Halaman 77 no 29.

$$y' = ry - ky^2 \quad , r>0 \;\&\; k>0 \;.\; \text{---}(i)$$

Pilih $r=1$, $k=1$, maka didapat

$$y' = y - y^2 \quad \text{---}(ii)$$

Persamaan (ii) dapat dipandang sbg separable diff eq$_s$ tetapi juga merupakan persamaan Bernoulli dgn $n=2$.

Selanjutnya akan kita selesai persamaan (ii) sebagai separable diff eq$_s$ sbb :

$$\frac{dy}{dt} = y - y^2 \quad , \quad y' = \frac{dy}{dt}$$

$$\frac{dy}{y-y^2} = dt$$

$$\int \frac{dy}{y-y^2} = t + c$$

$$\int \frac{dy}{y(1-y)} = t + c$$

$$\frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y} = \frac{A(1-y)+By}{y(1-y)}$$

$$\frac{1}{y(1-y)} = \frac{(B-A)y + A}{y(1-y)}$$

$B-A=0$, $A=1$

sbg $B=1$

$$\int \frac{dy}{y} + \int \frac{dy}{1-y} = t + c$$

$$\ln|y| - \ln|1-y| = t + c$$

$$\ln\left|\frac{y}{1-y}\right| = t + c$$

$$\left|\frac{y}{1-y}\right| = e^t \cdot e^c$$

$$\frac{y}{1-y} = \pm e^c \cdot e^t \quad \text{atau} \quad \frac{y}{1-y} = ke^t$$

$$y = ke^t - ke^t y$$

$$(1+ke^t)y = ke^t$$

$$y = \frac{ke^t}{1+ke^t} \quad \text{atau} \quad y = \frac{e^t}{1/k + e^t}$$

Sekarang $y' = y - y^2$ kita pandang sbg persamaan Bernoulli:

$$y' - y = -y^2 \quad \text{------ (iii)}$$

Misalkan $v = y^{1-2} = y^{-1}$

$$\frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt} = -y^{-2} \cdot \frac{dy}{dt}$$

$$\frac{dy}{dt} = -y^2 \frac{dv}{dt}$$

Disubstitusikan ke (iii) diperoleh

$$-y^2 \frac{dv}{dt} - y = -y^2 \quad \times -y^{-2}$$

$$\frac{dv}{dt} + y^{-1} = 1$$

$$\frac{dv}{dt} + v = 1 \quad \text{---- (iv)}$$

Dari persamaan (iv) $p(t) = 1$ sbg $\mu(t) = e^{\int p(t)dt} = e^{\int dt} = e^t$

Solusi dari (iv)

$$e^t \cdot v = \int 1 \cdot e^t \, dt + c$$

$$e^t v = e^t + c$$

$$v = \frac{e^t + c}{e^t}$$

$y' + p(t) y = q(t)$

$\downarrow$

$\mu y = \int \mu \cdot q(t) \, dt + c$

dengan $\mu = e^{\int p(t) dt}$

Karena $v = y^{-1}$ maka

$$y^{-1} = \frac{e^t + c}{e^t}$$

$$y = \frac{e^t}{e^t + c}$$