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Kinetic Theory of Gases I:

- Gas Pressure
- Translational Kinetic Energy
- Root Mean Square Speed

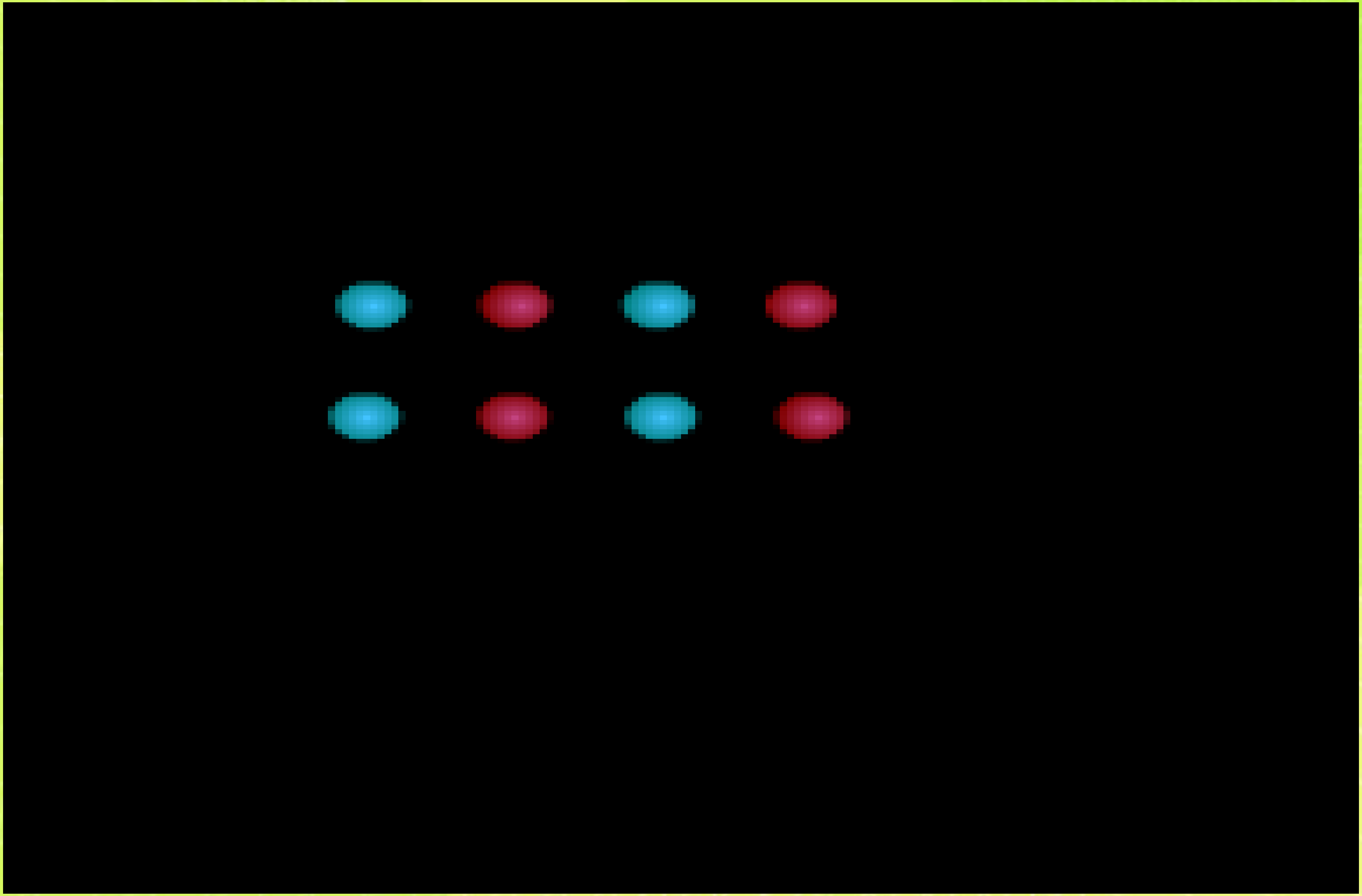
GASES

- Gases are one of the most pervasive aspects of our environment on the Earth. We continually exist with constant exposure to gases of all forms.
- The steam formed in the air during a hot shower is a gas.
- The Helium used to fill a birthday balloon is a gas.
- The oxygen in the air is an essential gas for life.

GASES

A windy day or a still day is a result of the difference in pressure of gases in two different locations. A fresh breeze on a mountain peak is a study in basic gas laws.

Mixture of gases



The Kinetic Molecular Model for Gases

- Gas consists of large number of small individual particles with negligible size
- Particles in constant random motion and collisions
- No forces exerted among each other
- Kinetic energy directly proportional to temperature in Kelvin

$$KE = \frac{3}{2} \cdot R \cdot T$$

The Ideal Gas Law

$$PV = nRT$$

← **in K**

n: the number of moles in the ideal gas

$$n = \frac{N}{N_A}$$

total number of molecules (→ N)

Avogadro's number: the number of atoms, molecules, etc, in a mole of a substance: N_A=6.02 x 10^{23}/mol. (→ N_A)

R: the Gas Constant: R = 8.31 J/mol · K

Pressure and Temperature

Pressure: Results from collisions of molecules on the surface

Pressure: $P = \frac{F}{A}$ ← Force, ← Area

Force: $F = \frac{dp}{dt}$ ← Rate of momentum given to the surface

Momentum: momentum given by each collision times the number of collisions in time *dt*

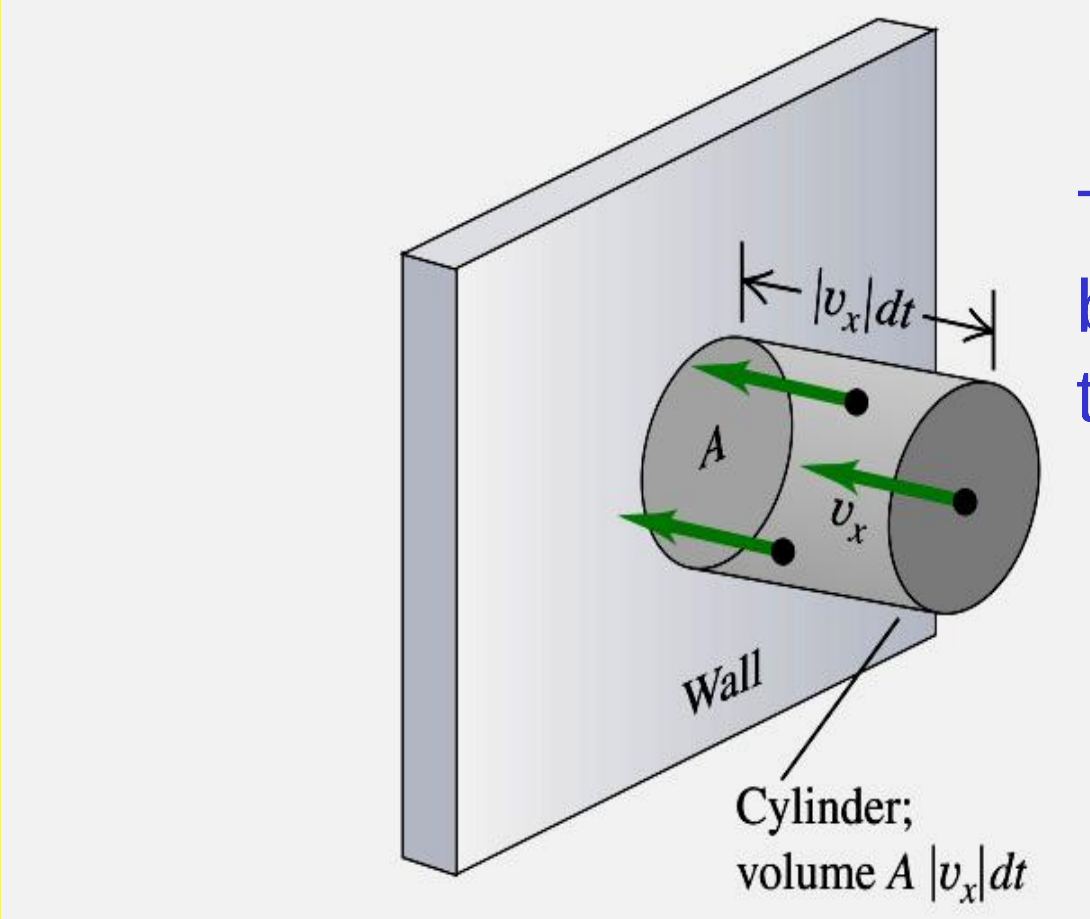
Only molecules moving toward the surface hit the surface. Assuming the surface is normal to the *x* axis, half the molecules of speed v_x move toward the surface.

Only those close enough to the surface hit it in time *dt*, those within the distance $v_x dt$

The number of collisions hitting an area *A* in time *dt* is

$$\frac{1}{2}\left(\frac{N}{V}\right)\cdot A\cdot v_x\cdot dt$$

Average density



The momentum given by each collision to the surface $2mv_x$

Momentum in time *dt*:

$$dp = (2mv_x)\cdot\frac{1}{2}\cdot\left(\frac{N}{V}\right)\cdot A\cdot v_x dt$$

Force

$$F = \frac{dp}{dt} = (2mv_x)\cdot\frac{1}{2}\cdot\left(\frac{N}{V}\right)\cdot A\cdot v_x$$

Pressure

$$P = \frac{F}{A} = \frac{N}{V} m v_x^2$$

Not all molecules have the same v_x $\Rightarrow$ average $\overline{v_x^2}$

$$P = \frac{N}{V} m \overline{v_x^2}$$

$$v_x^2 = \frac{1}{3}v^2 = \frac{1}{3}\left(v_x^2 + v_y^2 + v_z^2\right)$$

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2} = \frac{1}{3}\, v_{rms}^2$$

v_{rms} is the root-mean-square speed

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}}{3}}$$

Pressure: $$P = \frac{1}{3}\frac{N}{V}m\overline{v^2} = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\overline{v^2}\right)$$

Average Translational Kinetic Energy:

$$\overline{K} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}mv_{rms}^2$$

Pressure:

$$P = \frac{2}{3} \cdot \frac{N}{V} \cdot \overline{K}$$

From $PV = \frac{2}{3} \cdot N \cdot \overline{K}$ and $PV = nRT$

Temperature:

$$\overline{K} = \frac{3}{2} \cdot \frac{nRT}{N} = \frac{3}{2} \cdot k_B T$$

Boltzmann constant: $k_B = \frac{R}{N_A} = 1.38 \times 10^{-23}\ \text{J/K}$

From $$PV = \frac{1}{3} \cdot N \cdot m v_{rms}^2$$

and $$PV = nRT = \frac{N}{N_A} RT$$

Avogadro's number

$$N = nN_A$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Molar mass

$$M = mN_A$$

Pressure ☑ Density x Kinetic Energy

Temperature ☑ Kinetic Energy

(a) Compute the root-mean-square speed of a nitrogen molecule at 20.0 ˚C. At what temperatures will the root-mean-square speed be (b) half that value and (c) twice that value?

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

(a)

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{28.0\times10^{-3}\ \text{kg/mol}}} = 511\ \text{m/s}$$

(b) Since $v_{rms} \propto \sqrt{T}$

$$\frac{v'^2_{rms}}{v^2_{rms}} = \frac{T'}{T}$$

for 0.5 v_{rms} $\quad T' = 0.5^2 T = 73.3\text{K} = -200\,^\circ\text{C}$

for 2 v_{rms} $\quad T'' = 2^2 T = 1.17\times10^3\text{K} = 899\,^\circ\text{C}$

Please estimate the root mean square mean velocity of Hydrogen gas.

A. 2000 B. 1000 C. 500

E. 250 D. 100 m/s

What is the average translational kinetic energy of nitrogen molecules at 1600K, (a) in joules and (b) in electron-volts?

(a)

$$\overline{K} = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(1600\text{K}\right)$$

$$= 3.31\times10^{-20}\,\text{J}$$

(b) 1 eV = 1.60 x 10^{-19} J

$$\overline{K} = \frac{3.31\times10^{-20}\,\text{J}}{1.60\times10^{-19}\,\text{J/eV}} = 0.21\ \text{eV}$$

$$\overline{K} = \frac{3}{2}\cdot k_B T$$