldsymbol{\Gamma}^{(p)} = \begin{bmatrix} \boldsymbol{\Gamma}(0) & \boldsymbol{\Gamma}(-1) & \dots & \boldsymbol{\Gamma}(-p+1) \\ \boldsymbol{\Gamma}(1) & \boldsymbol{\Gamma}(0) & \dots & \boldsymbol{\Gamma}(-p+2) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Gamma}(p-1) & \boldsymbol{\Gamma}(p-2) & \dots & \boldsymbol{\Gamma}(0) \end{bmatrix} \quad (10)$$

where $\Gamma(s) = E[\mathbf{Z}(t)\mathbf{Z}'(t+s)]$.

Under some conditions Borovkova et.al. [3] have shown asymptotic multivariate normality of least square estimator in GSTAR model, i.e.

$$\sqrt{T}(\hat{\boldsymbol{\beta}}_T - \boldsymbol{\beta}) \xrightarrow{d} \mathbf{N}_d(\mathbf{0}, \mathbf{C}^*) \quad (11)$$

where $d = d = (p + \lambda_1 + \dots + \lambda_p)N$, and

$$\mathbf{C}^* = \left(\mathbf{M}(\mathbf{I} \otimes \Gamma^{(p)})\mathbf{M}' \right)^{-1} \mathbf{M}(\boldsymbol{\Sigma} \otimes \Gamma^{(p)})\mathbf{M}' \left(\mathbf{M}(\mathbf{I} \otimes \Gamma^{(p)})\mathbf{M}' \right)^{-1},$$

\mathbf{M} is defined in (9) and $\Gamma^{(p)}$ is defined in (10).

The parameter $\boldsymbol{\Sigma}$ can be estimated by

$$\hat{\boldsymbol{\Sigma}}_T = \frac{1}{T-p+1} \sum_{t=p}^T (\mathbf{Z}(t) - \hat{\mathbf{Z}}(t))(\mathbf{Z}(t) - \hat{\mathbf{Z}}(t))'$$

where

$$\hat{\mathbf{Z}}(t) = \sum_{s=1}^p \hat{\boldsymbol{\Phi}}_{s0} \mathbf{Z}(t-s) + \sum_{k=1}^{\lambda_s} \hat{\boldsymbol{\Phi}}_{sk} \mathbf{W}^{(k)} \mathbf{Z}(t-s)$$

The elements of matrix $\Gamma^{(p)}$ can be estimated

$$\hat{\boldsymbol{\Gamma}}_T(s) = \frac{1}{T-s+1} \sum_{t=0}^{T-s} \mathbf{Z}(t)\mathbf{Z}'(t-s)$$

and $\hat{\boldsymbol{\Gamma}}_T(-s) = \hat{\boldsymbol{\Gamma}}_T'(s)$, for $s \geq 0$. Cosequently the estimate of $\Gamma^{(p)}$ is obtained.

It is also proven in [3] that $\hat{\Sigma}_T$ dan $\hat{\Gamma}^{(p)}$ are consistent estimators for Σ and $\Gamma^{(p)}$ respectively.

The hypothesis to test the significantcy of parameter in GSTAR model can be written as

$$H_0 : \mathbf{R}\boldsymbol{\beta} = r \text{ against } H_1 : \mathbf{R}\boldsymbol{\beta} \neq r . \quad (12)$$

By using property (11) and the facts that $\hat{\Sigma}_T$ dan $\hat{\Gamma}^{(p)}$ are consistent estimators for Σ and $\Gamma^{(p)}$, and modifying Wald Statistic on linear model from White [15] we developpe Wald satictic for the hypothesis (12) .Let $m = \text{rank}(\mathbf{R})$, if $H_0 : \mathbf{R}\boldsymbol{\beta} = r$ holds, under some conditions,

$$i) \sqrt{T}(\mathbf{R}\hat{\boldsymbol{\beta}}_T - r) \xrightarrow{d} \mathbf{N}_d(\mathbf{0}, \boldsymbol{\Xi}) \quad (13)$$

where $d = d = (p + \lambda_1 + \dots + \lambda_p)N$, and

$$\boldsymbol{\Xi} = \mathbf{R}\mathbf{C}^* \mathbf{R}'$$

ii) The Wald statistic

$$\mathbf{W} = T(\mathbf{R}\hat{\boldsymbol{\beta}}_T - r)'(\hat{\boldsymbol{\Xi}})^{-1}(\mathbf{R}\hat{\boldsymbol{\beta}}_T - r) \xrightarrow{d} \chi_m^2 \quad (14)$$

where $\hat{\boldsymbol{\Xi}} = \mathbf{R}\hat{\mathbf{C}}^* \mathbf{R}'$

5. Check Diagnostic

After found the significant model, the final step is checking whether the model fulfill the assumption of *white noise* error vector. *White noise* means residuals of model form an uncorrelated sequence that leads model fit. Based on the estimated model, the error estimate can be written as

$$\hat{\mathbf{e}} = \mathbf{Z}(t) - \hat{\mathbf{Z}}(t),$$

The Portmanteau Multivariate test, a generalization of Ljung and Box (in [13]) for multivariate case, can be used to test *white noise* error vector. The null hypothesis of this test is

$$H_0: \boldsymbol{\rho}_1 = \dots = \boldsymbol{\rho}_m = 0, \quad (15)$$

where $\boldsymbol{\rho}$ is correlation matrix of error vector.

The test statistic is

$$Q_N(m) = T^2 \sum_{t=1}^m \frac{1}{T-t} \text{tr}(\hat{\boldsymbol{\Gamma}}_t \hat{\boldsymbol{\Gamma}}_0^{-1} \hat{\boldsymbol{\Gamma}}_t \hat{\boldsymbol{\Gamma}}_0^{-1}) \quad (16)$$

Under the null hypothesis (15), the statistic test (16) approximate Chi Square distribution with $N^2 m$ degree of freedom.

6. Empirical Study

The empirical study is carried out through the simulation study. It aims to explain the implementation of GSTAR modeling procedures that conducted based on the theory described above. Here, the GSTAR model of spatial order 1 and autoregressive order 2 GSTAR (2₁) is generated from VAR (2) by using

SAS program. The simulation is done for sample size 100. The simulation model is in the following expression

$$\begin{bmatrix} Z1_t \\ Z2_t \\ Z3_t \end{bmatrix} = \begin{bmatrix} 0.35 & 0.2 & -0.2 \\ -0.1 & 0.3 & 0.1 \\ 0.1 & -0.1 & 0.2 \end{bmatrix} \begin{bmatrix} Z1_{t-1} \\ Z2_{t-1} \\ Z3_{t-1} \end{bmatrix} + \begin{bmatrix} 0.25 & 0.1 & -0.1 \\ -0.1 & 0.25 & 0.1 \\ 0.1 & -0.1 & 0.15 \end{bmatrix} \begin{bmatrix} Z1_{t-2} \\ Z2_{t-2} \\ Z3_{t-2} \end{bmatrix} + \begin{bmatrix} e1_t \\ e2_t \\ e3_t \end{bmatrix} \quad (17)$$

As mentioned in Section 3 the spatial order 1 is used to build GSTAR model. And after running the simulation data by using PROCSTATSPACE, the minimum AIC value is in lag 2, so the autoregressive order 2 is the best choice. The result of cross-correlation between locations at the time lag 1 and 2, $r_{ij}(1)$ and $r_{ij}(2)$ where $i \neq j$ of simulation data followed equation (4) can be seen in Table 1.

Table 1. The result of cross-correlation between locations

Parameter	Coefficient estimated
$r_{12}^{(1)}$	0.317
$r_{13}^{(1)}$	-0.044
$r_{21}^{(1)}$	0.053
$r_{23}^{(1)}$	0.179
$r_{31}^{(1)}$	-0.030
$r_{32}^{(1)}$	0.142
$r_{12}^{(2)}$	0.433
$r_{13}^{(2)}$	0.023
$r_{21}^{(2)}$	0.028
$r_{23}^{(2)}$	0.346
$r_{31}^{(2)}$	0.027
$r_{32}^{(2)}$	0.040

The weights are attained by normalize that cross correlation using (5), i.e.

$$W_1 = \begin{bmatrix} 0 & 0.878 & -0.122 \\ 0,228 & 0 & 0,772 \\ -0.174 & 0.826 & 0 \end{bmatrix} \text{ dan } W_2 = \begin{bmatrix} 0 & 0.950 & 0.050 \\ 0,075 & 0 & 0,925 \\ 0.403 & 0.597 & 0 \end{bmatrix}$$

By using these weight, the resulted parameter estimates and significant test of GSTAR(2₁) model are shown in Table 2.

Tabel 2. The result of parameter estimates GSTAR(2₁) model

Parameter	Coefficient estimated	Standard Error	χ^2 -value	p-value
$\phi_{10}^{(1)}$	0.2833	0.1296	23.00258	0.030
$\phi_{20}^{(1)}$	0.2769	0.1056	47.11999	0.009
$\phi_{30}^{(1)}$	0.2307	0.0895	44.30766	0.010
$\phi_{11}^{(1)}$	0.0001	0.0001	0.6561	0.370
$\phi_{21}^{(1)}$	-0.0254	0.1129	0.002343	0.822
$\phi_{31}^{(1)}$	0.1227	0.1309	0.780749	0.349
$\phi_{10}^{(2)}$	0.1498	0.1272	1.938778	0.240
$\phi_{20}^{(2)}$	0.1851	0.1032	10.26626	0.074
$\phi_{30}^{(2)}$	0.2133	0.0893	5.7121	0.018
$\phi_{11}^{(2)}$	0.2298	0.1188	3.7249	0.054
$\phi_{21}^{(2)}$	0.2305	0.0957	5.8081	0.017
$\phi_{31}^{(2)}$	-0.0711	0.1126	0.3969	0.528

The last column in Table 2. reveals that not all parameters are significant. Then, the model re-estimated after excluding the insignificant parameters one by one. The final result only includes the significant parameters, and it is presented in Table 3.

Tabel 3. The result of significant parameter estimates GSTAR(2₁) model

Parameter	Coefficient estimated	Standard Error	χ^2 -value	p-value
$\phi_{10}^{(1)}$	0.3877	0.0992	15.2881	0.000
$\phi_{20}^{(1)}$	0.3319	0.0943	12.3904	0.000
$\phi_{30}^{(1)}$	0.2479	0.0874	8.0656	0.005
$\phi_{30}^{(2)}$	0.2133	0.0871	6.0025	0.015
$\phi_{11}^{(2)}$	0.3094	0.1012	9.3636	0.002
$\phi_{21}^{(2)}$	0.2669	0.0905	8.7025	0.003

By using that result and applying matrix operation, the GSTAR (2₁) model is:

$$\begin{bmatrix} Z1_t \\ Z2_t \\ Z3_t \end{bmatrix} = \begin{bmatrix} 0.3877 & 0 & 0 \\ 0 & 0.3319 & 0 \\ 0 & 0 & 0.2479 \end{bmatrix} \begin{bmatrix} Z1_{t-1} \\ Z2_{t-1} \\ Z3_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0.2939 & 0.0155 \\ 0.0200 & 0 & 0.2469 \\ 0 & 0 & 0.2133 \end{bmatrix} \begin{bmatrix} Z1_{t-2} \\ Z2_{t-2} \\ Z3_{t-2} \end{bmatrix} + \begin{bmatrix} e1_t \\ e2_t \\ e3_t \end{bmatrix} \quad (18)$$

Then, the procedure continues to the last step, that is checking the white noise assumption. Table 4. gives the result of **Portmanteau Multivariate** test using Chi-square distribution of error vector estimates yielded by model (18)

Tabel 4. The result of **Portmanteau Multivariate** Test

	Lag 1	Lag 5	Lag 6	Lag 10	Lag 12
χ^2 -value	8.21786	43.67469	52.32970	95.99643	111.49993
p-value	0.51235	0.52813	0.53905	0.31319	0.3894

It can be perceived from the p-value that the model is suitable for the white noise error vector assumption. Thus, how to proceed all the steps of GSTAR modeling as an implementation of theoretical result has illustrated clearly.

7. Conclusion

The method of GSTAR modeling is applied through the procedures in the spirit of Box-Jenkins. The procedure starts from model identification involving spatial and autoregressive orders determination. Only spatial order 1 is considered, while autoregressive order is specified by minimum value of AIC (2). Normalized cross correlation (5) is proposed as estimator of space weight and least square estimator (8) is derived as an estimate of autoregressive parameter. Wald test statistic (14) is constructed to select the significant autoregressive parameter. **The Portmanteau Multivariate** test (16) is considered as a tool to check the white noise error assumption in multivariate case. Simulation study has been performed to describe the procedures of GSTAR modeling. As a remark on space weight determination, other technique could be attempted to obtain better estimates.

References

1. Box, G.E.P., Jenkins, G.M. and Reinsel, G.C.: *Time Series Analysis, Forecasting and Control*. 3rd edition, Englewood Cliffs: Prentice Hall. (1994)
2. Borovkova, S.A., Lopuhaa, H.P., and Nurani, B.: Generalized STAR model with experimental weights. In M Stasinopoulos & G Touloumi (Eds.), *Proceedings of the 17th International Workshop on Statistical Modeling* (pp. 139-147). (2002)
3. Borovkova, S.A., Lopuhaa, H.P., and Nurani, B.: Consistency and asymptotic normality of least squares estimators in generalized STAR models. Working paper. (2008)

4. Cliff, A.D., Ord, J.: *Spatial Autocorrelation*. London. Pioneer. (1973)
5. Lopuhaa H.P. and Borovkova S.: Asymptotic properties of least squares estimators in generalized STAR models. *Technical Report*. Delft University of Technology. (2005)
6. Pfeifer, P.E. and Deutsch, S.J.: A Three Stage Iterative Procedure for Space-Time Modeling. *Technometrics*, Vol. 22, No. 1, pp. 35-47. (1980)
7. Pfeifer, P.E. and Deutsch, S.J.: Identification and Interpretation of First Order Space-Time ARMA Models. *Technometrics*, Vol. 22, No. 1, pp. 397-408. (1980)
8. Ruchjana, B.N.: Suatu Model Generalisasi Space-Time Autoregressive dan Penerapannya pada Produksi Minyak Bumi. (Disertasi, Institut Teknologi Bandung). (2002.)
9. Ruchjana, B.N.: Pemodelan Kurva Produksi Minyak Bumi Menggunakan Model Generalisasi S-TAR. *Forum Statistika dan Komputasi*, IPB, Bogor. (2002).
10. Suhartono dan Atok, R.M.: Pemilihan bobot lokasi yang optimal pada model GSTAR. *Prosiding Konferensi Nasional Matematika XIII*, Universitas Negeri Semarang. (2006)
11. Suhartono and Subanar.: The Optimal Determination of Space Weight in GSTAR Model by using Cross-correlation Inference. *JOURNAL OF QUANTITATIVE METHODS: Journal Devoted to The Mathematical and Statistical Application in Various Fields*, Vol. 2, No. 2, pp. 45-53. (2006)
12. Suhartono and Subanar.: Some Comments on the Theorem Providing Stationarity Condition for GSTAR Models in the Paper by Borovkova et al. *Journal of The Indonesian Mathematical Society (MIHMI)*, Vol. 13, No. 1, pp. 44-52. (2007)
13. Tsay, R.S.: *Analysis of Financial Time series*. John Wiley & Sons. New Jersey. (2005)
14. Wei, W.W.S.: *Time series Analysis: Univariate and Multivariate Methods*. Addison-Wesley Publishing Co., USA. (1990)
15. White H.: *Asymptotic Theory for Econometricians*, Academic Press Inc., New York. (1999)