**Bahan Ajar KALKULUS INTEGRAL**

**Oleh: ENDANG LISTYANI**

**ANTI TURUNAN**

**(hal 299)**

Anda tentu sudah mengenal invers atau balikan suatu operasi hitung

Invers dari operasi penjumlahan adalah pengurangan; perkalian dengan pembagian, pemangkatan dengan penarikan akar.

Demikian pula turunan merupakan invers atau balikan dari anti turunan dan sebaliknya.

**Definisi**

Suatu fungsi F disebut suatu anti turunan dari suatu fungsi f pada interval I jika F’(x) = f(x) untuk setiap x pada I

Ilustrasi

Jika F(x) = 3x3 + x2 – 2x – 7 , maka F’(x) = 9x2 + 2x – 2

Jika f adalah fungsi yang didefinisikan sebagai

f(x) = 9x2 + 2x – 2, maka f turunan dari F and F adalah anti turunan dari f

Jika G(x) = 3x3 + x2 – 2x + 5, maka G juga anti turunan dari f karena G’(x) = 9x2 + 2x – 2

Secara umum, fungsi yang didefinisikan sebagai 3x3 + x2 – 2x + C, dengan C adalah konstanta, merupakan anti turunan dari f

**Secara umum**, jika suatu fungsi F adalah suatu anti turunan dari f pada interval I dan jika G didefinisikan sebagai G(x) = F(x) + C dimana C adalah konstanta sebarang, maka G’(x) = F’(x) = f(x)

dan G juga merupakan anti turunan dari f pada interval I

**Notasi untuk Anti Turunan**

Ax(9x2 + 2x – 2) = 3x3 + x2 – 2x + C

Bagaimana dengan

Ax(x3 + 2x2 – 2) = ?

Tentu bukan pekerjaan mudah menentukan suatu fungsi yang jika diturunkan berbentuk

x3 + 2x2 – 2

Untuk itu diperlukan notasi dan aturan-aturan yang dapat mempermudah menentukan anti turunan

Anti turunan dinotasikan sebagai .... dx

  = F(x) + C

**Teorema A (hal 301)**

** =  untuk r bilangan rasional dan r ≠ - 1**

**Sifat-sifat**

**1. ** = x + C

**2. ** = a

**3.  =** + 

**Dengan demikian**

Ax(x3 + 2x2 – 2) = (x3 + 2x2 – 2) **dx**

 **=**  +  - 

 =  + C1 + + C2 – 2( + C3 )

 =  +  - 

**Soal-soal 5.1 halaman 307. Tugas individu**

**Untuk dikerjakan hari kamis tanggal 14 Feb 2013**

**No 2 – 26 no genap saja**

**No 27 – 50 semua**

**FUNGSI-FUNGSI APA YANG DAPAT DIINTEGRALKAN?**

**Sebarang fungsi yang terintegralkan pada [a,b] harus terbatas di [a , b]. Yaitu: terdapat konstanta M sedemikian sehingga** 

**Contoh**

****

Perhatikan fungsi  pada selang [-2 , 2]

merupakan fungsi tak terbatas.

Tidak terdapat suatu bilangan M sedemikian sehingga



Dengan demikian  tidak terintegralkan pada

 [-2 , 2]

**Lihat soal no 21 hal 347**

****

 ****

f(x) = x3 + sin x

Ada M = 9 sehingga  untuk setiap x pada

[-2 , 2]

Jadi f(x) = x3 + sin x terintegralkan pada [-2 , 2]

****

f(x) = 1/(x-1)

pada [-2 , 2] maksimum dan minimum di  dan di -

**tidak ada nilai M** sehinngga 

jadi f(x) = 1/(x-1) tidak terintegralkan pada [-2 , 2]

f(x) = 1/(x+3)

Ada M = 1 sehingga  untuk setiap x pada [-2 , 2]

Jadi f(x) = 1/(x+3) terintegralkan pada [-2 , 2]

****

f(x) = tan x

pada [-2 , 2] maksimum dan minimum di  dan di -

f(x) = x3 + sin x

Ada M = sehingga  untuk setiap x pada

[-2 , 2]

Jadi f(x) = x3 + sin x terintegralkan pada [-2 , 2]

f(x) = 1/(x+3)

Ada M = 1 sehingga  untuk setiap x pada [-2 , 2]

Jadi f(x) = 1/(x+3) terintegralkan pada [-2 , 2]

f(x) = 1/(x-1)

pada [-2 , 2] maksimum dan minimum di  dan di -

**tidak ada nilai M** sehinngga 

jadi f(x) = 1/(x-1) tidak terintegralkan pada [-2 , 2]

f(x) = tan x

pada [-2 , 2] maksimum dan minimum di  dan di -

**tidak ada nilai M** sehinngga 

jadi f(x) = tan x tidak terintegralkan pada [-2 , 2]

**TEOREMA A (hal 342)**

**(Teorema keintegralan). Jika f terbatas pada [a , b] dan kontinu di [a , b] kecuali pada sejumlah terhingga titik, maka f terintegralkan pada [a , b].**

**Khususnya jika f kontinu pada seluruh selang [a , b], maka f terintegralkan pada [a , b]**

****

f(x) = sin(1/x)

Ada M =..... sehingga  untuk setiap x pada [-2 , 2]

Jadi f terintegralkan pada [-2 , 2]

****

f(x) = 

Ada M = 4 sehingga  untuk setiap x pada [-2 , 2]

Jadi f terintegralkan pada [-2 , 2]

FUNGSI LOGARITMA ASLI





Teorema (hal 453)

ln 1 = 0



19) 

Misal ln x = u



 = 

20) 

Misal ln x = u



 = 

 = u-1 + C =  + C =  + C

Hitunglah



1



2)0)))

FUNGSI BALIKAN/INVERS

Misalkan y = f(x) = x3 + 1

 x = f-1(y) = 



Apakah setiap fungsi mempunyai invers?

Perhatikan fungsi y = f(x) = x2



x =  bukan fungsi , jadi y = f(x) = x2 tidak mempunyai balikan/INVERS.

Tetapi jika domainnya di batasimisalnya y = f(x) = x2 didefinisikan pada [0 , ] maka y = f(x) = x2 mempunyai invers yaitu x = f-1(y) = 



y = f(x) mempunyai invers jika y merupakan fungsi satu-satu atau fungsi monoton

Jika f memiliki invers, maka y = f(x) x = f-1(y)

Grafik y = f(x) sama/identik dengan grafik x = f-1(y)

Pembahasan lebih lanjut yang terkait dengan fungsi invers adalah menggunakan bentuk y = f-1(x). Perhatikan bahwa posisi x dan y dipertukarkan

Dengan demikian grafik fungsi y = f-1(x) dapat diperoleh dengan mencerminkan grafik y = f(x) terhadap garis y = x

 y = x

y = f-1(x)



y = f(x)

FUNGSI EKSPONEN ASLI

Invers dari fungsi logaritma asli adalah fungsi eksponen asli

y = ln x x = ey

Grafik y = ln x identik dengan grafik x = ey



y = ln x

x = ey

Grafik y = ex diperoleh dengan mencerminkan grafik y = ln x terhadap garis y = x (hal 468)



y = ex

y = ln x

x = ey

y = f(x) = ex disebut fungsi eksponen asli

Sifat-sifat:

 ln e = 1



Contoh





**Turunan dari ex**







Contoh









Contoh

Integralkan



Misal 2x+1 = u

 2 dx = du

 = 

Fungsi eksponen umum





ecara Umum



Contoh



Tentukanlah 

1. y = 52x+3 (2) y = 





Contoh



u=sin x

du = cos x dx

Fungsi logaritma terhadap basis a

Definisi (hal 479)











Fungsi Invers Trigonometri (hal 494)



Bagaimana grafik fungsi y = arc sin x ?









 1

x

 yy

y







**Introduction to Differential Equation**

We know that the expression F’(x) = f(x) is equivalent with dF(x) = f(x) dx,

so we can write  

this formula will help us to solve differential equation.

**What is differential equation ?**

Let start with an example.

Suppose we want to find out xy*-*equation of a curve passing through a point (-1, 2) with the gradient on each point of the curve is equal to twice the absis of the point.

Hence  = 2x on each point of the curve.

Now, we will find a function *y* = f(x) satisfied that condition.

Method 1. If the equation is on the form

= *g* (*x*), then y =  ,

 y =  = x2 + C

Method 2

Think  as dy is divided by dx, so we can write



Integrate two sides  = 

 y + C1 = x2 + C2

y = x2 + C2 - C1 y = x2 + C

If the curve pass the point (−1,2), we can find C :

2 = (−1)2 + C

So C = 1 and the xy-equation is y = x2 + 1

The expression = 2x is called differential equation.

**Other examples of differential equation are**

 = 2xy

y dy = (x2 + 1) dx

 + 2 - 3xy = 0

**An equation that contains an unknown function and some of its derivatives** is called **differential equation**.

In this lecture, we only consider separable first order differential equation.

Notify that the equation

  = 

can be written as

 y2 dy = (x + 3x2) dx

Here, x and y term are separated. To solve this equation, we use method 2

= 

 + C1 = + x3 + C2

y3 = + 3x3 + 3C2 – 3C1

y3 = + 3x3 + C

y = 

Supposed we have *y* = 6 for *x*  = 0,

then we can find C:

 6 = 

C = 216

Hence,

 y = 

Check this result by substituting it in differential equation. The left side of the differential equation becomes

  = (3x + 9x2)

 = 

and the right side of the differential equation becomes

 = 

.

Those give the same expression.

Evaluate

1. The point (3,2) is on a curve, and at any point (x,y) on the curve the tangent

line has a slope equal to 2x – 3. Find an equation of

the curve

**Solution**

Suppose *y* = f(x) is the equation of the curve

= 2x – 3

dy = (2x – 3)dx

y = (2x – 3)dx = x2 – 3x + C

The point (3,2) is on a curve, so 2 = 32 – 3.3 + C C = 2

Hence the equation of the curve is y = = x2 – 3x + 2

1. The points (-1,3) and (0,2) are on a curve, and at any point (x,y) on the curve

Dx2y = 2 – 4x. Find an equation of the curve

**Solution**

Suppose *y* = f(x) is the equation of the curve

= (2 – 4x) dx = 2x – 2x2 + C1

y = (2x – 2x2 + C1) dx = x2 - x3 + C1x + C2

The points (-1,3) and (0,2) are on a curve so,

3 = (-1)2 - (-1)3 + C1(-1) + C2 …………..(1)

2 = (0)2 - (0)3 + C1(0) + C2 ……………..(2)

From (1) and (2) : C2 = 2 , C1 =

Hence the equation of the curve is

y = x2 - x3 + x + 2

1. An equation of the tangent line to a curve at the

point (1,3) is y = x +2. If at any point (x,y) on the curve, Dx2y = 6x, find an equation of the curve

**Solution**

Suppose *y* = f(x) is the equation of the curve

= 3x2 + C1

y = x3 + C1x + C2

 The slope of the tangent line to the curve at the

point (1,3) is1, so 3(1)2 + C1 = 1, C1 = -2

The point (1,3) is on the curve, so

3 = 13 + (-2)(1) + C2 , C2 = 4

Hence the equation of the curve is y = x3 - 2x + 4

1. The Volume of water in a tank is V cubic meters

when the depth of the water is h meters . If the rate

of chane ofV with respect to h is given by

DhV = (2h+3)2, find the volume of water in the tank

when the depth is 3 m

**Solution**

Suppose V = f(h)

 = (2h+3)2 , V = (2h+3)2 dh

V = (2h+3)2 d(2h+3)

 = .(2h+3)3 + C

When h = 0, V = 0, so 0 = .(2.0+3)3 + C,

C = - 

V = (2h+3)3 - 

If h = 3, V = (2.3+3)3 -  = 117

The volume of water in the tank when the depth

is 3 m = 117 m3

Evaluate

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Introduction to Area

Consider a region R in the plane as shown in Fig. 1. The region R is bounded by the x axis, the lines x = a and x = b, and the curve having the equation y = f(x), where f is a function continuous on the closed interval [a,b].

R

 a b

 Fig. 1

Divide the closed interval [a,b] into n subintervals

For simplicity, now we take each of these subintervals as being of equal length, for instance, x. There for x = 

Denote the endpoints of these subintervals by x0 , x1 , x2, . . . , xn-1 , xn

where x0 = a , x1 = a +x , xi = a + ix

xn = b

Let the ith subinterval be denote by

[xi-1,xi].

Because f is continuous on the closed interval [a,b], it is continuous on each closed subinterval.

By the extreme-value theorem, there is a number in each bsubinterval for which f has an absolute minimum value.

In the ith subinterval, let this number be ci , so that f(ci) is the absolute minimum value of f on the subinterval [xi-1,xi].

Consider n rectangles, each vhaving a width x units and an altitude f(ci) units see Fig 2.

 f(ci)

R

 a x b

 Fig. 1

Let the sum of the areas of these n rectangle be given by Sn square units, then

 Sn = f(c1)x + f(c2)x + . . . + f(cn)x

 =  f(ci)x …………….(\*)

The summation on thr right side of (\*) gives the sum of measures of vthe areas of n inscribed rectangles. Thus however we define A, it must be such that A  Sn

DEFINITION

Suppose that the function f is continuous on the closed interval [a,b], with f(x)  0 for all x in [a,b], and that R is the region bounded by the curve y=f(x), x axis, and the lines x = a and x = b. Divide the closed interval [a,b] into n subintervals

each of lengthx = , and denote the ith subinterval by [xi-1,xi]. Then if f(ci) is the absolute minimum function value on the ith subinterval, the measure of the area of region R is given by

 A = x

Example

Find the area of the region bounded by the curve y = x2, the x axis, and the line

x = 3 by taking inscribed rectangle.

Solution

Devide the interval [0,3] into n subinterval, each of length x;

x0 = 0 , x1 = x, x2 = 2x, . . . , xi = ix

 xn-1= (n-1) x , xn = 3

x =  = 

Because f is increasing on [0,3], the absolute minimum value of f on the ith subinterval [xi-1,xi] is f(xi-1)

There for A = x

Because xi-1 = (i-1)x and f(x) = x2,

f(xi-1) = [(i-1)x]2

Therefore

x = (x)3

 = 

 = ………….lanjutkan

Gunakan (pilih) rumus sbb

 ; 

 ; 

**THE DEFINITE INTEGRAL**

In the preciding section the measure of the area of a region was defined as the following limit:

x ……………………..(\*)

To define the definite integral we need to consider a new kind of limiting process, of which the limit given in (\*) is a special case.

Let f be a function defined on the closed interval [a,b]. Divide thus interval into n subintervals by choosing any (n-1) intermediate points between a and b.

Let x0 = a , and xn = b , and let

x0 < x1 < x2 < . . . < xn-1 < xn

The points x0 , x1 , x2, . . . , xn-1 , xn are not necessarily equidistant. Let x be the length of the ith subinterval so that x = xi – xi -1

A set of all such subintervals of the interval [a,b] is called a **partition** of theinterval [a,b].

One (or more) of these subintervals is longest. The length of the longest subinterval of the partition called the **norm of the partition, is denoted by **

Choose a point in each subinterval of the partition P. Let xi\* be the point chosen in [xi -1 , xi]

Form the sum f(xi\*)x + f(x2\*)x + . . . + f(xi\*)x + . . . + f(xn\*)x = x

Such a sum is called a **Riemann sum,** named for the mathematician George Friedrich Bernhard Riemann (1826 – 1866)

 Y

 x2\* x3\* xn\*

 x1\* xi\* X

**Definition**

If f is a function defined on the closed interval [a,b], then the definite integral of f from a to b, denote by

, is given by

 = x if the limit exists

*Note*

That the statement “the function f is integrable on the closed interval [a,b]” is synonymous with the statement : the definite integral of f from a to b exists

In the notation for the definite integral ,

 is called the **integrand**, **a** is called the **lower limit**, and **b** is called the **upper limit**.

The symbol is called an **integral sign**

**Definition**

* If a >b, then  = - 
*  = 0

**THE FUNDAMENTAL THEOREM OF THE CALCULUS**

**Theorem**

If f is a continuous function on the closed interval [a,b], and F is an any antiderivative of f on [a,b], then

 = F(b) – F(a)

We should write F(b) – F(a) = 

**Example**

Evaluate 

Solution

 =  =  -  = 4 -  = 3

**Properties of The Definite Integral**

**Theorem**

*  = 
*  =  + 
*  =  - 

Kerjakan secara kelompok, 2 – 3 orang

Tentukan panjang busur

1. y3 = 8x2, dari x = 1 ke x = 8
2. 6xy = x4 + 3 , dari x = 1 ke x = 2
3. 27y2 = 4(x-2)3 , dari x = 2 ke x = 11
4. x = 5 cos , y = 5 sin 02
5. x = 2 cos + cos(2) + 1 , y = 2 sin + sin (2) , 0

Jawab

Kerjakan secara kelompok, 2 – 3 orang

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