

**LESSON STUDY:  
Promoting Student Thinking on the Concept of  
Least Common Multiple (LCM) Through Realistic Approach in the 4<sup>th</sup> Grade of  
Primary Mathematics Teaching**

**Marsigit, Atmini Dhoruri, Sugiman, Ali Mahmudi  
The State University of Yogyakarta, Indonesia**

*The aim of this paper is to encapsulate, through Lesson Study, the picture of mathematical thinking that is students thinking on the concept of Lowest Common Multiple (LCM) at the 4<sup>th</sup> Grade Students of Primary School in Indonesia. With the ground of the New School-Based Curriculum in which we defined our mathematics, we, in collaboration with teacher, prepared teaching learning of LCM with Realistic Approach. The results of the study indicated that the characteristics of realistic approach in which the students thinking the concept of LCM were uncovered.*

## **INTRODUCTION**

Based on the Ministerial Decree No 22, 23, 24 year 2006, starting June 2006, Indonesia Government committed to implement the new curriculum for primary and secondary education, called KTSP “School-Based Curriculum”. This School-based curriculum combines two paradigms in which, one side, stressing on students competencies but also, on the other side, concerning students’ learning processes. The National Standard of Competencies is then to be elaborated to be the Basic-Competencies that is the minimum competencies that should be performed by the students, covering affective, cognitive and psychomotor competencies. Indonesian Government has developed Contextual teaching and learning (CTL) and Realistic to support School-Based Curriculum implementations; it means that the government encourages the teachers to develop students life skills by employing optimally the environment to support students’ activities. The scope of primary mathematics: Numbers, Geometry and Measurements, and Analysing Data

In the National Standard for the content of our New School-Based Mathematics Curriculum for Primary School, it was stated that mathematics is the universal science and it is the ground of other sciences. Currently fascinating achievement in information technology and other subjects are the effects of good achievements in mathematics. Therefore, to gain the better future it needs good achievements in mathematics since the youth.

In the Guide Line for developing School-Based Curriculum, it was stated that mathematics in primary school have its function to encourage the students to think logically, analytically, systematically, critically, creatively and be able to collaborate with others. Those competencies are needed for the students in order that they can get, access and employ information to preserve their live. Mathematical thinking is defined as students’ activities to communicate mathematical ideas in

which it involves the using of symbols, tables, diagrams and other sources in other that the students are able to solve their problems.

The implementations of primary mathematics curriculum in class-rooms thus need to develop problem solving skills covering both closed and open problems. In solving the problems, students need to creatively develop many ways and alternatives, to develop mathematical models, and to estimate the results. It was suggested that in teaching learning of primary mathematics, students have the chances to identify mathematical problems contextually and realistically.

Contextual and realistic approaches are recommended to be developed by the teachers to encourage mathematical thinking in primary schools. With these approaches, there is a hope that the students step-by-step learn and master mathematics enthusiastically. To make their teaching learning of primary mathematics more effective, teachers also need to develop resources such as information technology, teaching aids and other media.

Specifically, our primary mathematics curriculum outlines the aims of teaching learning of mathematics are as follows:

1. to understand the concepts of mathematics, to explain the relationships among them and to apply them to solve the problems accurately and efficiently.
2. to develop thinking skills to learn patterns and characteristics of mathematics, to manipulate them in order to generalize, to proof and to explain ideas and mathematics propositions.
3. to develop problems solving skills which covers understanding the problems, outlining mathematical models, solving them and estimating the outcomes.
4. to communicate mathematics ideas using symbols, tables, diagrams and other media.
5. to develop appreciations of the uses of mathematics in daily lifes, curiosity, consideration, and willingness to learn mathematics as well as tough and self-confidence.

As the Programme for International Student Assesment (PISA) concerned with the capacities of students to analyse, reason, and communicate ideas effectively as they pose, formulate and interpret mathematics in a variety of situations. In line with the effort to prepare students for society and for future schooling and work; and thus, it is reasonable for Indonesian government to implement Realistic Mathematics Education (RME) in the schema of Lesson Study, in which the extents of above assues are able to be handled. Realistic Mathematics Education (RME) in the schema of Lesson Study let the teachers to improve instructional approach from traditional to progressive one.

## **THEORETICAL FRAMEWORK**

Isoda, M. (2006), outlined that mathematical thinking is open ideas; thus, it is very difficult to discuss its development without having a window to discuss. When we focus on each lesson, we easily focus on specific knowledge and skills (Understanding), and easily forget to develop Attitude, Mathematical Thinking and Representation. The present form of RME is mostly determined by

Freudenthal's view on mathematics (Freudenthal, 1991). Two of his important points of views are mathematics must be connected to reality and mathematics as human activity.

First, mathematics must be close to children and be relevant to every day life situations. However, the word 'realistic', refers not just to the connection with the real-world, but also refers to problem situations which real in students' mind (Zulkardi, 2006). For the problems to be presented to the students this means that the context can be a real-world but this is not always necessary. De Lange (1996) stated that problem situations can also be seen as applications or modeling.

Second, the idea of mathematics as a human activity is stressed. Mathematics education organized as a process of *guided reinvention*, where students can experience a similar process compared to the process by which mathematics was invented. The meaning of invention is steps in learning processes while the meaning of guided is the instructional environment of the learning process. Moreover, the reinvention principle can also be inspired by informal solution procedures. Informal strategies of students can often be interpreted as anticipating more formal procedures (Zulkardi, 2006).

Two types of mathematization which were formulated explicitly in an educational context by Treffers, 1987, in Zulkardi, 2006, are horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. The following activities are examples of horizontal mathematization: identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known mathematical problem (Zulkardi, 2006). On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. The following activities are example of vertical mathematization: representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing.

*Realistic approach*, a real-world situation or a context problem is taken as the starting point of learning mathematics. And then it is explored by horizontal mathematization activities. This means students organize the problem, try to identify the mathematical aspects of the problem, and discover regularities and relations. Then, by using vertical mathematization students develop mathematical concepts. Following figure illustrates the process of reinvention in which both the horizontal and vertical mathematization take place in order to develop basic concepts of mathematics or formal mathematical language.

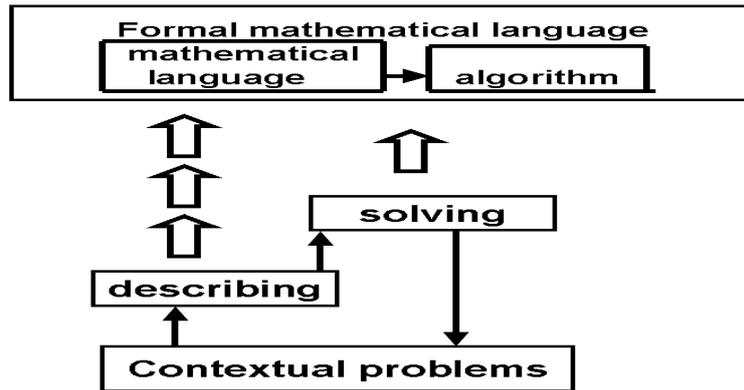
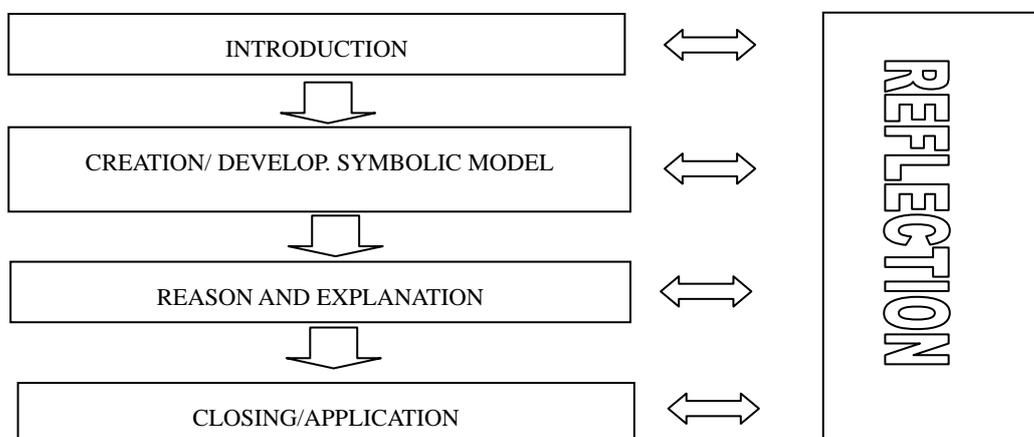


Figure 1. Guided Reinvention model (Gravenmeijer, 1994)

The learning process starts from contextual problems. Using activities in the horizontal mathematization, for instance, the student gains an informal or a formal mathematical model. By implementing activities such as solving, comparing and discussing, the student deals with vertical mathematization and ends up with the mathematical solution. Then, the student interprets the solution as well as the strategy which was used to another contextual problem.

The mathematics lessons that will be designed in *Realistic Approach* should represent the characteristics of how the students do mathematization. The way of embed these characteristics into the lesson plan components can be seen in the following diagram (Zulkardi, 2006):



## PROMOTING STUDENT THINKING ON THE CONCEPT OF LEAST COMMON MULTIPLE (LCM) THROUGH REALISTIC APPROACH IN THE 4<sup>TH</sup> GRADE OF PRIMARY MATHEMATICS TEACHING

### Search Method

The search began with two series of discussions between teachers and lectures and followed by observing and reflecting two lesson activities in the class, as the following description:

**Primary School** : SD Percobaan 2 Yogyakarta, Indonesia  
**Grade/Sem/year** : IV/Sem I/2006  
**Teacher** : Budiyati  
**Number of Students** : 44  
**Standard Competency** : To Understand and to apply factors and multiple of numbers to solve problems.  
**Base Competencies** : 1. to understand the Least Common Multiple (LCM)  
 2. to determine the Least Common Multiple (LCM)  
 3. to solve problems which is related to LCM  
  
**Lesson I** : **Students are to understand Common Multiple (CM)**  
**(Day: Tuesday, 12 October 2006, Time: 07.00 – 09.00)**

Activities:

1. Introduction
2. Describing *prepared contextual problems* (problems situated in reality as follow):  
*Since the early of the year 2006, Shinta has two activitis i.e. swimming and gardening. She is periodically going to swim once a week and gardening every 8 days, as shown in the following callendar:*

November 2006						
Monday	Tuesday	Wednesd	Tuesday	Friday	Saturday	Sunday
		1	②	3	△4	5
6	7	8	⑨	10	11	△12
13	14	15	⑩	17	18	19
△20	21	22	⑪	24	25	26
27	△28	29	⑫			

 : Swimming (once a weeks)  
 : Gardening (once in 8 days)

**Question:**

*When Shinta is going for swimming and gardening on the same days?*

3. Developing (group discussion)
4. Reason and explanation (presentation)
5. Conclusion (homework)
6. Closing

**Lesson II : Students are to understand Least Common Multiple (LCM)  
(Day: Friday, 13 October 2006, Time: 07.00-08.30)**

Activities:

1. Introduction
2. Continue to developing (group discussion)
3. Reason and explanation (presentation)
4. Conclusion (homework)
5. Closing

### ANALYSES VIDEOTAPED LESSON I



Introduction:

Apperception (prepare for emotional and awareness)

- Teacher started the lesson by delivering the question of whose the students have routine activities.

- Analyses:

*For a moment teacher could not identify the satisfied problems from students, and therefore she posed the prepared problem*

Preparing for group discussion:

- The teacher distributed prepared problem written in Worksheet to every student. The teacher let the student first worked individually i.e. read and learn the contextual problem in calendar format:

- Analyses:

*The calendar format lead the students to develop their solution also in calendar format.*



Developing in Group Discussion:

- The teacher let the students to have group discussion to solve the problem..

- Analyses:

*Most of the group employed calendar to find the multiple of 7 days and the multiple of 8 days in one year.*

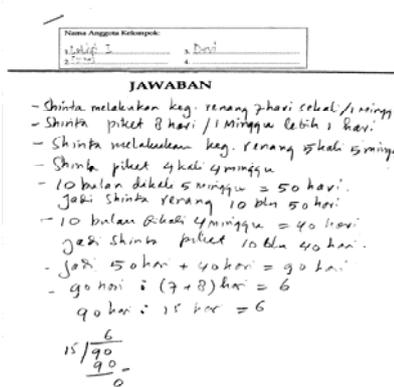
*Most of the group constructed the complete one year calendar.*





### Monitoring developing in group discussion

- The teacher let the students to develop their own methods and resources
- Analyses:  
*The students by themselves employed various aids to solve the problems such as calendar, hand-phone, and blank table prepared by the teacher.*
- The teacher let the students to continue to solve the problem as home-work.



Translation of one of the the result of group discussion:

- Shinta goes to swim once every 7 days and to gardening once every 8 days
- Shinta goes to swim 5 times a month and to gardening 4 times a month
- For 10 month, Shinta goes to swim  $10 \times 5 = 50$  times
- For 10 month, Shinta goes to gardening  $10 \times 4 = 40$  times
- Total number of Shinta's activities = 50 times + 40 times = 90 times
- 90 times : (7 + 8) days = 6
- 90 times : 15 days = 6

Analyses:

*The series of sentences produces by the group indicated first horizontal mathematization then followed by vertical mathematizaion.*

## ANALYSES VIDEOTAPED LESSON II



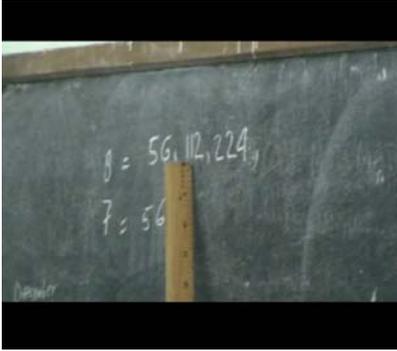
Reflection (reason and explanation):

- Representatives of selective group presented their results of yesterday discussion.
- Analyses:  
*Most of the students employed subtraction, addition, multiplication and division to list multiple of 7 and 8.*  
*Most of the students indicated Common Multiple , as the mathematical concepts to answer the common Shintas activities in one year.*



Reflection (reason and explanation):

- Student explaining their finding/solution of the problems
- Analyses:  
*There was a transition from daily language of mathematical language i.e. from common activities to common multiple.*  
*There was a student who jumped their concept to LCM due to he got it from "informal private lesson".*



### Vertical Mathematization → Conclusion

- Analyses:
  - *The teacher encouraged the students to list more the multiple of 7 and the multiple of 8*
  - *The teacher encouraged the students to indicate the common multiple of 7 and 8.*
  - *The teacher let the students to find the Least Common Multiple of 7 and 8, i.e 56*
  - *The teacher delivered class assignment to find the LCM of 2 and 3, and pointed out some students to write their answer in front of the class*

## Discussion

The search in this lesson study strived to uncover the idea of mathematics as a human activity that is stressed in realistics approach. Teacher organized the class as a process of *guided reinvention* (De Lange, 1996, in Zulkardi, 2006) that is to step in learning LCM by developing instructional environment e.g. let the students to freely chose and develop their methods and aids to solve the problems. The teacher let the students to work individually and in group less formally to perform horizontal mathematization; and then anticipating the structure to more formal raise mathematization activities.

From the analyses of videotaped lesson, it was indicated that the students strived to develop horizontal mathematization through some following activities:

- a. identifying or describing the specific mathematics :
  - in the routine activities there are the concept of addition and subtraction i.e.
    - $7 + 7 + 7 + 7 \dots$  or subtracting by 7 (for swimming)
    - $8 + 8 + 8 + 8 + 8 \dots$  or subtracting by 8 (for gardening)
  - in the question of “ how many times common activities” there is the concept of “frequency” or “repeating addition or subtraction” i.e. th concept of multiple of number:
    - For 10 month, Shinta goes to swim  $10 \times 5 = 50$  times
    - For 10 month, Shinta goes to gardening  $10 \times 4 = 40$  times
- b. schematizing, formulating and visualizing a problem in different ways
  - there are various ways in determining the multiple number of 7 and 8 e.g. using calendar, using series of numbers, using calculator and manipulating different symbols for 7 and 8.
  - There are different schemas on determining the common multiple of 7 and 8 i.e. some students calculate the multiple of 7 for the whole year first then multiple for 8; and followed by counting the number of common activities in one year. Some students

indicated first the common multiple of 7 and 8 (i.e. 56) and then counting the number of common activities in one year.

- c. discovering relations
  - the students discovered the relationship between “common activities” and ”common multiple” i.e. 7 days and 8 days compare with “multiple of 7 and 8 = 56”
- d. discovering regularities
  - The concepts of regularities arise from the concepts of “routine activities”
- e. recognizing isomorphic aspect in different problems
  - The students identified that the activities to be manipulated were not only about “swimming” and “gardening”, but also for others their daily activities such as “study club”, “laboratory activities” or “going to library”
- f. transferring a real world problem to a mathematical problem
  - There are the key concepts reflecting by the key word of how the students can transfer the real world problems to mathematical problem e.g. the concepts of “common”, “regular”, “routine”, “number of”, etc.

regular	→ to add regularly: $7 + 7 + 7 + 7 + 7 \dots$
common activities	→ common multiple (56)
number of common activities	→ number of common multiple.

On the other hand, the following results indicated that the students seemed to develop vertical mathematization through some aspects:

- Shinta goes to swim once every 7 days and to gardening once every 8 days
- Shinta goes to swim 5 times a month and to gardening 4 times a month
- For 10 months, Shinta goes to swim  $10 \times 5 = 50$  times
- For 10 months, Shinta goes to gardening  $10 \times 4 = 40$  times
- Total number of Shinta's activities = 50 times + 40 times = 90 times
- 90 times : (7 + 8) days = 6
- 90 times : 15 days = 6

- a. The students strived to represent daily problems in a related mathematical formula
- b. The students strived to prove regularities of consisting concepts
- c. Some students refined and adjusting models by not continuing to complete the whole year calendar; and found effective ways to count the common multiple.
- d. Some students performed vertical mathematization by employing different models
- e. Some students formulating a mathematical model, and generalizing in such way that they found  $90 \text{ times} : 15 \text{ days} = 6$ .

## Conclusion

In this Lesson Study, the researchers had sought to encapsulate the picture in which the teacher strived to promote Student Thinking on the Concept of Least Common Multiple (LCM) Through Realistic Approach. The striking results of the study illustrated that :

1. Students' thinkings of the concept of LCM were much contributed by teacher's employing real-life contexts as a starting point for their learning.
2. The "calendar format problem" was the useful models for the students to bridge mathematical thinking between abstract and real, and helped students to learn LCM at different levels of abstractions.
3. Students' thinking of the concept of LCM simultaneously affected by the use of their own productions of formulas and strategies
4. In thinking the concept of LCM, interactions between teacher and students, students and students are the essential activities.
5. Students' thinkings of the concepts of LCM were influenced by the connection among the strands of mathematical concepts developed previously e.g. the concept of factor of numbers, concept of common multiple (CM) and by the connection with meaningful problems in the real world.

## Reference :

- Ernest, P. (1991). *The Philosophy of Mathematics Education*. Hampshire: The Falmer Press.
- Freudenthal, H. (1991). *Revisiting Mathematics Education. China Lectures*. Dordrecht: Kluwer Academic Publishers.
- Gravemeijer, K.P.E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β
- Isoda, M. (2006). *First Announcement : APEC-Tsukuba International Conference on Innovative Teaching Mathematics Through Lesson Study (II) – Focussing on Mathematical Thinking- December 2-7, 2006, Tokyo & Sapporo, Japan*
- Lange, J. de (2006). *Mathematical Literacy for Living From OECD-PISA Perspective, Tokyo: Symposium on International Cooperation*
- Sugiman (2005), *Changing Instructional Approach of Indonesian Mathematics Teachers with RME. ITB: Seminar on RME*
- Zulkardi (2006). *How to Design Mathematics Lessons based on the Realistic Approach?* Retrieved 2006 <<http://www.google.com>>

## Appendix 1: Lesson Plan

SCENARIO OF TEACHING	
Topic	: Multiple Numbr
Class/Sem	: IV/I
Time	: 2 x 60'
A	Standard Competencies
	<ol style="list-style-type: none"> <li>1. Understand multiple number</li> <li>2. Use multiple number to solve problems</li> </ol>
B	Base Competencies
	<ol style="list-style-type: none"> <li>1. Determine multiple number</li> <li>2. Solve problems which are related to multiple number</li> </ol>
C	Objective
	<ol style="list-style-type: none"> <li>1. Students are able to determine multiple number</li> <li>2. Students are able to solve problems using multiple number</li> </ol>
D	Teaching Material : Story Problems
E	Teahing Learning Processes
	<ol style="list-style-type: none"> <li>I. Introduction (10')               <ol style="list-style-type: none"> <li>a. praying</li> <li>b. question-answer</li> <li>c. grouping</li> </ol> </li> </ol>
	<ol style="list-style-type: none"> <li>II. Main Activity (30')               <ol style="list-style-type: none"> <li>a. the students get the problems and should be done in the group</li> <li>b. group discussion</li> </ol> </li> </ol>
	<ol style="list-style-type: none"> <li>III. Closing (20')               <ol style="list-style-type: none"> <li>a. students' reflection on the their process and results</li> <li>b. conclusion: LCM?</li> </ol> </li> </ol>
F	Resources/Media
	<ol style="list-style-type: none"> <li>a. Resources: School-Based Curriculum 2006</li> <li>b. Media: worksheet, calendar</li> </ol>

## Appendix 2: Teaching Material

Since the early of the year 2006, Shinta has two activities i.e. swimming and gardening. She is periodically going to swim once a week and gardening every 8 days, as shown in the following calendar:

November 2006						
Senin	Selasa	Rabu	Kamis	Jum'at	Sabtu	Minggu
		1	②	3	△4	5
6	7	8	⑨	10	11	△12
13	14	15	⑩16	17	18	19
△20	21	22	⑪23	24	25	26
27	△28	29	⑫30			

○ : Swimming (one a week)  
 △ : Gardening (once in eight days)

*Question:*

When Shinta is going for swimming and gardening on the same days?

## Appendix 3:

### National Standard Competency

#### Grade IV, Semester 1

Standard Competency	Base Competency
<b>Numbers</b> 1. To understand and use the characteristic of numbers operation to solve problems.	1.1 To identify the characteristics of numbers operation 1.2 To order the numbers 1.3 To do multiplication and division operations. 1.4 To do mix number operation 1.5 Estimating and rounding 1.6 To solve problems related to money
2. To understand and use factors and multiple to solve problems	2.1 To describe the concepts of factors and multiple 2.2 To determine multiple and factors of numbers 2.3 To determine Least Common Multiple and Greatest Common Factors 2.4 To solve problems related to Least Common Multiple and Greatest Common Factors
<b>Geometriy and Measurement</b> 3. To use the measurement of angles, distance and weight to solve problems.	3.1 To determine the size of angle with non-standar unit and degree unit 3.2 To determine the relationship between time unit, length unit and weight unit 3.3 To solve problems related to time unit, length unit and weight unit 3.4 To solve problems related to quantity units
4. To use the concepts of perimeter and area of simple plane geometrical shape to solve problems.	4.1 To determine perimeter and area of paralellogram and triangle. 4.2 To solve problems related to perimeter and area of paralellogram and triangle.