Philosophical Grounds for Mathematics Research

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ABSTRACT

Some mathematicians have, for a long time, repeatedly been engaged in debates over paradoxes and difficulties they have seen emerging from the midst of their strongest and most intuitive mathematical research. From the rise of non-Euclidean geometry, to present-day problems in the analytic theory of the continuum, and from Cantor's discovery of a transfinite hierarchy to the fall of Frege's system, mathematicians have also voiced their concern at how we research our everyday intuitions in unfamiliar domains, and wildly research our mathematics where intuition either has given out, or becomes prone to new and hitherto unforeseen pitfalls, or outright contradiction. At the heart of mathematical research lies the task of isolating precisely what it is that our intuition provides us with, and deciding when we should be particularly circumspect about applying it; nevertheless, those who research an epistemologically satisfying account of the role of intuition in mathematics are often faced with an unappealing choice, between the smoky metaphysical research of Brouwer, and the mystical affidavit of Gödel and the Platonists that we can intuitively research the realm of mathematical truth.

Key Words: philosophical grounds, mathematics research

A. BACKGROUND

It was indicated that, in term of the research of its foundations, mathematics is perceived as logical science, cleanly structured, and well-founded or in short mathematics is a highly structured logical science; however if we dig deep enough and in depth research, we still find some sand that makes the discursion involves mathematical philosophy. It is the fact that, in term of the research of its foundations, an assortment of historical came, starting in ancient Greece, running through the turbulent present into an exiting future; while in term of logical foundation systems, the methods of mathematics are deductive, and logic therefore has a fundamental role in the development of mathematics. Suitable logical frameworks in which mathematics can be researched can therefore be called logical foundation systems for mathematics.

Some problems still arises: in term of meaning, we are wondered about the use of special languages for researching mathematics, whether they strange things or out of this world and what does it all mean?; and then, in the sense of ontology, we may wonder whether mathematicians talk about strange thing, whether they really exist, and how they can we tell or does it matter? In epistemological research, mathematics has often been presented as a paradigm of precision and certainty, but some writers have suggested that this is an illusion. How can we research the truth of mathematical propositions?; and in term of application, how can knowledge of abstract mathematics be applied in the real world?; what are the implications for mathematics of the information revolution?; and what can mathematics contribute?.

Thompson, P.,1993, insisted that the analysis combines a cognitive, psychological account of the great "intuitions" which are fundamental to research in mathematics, with an epistemic account of what role the intuitiveness of mathematical propositions should play in their justification. He examined that the extent to which our intuitive research are limited both by the nature of our sense-experience, and by our capacity for conceptualization

B. Mathematical Research among the Un-stability of Mathematics Foundations

Litlangs, 2004, confronted that Aristotle disagreed with Plato; according to Aristotle, forms were not entities remote from appearance but something which entered into objects of the world. Aristotle claimed that when we can abstract oneness or circularity, it does not mean that these abstractions represent something remote and eternal. For Aristotle, mathematics was simply reasoning about idealizations; and he looked closely at the structure of mathematics, distinguishing logic, principles used to demonstrate theorems, definitions and hypotheses. Plato also reflected on infinity, perceiving the difference between a potential infinity e.g. adding one to a number ad infinitum and a complete infinity e.g. number of points into which a line is divisible.

Bold, T., 2004, claimed that both the intuitionist and the formalist assured that research in mathematics are just inventions and do not inform us with anything about the world; both take this approach to explain the absolute certainty of mathematics and reject the use of infinity. Bold noted that intuitionist researchers admit this major similarity the formalist and note the difference as a disagreement on where mathematical exactness exist; the intuitionist says in the human intellect and the formalist says on paper. According to Arend Heyting, mathematics research is a production of the human mind; he claimed that intuitionism claims mathematical research inherit their certainty from human knowledge that is based on empirical experience.

Bold maintained that since, infinity can not be experienced, the intuitionist refuses to push application of mathematics beyond finite; while Heyting declared that faith in transcendental existence, unsupported by concepts, must be rejected as a means of mathematical proof. Similarly, Bold found that Hilbert wrote that for logical inferences to be reliable it must be possible to research these objects completely in their parts; since there is no such survey for infinity a reliable inference can only be based on a finite system. According to the formalist researchers, the whole of mathematics consists of only arbitrary rules like those of chess. Further, Bold, T., 2004, indicated that, on the other hand, the logicist researchers came close to proving that mathematics was a branch of logic.

According to Bold, the logicist researchers want to define mathematical concepts in terms of logical concepts and deduct mathematical propositions from logical axioms; as the basic elements of logic are sets and their properties, the logicists use sets to define mathematical concepts. Hilbert actually put a structure on the intuitive part of mathematics, essentially that of finitary thought and formal systems; with Gödel's work. Thompson, P.,1993, argued that the Gödelian brand of Platonism, in particular, takes its research from the actual experience of doing mathematics, and Gödel accounts for the obviousness of the elementary set-theoretical axioms by positing a faculty of mathematical intuition, analogous to sense-perception in physics, so that, presumably, the axioms 'force themselves upon us' much as the assumption of 'medium-sized physical objects' forces itself upon us as an explanation of our physical experiences.

However, Thompson stated that counterintuitive researcher has acquired an ambiguous role in our language use that is when applied to a strange but true principle; counterintuitive can now mean anything on a continuum from intuitively false to not intuitively true, depending on the strength of the conjecture we would have been predisposed to make against it, had we not seen, and been won over by, the proof; and indeed, to our surprise, we often find out, in times of paradox, how weak and defeatible our ordinary intuitions are. Thompson claimed that the very idea that our intuitions should be both decisive and failsafe, derives historically from the maelstrom of senses which the term 'intuition' has acquired in a series of primitive epistemic theories in which some of these senses have been inherited from the large role introspection played in the indubitable bedrock of Cartesian-style philosophy, and some simply from the pervasiveness of out-moded theological convictions which seek to make certain modes of justification unassailable.

On the other hand, Hilbert's formal system fits the theory of recursive functions. Brouwer was very much opposed to these ideas, especially that of formalizing systems; he even opposed the formalization of logic; Brouwer had a very radical view of mathematics and language's relationship. According to Brouwer, in language, we can communicate the output of mathematical research, thus helping others recreate the mathematical experience; however, the proof itself is a pre-linguistic, purely conscious activity which is much more flexible than language. Brouwer thought formal systems could never be adequate to cover all the flexible options available to the creative mathematician; and thought that formalism was absurd. He thought that it was crazy to think that codified logic could capture the rules for correct mathematical thought. Brouwer showed particular rules of logic are inadequate with the most famous of the law of the excluded middle.

Brouwer beliefs that the research to applicability of traditional logic to mathematics was caused historically; he next stated that by the fact that, firstly, classical logic was abstracted from the mathematics of subsets of a definite finite set, that, secondly, an a priori existence independent of mathematics was ascribed to this logic, and finally, on the basis of this suppositious apriority, it was unjustifiably applied to the mathematics of infinite sets. Brouwer’s hypothesis about the reason why philosophers and mathematicians included the law of the excluded middle; according him, logic was codified when the scientific community was concerned only with finite objects. Brouwer said that, considering only finite objects, the law of the excluded middle holds; however, a mistake was made when mathematics moved into the infinitary in which the rigid rules of logic were maintained without question.

Brouwer suggested that no rigid codification should come before the research of mathematics. The major distinction between Brouwer and Hilbert was that they disagreed on the position of logic in which Hilbert thought logic was an autonomous, finished science that could be freely applied to other mathematics, Brouwer argued that logic should only come after the mathematics is researched. Litlangs, 2004, in his overview, insisted that profound questions of how varied of intellect faces difficulties in research mathematics internally i.e. their gaps, contradictions and ambiguities that lie beneath the most certain of procedures, leads to rough conclusion that mathematics may be no more logical than poetry; it is just free research of the human mind that unaccountably give order to ourselves and the natural world.

Litlang, 2004, perceived that though mathematics might seem the clearest and most certain kind of knowledge we possess, there are problems just as serious as those in any other branch of philosophy about the nature of mathematics and the meaning of its propositions. He found that Plato believed in forms or ideas that were eternal, capable of precise definition and independent of perception; among such entities Plato included numbers and the objects of geometry such as lines, points, circles, which were therefore apprehended not with the senses but with reason; he deals with the objects of mathematics with specific instances of ideal forms. According to Plato, since the true propositions of mathematics were true of the unchangeable relations between unchangeable objects, they were inevitably true that is mathematics discovered pre-existing truths "out there" rather than created something from our mental predispositions; and as for the objects perceived by our senses, they are only poor and evanescent copies of the forms.

Meanwhile, Litlangs, 2004, insisted that Leibniz brought together logic and mathematics; however, whereas Aristotle used propositions of the subject i.e. predicate form, Leibniz argued that the subject contains the predicate that is a view that brought in infinity and God. According to Leibniz, mathematical propositions are not true because they deal in eternal or idealized entities, but because their denial is logically impossible; they are true not only of this world, or the world of eternal forms, but of all possible worlds. Litlangs insisted that unlike Plato, for whom research were adventitious aids, Leibniz saw the importance of notation, a symbolism of calculation, and so began what became very important in the twentieth century that is a method of forming and arranging characters and signs to represent the relationships between mathematical thoughts.

Litlangs, 2004, further stipulated that Immanuel Kant perceived mathematical entities as a-priori synthetic propositions, which of course provide the necessary conditions for objective experience; time and space were matrices, the containers holding the changing material of perception. According to Kant, mathematics was the description of space and time; if restricted to thought, mathematical concepts required only self-consistency, but the construction of such concepts involves space having a certain structure, which in Kant's day was described by Euclidean geometry. Litlangs noted that for Kant, the distinction between the abstract "two" and "two pears" is about construction plus empirical matter; in his analysis of infinity, Kant accepted Aristotle's distinction between potential and complete infinity, but did not think the latter was logically impossible. Kant perceived that complete infinity was an idea of reason, internally consistent, though of course never encountered in our world of sense perceptions.

Frege and Russell and their followers research Leibniz's idea that mathematics was something logically undeniable; Frege used general laws of logic plus definitions, formulating a symbolic notation for the reasoning required. However, through the long chains of reasoning, these symbols became less intuitively obvious, the transition being mediated by definitions. Russell saw them as notational conveniences, mere steps in the argument; while Frege saw them as implying something worthy of careful thought, often presenting key mathematical concepts from new angles. Litlangs found that while in Russell's case the definitions had no objective existence, in Frege's case the matter was not so clear that is the definitions were logical objects which claim an existence equal to other mathematical entities. Russell researchs, resolving and side-stepping many logical paradoxes, to create with Whitehead the monumental system of description and notation of the Principia Mathematica.

Meanwhile, Thompson, P.,1993, exposed the critical movement of Cauchy and Weierstrass to have been a caution or reserve over the mathematical use of the infinite, except as a façon de parler in summing series or taking limits, where it really behaved as a convenient metaphor, or mode of abbreviation, for clumsier expressions only involving finite numbers. Thompson claimed that when Cantor came on the scene, the German mathematician Leopold Kronecker, who had already 'constructively' re-written the theory of algebraic number fields, objected violently to Cantor's belief that, so long as logic was respected, statements about the completed infinite were perfectly significant. According to Thompson, Cantor had further urged that we should be fully prepared to use familiar words in altogether new contexts, or with reference to situations not previously envisaged.

Kronecker, however, felt that Cantor was blindly cashing finite schemas in infinite domains, both by attributing a cardinal to any aggregate whatsoever, finite or infinite, and worse still, in his subsequent elaboration of transfinite arithmetic. Thompson insisted that although the interim strain on the intuition, at the time, was crucial to Euler's heuristic research, this particular infinite detour had been analyzed out of his subsequent proofs of the result, which appeared almost 10 years after its discovery. Thompson, P.,1993, clarified that Gödel's feeling is that our intuition can be suitably extended to a familiarity with very strongly axiomatic domains, such as extensions of ZF, or calculus on smooth space-time manifolds, thereby providing us with backgrounds for either accepting or rejecting hypotheses independently of our pre-theoretic prejudices or preconceptions about them.

C. Mathematical Research Insides the Blow of Epistemology

1. The Cartesian Doubt and Kant's Synthetic A Priori

Turan, H., 2004, elaborated that Descartes called mathematical propositions into doubt as he impugned all beliefs concerning common-sense ontology by assuming that all beliefs derive from perception seems to rest on the presupposition that the Cartesian problem of doubt concerning mathematics is an instance of the problem of doubt concerning existence of substances. Turan argued that the problem is not whether we are counting actual objects or empty images but whether we are counting what we count correctly; he argued that Descartes's works is possible to expose that the proposition '2+3=5' and the argument 'I think, therefore I am,' were equally evident. According to Turan, Descartes does not found his epistemological investigation upon the evidence of mathematical propositions; and the doubt experiment does not seem to give positive results for mathematical operations.

According to Turan, consciousness of carrying out a mathematical research is immune to doubt; and statements of consciousness of mathematical or logical operations are instances of 'I think' and hence the argument 'I count, therefore I am' is equivalent to 'I think, therefore I am'. Turan indicated that if impugning the veridicality of mathematical propositions could not pose a difficulty for Descartes's epistemology which he thought to establish on consciousness of thinking alone, then he cannot be seen to avoid the question. Turan concluded that discarding mathematical propositions themselves on the grounds that they are not immune to doubt evoked by a powerful agent does not generate a substantial problem for Descartes provided that he believes that he can justify them by appeal to God's benevolence.

Turan, H., 2004, insisted that a relation between perception and mathematics is incontestable, however confining our thoughts to a context where the ontological presuppositions of un-philosophical reflection on perception are at stake; according to him, we must note the significance of perception with respect to the nature of existence that Descartes considers primarily for epistemological purposes. Turan noted that Descartes seems to abandon the deceiving God argument for the demon assumption and this last hypothesis seems to call into doubt exclusively beliefs related to existence of an external world; therefore, it is possible to argue that Descartes gave up researching the question concerning the veracity of the mathematical judgments; and Descartes seems to endow the evil genius solely with the power of deceiving him in matters related to judgments on the existence of external things.

Turan found that Descartes always considered mathematical research among the most evident truths that human mind can attain, and referred to them as examples of objects which can be intuited clearly and distinctly; Descartes perceived that arithmetic and geometry alone are free from any taint of falsity or uncertainty. According to Descartes, mathematical research is concerned with an object so pure and simple that they make no assumptions that experience might render uncertain; it consists in deducing conclusions by means of rational arguments. Next, Turan, H., 2004, insisted that

Descartes put on external existence of the objects in question; he considered both deduction and intuition as legitimate methods of researching mathematics.

For Descartes, intuition is simple indubitable conception of a clear and attentive mind which proceeds solely from the light of reason and on that account more certain than deduction, but deduction is not epistemologically inferior to intuition for the attentive human mind. Descartes claimed that although mathematics makes extensive use of deduction. Descartes does not say that deduction is the sole legitimate method of this domain and holds that intuition is as indispensable as deduction for the mathematical research; and mathematical research had the same degree of certainty as the indubitable ontological argument cogito. Turan indicated Descartes none the less always related the evidence of mathematical research to exactitude of mathematics which he thought to be deriving from the simplicity of their objects and hence to its ontological un problematical nature.

For Descartes mathematics is invariable with respect to ontological presuppositions, but once researched into the context of the doubt experiment it is seen that it bears crucial ontological implications that is it appears that mathematical objects and operations presuppose existence. Further, Turan, H., 2004, insisted that functional and ontological dependence of number and other universals, renders cogito in which an instance of thought where both evidence and ontological certainty could be researched in a single step; epistemologically prior to mathematical propositions which may, it researched apart from the context of the doubt experiment and seen to embody evidence. According to Turan, 'I count, therefore I am' is epistemologically equivalent to 'I think, therefore I am'; both arguments are immune to doubt; however, the evil genius can indeed make me go wrong as I count my thoughts or the appearances, but cannot deceive me in the inference I draw there from the fact that I am counting is sufficient to prove that I exist regardless of whether or not I count or add or perform any mathematical operation erroneously.

Turan concluded that the ontological research established by the Cartesian experiment of doubt brings in serious epistemological constraints; the researcher discovers that any epistemological means he may want to employ for a further ontological research must necessarily be one available from the proper resources of the ontological situation he has confined himself to for epistemological research; in other words, the epistemological standards of the research must conform to are determined by the ontological setting of the doubt experiment. Turan noted that hence the mathematical researcher finds him self alone with things which we may call perceptions or thoughts, at a standpoint from where he attests to happenings of perceptions and thoughts and cannot know well how they are procured; while Descartes could therefore depend solely on the thought that he has perceptions or thoughts in his epistemological research to establish a certainty which may not be affected by the arguments of the doubt experiment.

Posy, C., 1992, elaborated that prior to Kant, mathematics is about the research of the empirical world, but it is special in one important way that necessary properties of the world are found through mathematical proofs; however to prove something is wrong, one must show only that the world could be different. In term of epistemological research, Posy notified that sciences are basically generalizations from experience, but this can provide only contingent, possible properties of the world that is it could have been otherwise. On the other hand, science simply predicts that the future will mirror the past; while mathematics is about the empirical world, but usually methods for deriving knowledge give contingent knowledge, not the necessity that pure mathematics gives us; in sum, Posy concluded that Kant wants necessary knowledge with empirical knowledge.

Posy then exposed Kant’s solves the problem in a couple of steps: first, that objects in the empirical research are appearances or phenomenon in which, by their nature, they have only the properties that we come to know of them from experiences; they are not things in themselves. Posy found that Kant said we must become an idealist researcher in which object's properties are only what is perceivable; there are no non-experienceable properties of objects. Second, Kant[1] suggested to build into our minds two forms of intuition and perception such that every perception we have is shaped by the forms of Space and Time; according to Kant, these are, in fact, parts of the mind, and not something the mind picks up from experience; and thus, empirical objects are necessarily spacio-temporal objects.

Next, Posy, C., 1992, indicated that, according to Kant, we come to research spacio-temporal properties in an a priori fashion; and in studying spacio-temporal properties, we are merely researching ourselves, and our perceptual abilities. According to Kant, mathematics is simply the science that research the spacio-temporal properties of objects by studying the nature of space and time; and thus, mathematics is the researching of the abstract form of perception. In term of infinitary ideas that things is not subject to perception, Kant, as it was indicated by Posy, makes a distinction between empirical intuition that is the intuition from the senses which is always finite and pure intuition. Posy indicated that the research of possibilities for empirical intuition where finite limits are not introduced in either direction; and mathematics doesn't deal with this.

According to Kant mathematical research can allow the division of small intervals and the expansion of large intervals; this means we can discuss smaller and smaller quantities without introducing the smallest quantities e.g. if we want to prove an interval is divisible, we can do this by picking an interval; showing it is divisible; and abstracting from its actual size, and let it represent the notion of a perceivable interval. Kant claimed that pure mathematics, as synthetical cognition a priori, is only possible by referring to no other objects than those of the senses, in which, at the basis of their empirical intuition lies a pure intuition (of space and of time) which is a priori. Kant claimed that this is possible, because the latter intuition is nothing but the mere form of sensibility, which precedes the actual appearance of the objects, in that it, in fact, makes them possible; and yet this faculty of intuiting a priori affects not the matter of the phenomenon.

Kant illustrated that in ordinary and necessary procedure for geometrical research, all proofs of the complete congruence of two given figures come ultimately to this that they may be made to coincide; which is evidently nothing else than a synthetical proposition resting upon immediate intuition, and this intuition must be pure, or given a priori, otherwise the proposition could not rank as apodictically certain, but would have empirical certainty only. Kant further claimed that everywhere space has three dimensions, and that space cannot in any way have more, is based on the proposition that not more than three lines can intersect at right angles in one point. Kant argued that drawing the line to infinity and representing the series of changes e.g. spaces travers by motion can only attach to intuition, then he concluded that the basis of mathematics actually are pure intuitions; while the transcendental deduction of the notions of space and of time explains, at the same time, the possibility of pure mathematics.

Because it would be absurd to base an analytical judgment on experience, as our concept suffices for the research without requiring any testimony from experience, Kant concluded that Empirical judgments are always synthetical, e.g. “That body is extended” is a judgment established a priori, and not an empirical judgment. And also, for before appealing to experience, we already have all the conditions of the judgment in the concept, from which we have but to elicit the predicate according to the law of contradiction, and thereby to become conscious of the necessity of the judgment, Kant concluded that which experience could not even teach us. According to Kant, Mathematical judgments are all synthetical and he argued that this fact seems hitherto to have altogether escaped the observation of those who have analyzed human reason; it even seems directly opposed to all their conjectures, though incontestably certain, and most important in its consequences.

Further Kant claimed that for as it was found that the conclusions of mathematical research all proceed according to the law of contradiction, men persuaded themselves that the fundamental principles were known from the same law. “This was a great mistake”, he said. He then delivered the reason that for a synthetical proposition can indeed be comprehended according to the law of contradiction, but only by presupposing another synthetical proposition from which it follows, but never in itself. Similarly, Kant argued that all principles of geometry are no less analytical. He illustrated that the proposition “a straight line is the shortest path between two points”, is a synthetical proposition because the concept of straight contains nothing of quantity, but only a quality. He claimed that the attribute of shortness is therefore altogether additional, and cannot be obtained by any analysis of the concept; and its research must come to aid us; and therefore, it alone makes the synthesis possible.

2. Fallacies Of Intuition and the Ways Out

Thompson, P.,1993, elaborated that those who are eager to research how futile it is to try and demarcate or even seek out an epistemologically safe subsystem of pure intuitive propositions which could be used as the basis for an unproblematic branch of mathematics, also tend to emphasize how often we fail to discriminate reliable intuitions from processes known, post facto, to lead to false beliefs. Thompson argued that in his attack on various popular accounts of intuition, insofar as they claim that intuition provides us with an incorrigible a priori knowledge of mathematics, Philip Kitcher cites several episodes from the history of mathematics when mathematicians have hailed something as intuitively self-evident to give it much the same status as we give to the Zermelo-Fraenkel axioms of set theory, subsequently turned out to be false.

Thompson exposed that when Frege toke any property to determine a set in which this is by no means the only case of its kind; and the great Gauss and Cauchy went astray by surrendering themselves to the guidance of intuition, and earlier still many mathematicians of the 18th century believed in the self-evidence of the law of continuity which states that what holds up to the limit, also holds at the limit; this also turns out to be a natural fallacy. Thompson argued that this lead to disconcerting cases show that we cannot always apply Gödel's wedge and discriminate reliable or even a priori intuitions from processes known to lead to false beliefs.

Further, Thompson, P.,1993, insisted that in cases where experience suggests that the intuitive belief we have formed is misguided and this provides a stumbling-block for the thesis that our intuitions occupy the position of being a privileged warrant, by their very nature, for our beliefs, and somehow continue to justify them, whatever recalcitrant experience we come up against; similarly, the set-theoretical paradoxes threaten not so much the possibility of mathematical knowledge, as they now threaten either an a priori, or any other unduly perspicuous account of its nature. Thompson concluded that these fallacies of intuition then, have gained a significant in the contemporary epistemological research of mathematics, in which Georg Kreisel suggests that it has been somewhat overplayed; he claimed that this, no doubt, results from our memory bias which makes us, for the most part, recall surprises, memorable cases in which strong initial impressions were later disconfirmed, and ultimately it also leads to an overestimate of the dangers of intuitive thinking.

Next, Thompson, P.,1993, in the sense of catching strong postulates in a broader intuitive net, insisted that there are several types of cut-off arguments which seem devastating against any ramifying plan such as that advocated by Gödel; by way of illustration, one of Gödel's original arguments in favor of the un-solvability of the generalized continuum problem, seemed to indicate intuitively that the continuum hypothesis will ultimately turn out to be wrong, while, on the other hand, we know that its disproof is demonstrably impossible, on the basis of the axioms being used today. Thompson indicated that even our schematic means of definition in creating an apparently substantial hierarchy by recursion of our intuitive operations over the countable ordinals, guarantees that we have insidiously conferred an unwanted simplicity on what point-sets we are equipped with, to act as feedstock for ramifying our intuition.

Thompson also indicated that in-building the cognitive tendency, which hampers our attempts to ramify our intuition, we research our mathematics into strongly-axiomatized domains, where new principles have a much freer rein than before, so that the potential domain of their application outstrips what we can readily specify using our old schemas, even suitably bolstered by using transfinite induction, or recursion, as ramifiers. Thompson argued that, consequently, any familiarity we pretend to research with these domains will be largely mediated by schemas developed on the subsystem, which we must therefore guard ourselves against cashing - as far as is consciously possible that is in the surrounding global domain. Thompson summed up that inability to escape from intuiting formally simple subsystems of those domains into which we research our mathematics, guarantees that the progress of ramifying our intuition will inevitably be skeletal.

Further, Thompson, P.,1993, argued that the progressive insinuation into the epistemologically-safer sub-domains of mathematics, can be partially held back by a revisionist struggle, such as that advocated by Hermann Weyl whici consists of successively: updating, altering and refining our naive intuitions to diminish Frege's qualm; and subsequently decreasing the shortfall between our formal systems and the intuitions of the day, which they claim to represent i.e. reducing Brouwer's qualm. Thompson claimed by this way, the conclusions of mathematical research will not be intuitively false, but simply not intuitively true, and the candidates for appraisal will behave like targets which are no longer just very far from the archer, but no longer even visible at all. In the sense of the analysts distance themselves from geometrical research and its role in extension problems, Thompson insisted that the 19th century belief that our geometrical prejudices should be isolated and withdrawn from the formal presentation of proofs in analysis, led to the idea that our basic intuitions were too weak to have any decisive role to play in the subsequent development of mathematics; this, however, often meant that we had now begun to notice when inappropriate schemas of research were being used, or that we had become impatient on noticing that their unquestionable success at the conjectural stage.

3. Mathematical Research by Nurturing Intuition

Gödel explained our surprise at the emergence of paradoxes such as Peano's construction of space-filling curves, or Weierstrass's discovery of continuous but nowhere-differentiable functions, by accusing us of carelessly mixing our pre-theoretic intuitions, with our more refined, analytic and topological ones; such a clash, between familiar geometry, say, and the set-theoretic reduction of point-sets, will undoubtedly arise at some stage of mathematical research; the paradoxical appearance can be explained by a lack of agreement between our intuitive geometrical concepts and the set-theoretical ones occurring in the theorem; therefore, he suggested that we must drive a wedge between our pre-formal and formal intuitions, in the hope of separating out errors coming from using the pre-theoretical intuition.

Thompson, P.,1993, insisted that Gödel suggestion exercises in discrimination seems notoriously difficult to research, especially when it is tempting to refine intuitions of one generation; he claimed that far from being a once-and-for-all clarification of our logical optics, they have historically either turned out to be fallacies, or at best become the naivest intuitions of the next. On the other hand, Thompson indicated that when Frege speaks that the truths of Euclidean geometry, as governing all that is spatially intuitable, looks as though, at last, we may have found a domain in which our intuitions are constrained and held within strict and well-defined bounds. He insisted that while the patterns we are trained to recognise are codified as schemas of mathematical research, the schemas we are most keen to apply are occasionally poorly-tuned, not suitable for the context, or totally in default when we project them into new context of the research; they may be indispensable as a heuristic, but the fact that they are so familiar often seduces us into the jaws of paradox.

Thompson, P.,1993, further claimed that if intuition in mathematics is properly characterized as a living growing element of our intellect, an intellectual versatility with our present concepts about abstract structures and the relations between these structures, we must recognize that its content is variable and subject to cultural forces in much the same way as any other cultural element. Thompson insisted that even the symbols designed for the expression and research of mathematics have variable meanings, often representing different things in the 19th and 20th centuries, by virtue of the underlying evolution of mathematical thought; it must therefore remain an important strategy to aim to research an increasingly versatile and expressive medium for the representation of familiar ideas. Further, Thompson argued that as researcher of mathematics, with increasingly abstract material, it seems that the ability to reason formally, which requires the explicit formulation of ideas, together with the ability to show ideas to be logically derivable from other and more generally accepted ideas, are great assets in broadening the scope and range of the schemas of the research which become second nature to us, and are instrumental in extending the familiar territory of our intuition.

4.Mathematical Research in the Perspective of Quasi-empiricism

It was elaborated that Quasi-empiricism in mathematics is the movement in the philosophy of mathematics to reject the foundations problem in mathematics, and re-focus philosophers on mathematical practice itself, in particular relations with physics and social sciences; a key argument is that mathematics and physics as perceived by humans have grown together, may simply reflect human cognitive bias, and that the rigorous application of empirical methods or mathematical practice in either field is insufficient to disprove credible alternate approaches. Hilary Putnam argued convincingly in 1975 that real mathematics had accepted informal proofs and proof by authority, and made and corrected errors all through its history, and that Euclid's system of proving theorems about geometry was peculiar to the classical Greeks and did not evolve in other mathematical research in China, India, and Arabia.

Further, it was indicated that this and other evidence led many mathematicians to reject the label of Platonists, along with Plato's ontology and the methods and epistemology of Aristotle, had served as a foundation ontology for the Western world since its beginnings. On the other hand, Putnam and others argued that it necessarily be at least 'quasi'-empirical that is embracing 'the scientific method' for consensus if not experiment. However, Koetsier, T., 1991, indicated that Mac Lane encouraged philosophers to renew the research of the philosophy of mathematics, a subject which he described as being "dormant since about 1931"; while Putnam concluded that none of the existing views on the nature of mathematics were valid and Goodman argued that the four major research in the philosophy of mathematics that are formalism, intuitionism, logicism and platonism, arise from an oversimplification of what happens when we research mathematics.

Koetsier noted that from Goodman's point of view a more adequate philosophy of mathematics had yet to be formulated; on the other hand, Tymoczko stated that previous anthology delineates quasi-empiricism as a coherent and increasingly popular approach to the philosophy of mathematics. For Tymoczko, quasi-empiricism is a philosophical position, or rather a set of related philosophical positions, that attempts to research the mathematical experience by taking the actual practice of mathematics seriously; he claimed that if we look at mathematics without prejudice, many features will stand out as relevant that were ignored by the foundationalists i.e. informal proofs, historical development, the possibility of mathematical error, mathematical explanations, communication among mathematicians, the use of computers in modern mathematics, and many more.

Further, Koetsier, T., 1991, indicated that Lakatos researched two different kinds of theories i.e. quasi-empirical theories and Euclidean theories; Lakatos defined Euclidean theories as theories in which the characteristic truth flow inundating the whole system goes from the top, the axioms, down to the bottom; and defined quasi-empirical theories as theories in which the crucial truth flow is the upward transmission of falsity from the basic statements to the axioms. Koetsier noted that, attacking the foundationalist illusion that there exists a means of finding a foundation for mathematics which will be satisfactory once and for all, Lakatos argued that mathematics is not Euclidean, but instead quasi-empirical; carried away by his own reasoning and wishing to show the fallibility of mathematics in the sense of Popper's falsificationism, Lakatos researched that there is an upward transmission of falsity in mathematics, but it is not the crucial truth flow.

Koetsier found that Putnam defending the point of view that mathematics is quasi-empirical; Putnam argued that mathematical knowledge resembles empirical knowledge in which the criterion of truth in mathematics just as much as in physics is success of our ideas in practice, and that mathematical knowledge is corrigible and not absolute. Next, Koetsier, T., 1991, found that Putnam presented his quasi-empirical realism as a modification of Quine's holism in which it consists of the view that science as a whole is one comprehensive explanatory theory, justified by its ability to explain sensations; according to Quine, mathematics and logic are part of this theory, differing from natural science in the sense that they assume a very central position. Kotsier insisted that since giving up logical or mathematical truths causes great upheaval in the network of our knowledge, they are not given up; according to him, mathematics and logic are no different from natural science.

Koetsier insisted that Putnam's quasi-empirical realism consisted of Quine's view, but with two modifications; first, Putnam added combinatorial facts e.g. the fact that a finite collection always receives the same count no matter in what order it is counted, to sensations as elements that mathematical theorems must be researched.; secondly, Putnam required that there be agreement between mathematical theory and mathematical intuitions whatever their source e.g. the self-evidence of the Comprehension Axioms in set theory. Koetsier notified that both Lakatos and Putnam researched mathematical theories to be interrelated sets of statements that are considered to be true. Koetsier concluded that the quasi-empirical element in their positions is the fact that they reject the view that, in principle, mathematics could be researched in an Euclidean way in Lakatos's sense of the term.

Further, Koetsier, T., 1991, maintained that both Lakatos and Putnam argue that, to a certain extent, mathematical theories always possess a hypothetical status; in that respect mathematical knowledge resembles empirical knowledge. According to Koetsier, Lakatos's position can be summarized in the form of two theses i.e. Lakatos’s fallibility thesis and Lakatos’s rationality thesis. Koetsier described that in the Lakatos's fallibility thesis, fallibility is an essential characteristic of mathematical knowledge and most philosophies of mathematics are infallibilist; infallibilists argue that, although in practice mathematicians make mistakes, mathematical knowledge is essentially infallible. In fact Lakatos's quasi-empiricism consists in the fallibility thesis; although their subject matter is different, mathematical theories and empirical theories have in common the fact that they are fallible.

D. Conclusion

Thompson indicated that the mathematical research, as in the Gödel and Herbrand sense, with regard to their claims to be collectively demarcating the limits of intuitive computability, is a feature of this particular problem that it is susceptible to a diversity of equally restrictive intuitive re-characterizations, whose unexpected confluence gives each of them a strong intuitive recommendation and this confluence turns out to be a surprisingly valuable asset in appraising our rather more recondite extensions of our intuitive concepts. Thompson concluded that Gödel, with his basic trust in transcendental logic, likes to think that our logical optics is only slightly out of focus, and hopes that after some minor correction of it, we shall research sharply, and then everyone will agree that we are right; however, he who does not share such a trust will be disturbed by the high degree of arbitrariness in a system like Zermelo's, or even in Hilbert's system. Thompson suggested that Hilbert will not be able to assure us of consistency forever; therefore we must be content if a simple axiomatic system of mathematics has met the test of our mathematical research so far.

Kant researched the previous geometers assumption which claimed that other mathematical principles are indeed actually analytical and depend on the law of contradiction. However, he strived to research that in the case of identical propositions, as a method of concatenation, and not as principles, e. g., a=a, the whole is equal to itself, or a + b > a, the whole is greater than its part. He then claimed that although they are recognized as valid from mere concepts, they are only admitted in mathematics, because they can be represented in some visual form. Posy concluded that there are two consequences of Kantian view that no such thing as unapplied mathematics i.e.

mathematics is, by nature, about the world; if it's not, it's just an abstract game; and that there is exactly one right mathematical theory of time, space, and motion.

Thompson then summed up that during all but a vanishingly small proportion of the time spent in mathematical research, we seem to be somewhere between having no evidence at all for our conclusions of mathematical research, and actually knowing them; second, that during this time, intuition often comes to the forefront, both as a source of mathematical research, and of epistemic support; third, that our intuitive judgments in these situations are often biased, but in a predictable manner. Ultimately, Thompson concluded that although any satisfactory of research analysis of the role of intuition in mathematics should recognize it as a versatility in measuring up new situations, or even conjecturing them, using a rich repository of recurrent and strategically-important schemas or conceptual structures, painstakingly abstracted from sensory experience by the intellect, constrained by the languages available to us at the time, and influenced by the accumulated resources of our cultural and scientific heritage.

What intuition does not do is constitute a research gained by reason, through some remarkable clairvoyant power that is an insight, which, for Gödel, seemingly paved the way towards a crystal-clear apocalyptic vision of mathematical research, or, for Descartes, paved the research into the ultimate structure of the human mind. In Lakatos's rationality thesis, as it is characterized as fallible, the development of mathematical research is not completely arbitrary, but possesses its own rationality. Koetsier insisted that fallible mathematical knowledge is replaced by other fallible knowledge in accordance with certain norms of rationality; most of Lakatos's work with respect to mathematics concentrates on the rationality thesis; a rational reconstruction is a reconstruction that is explicitly based on a particular method of mathematical research.

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