PROMOTING PRIMARY AND SECONDARY MATHEMATICAL THINKING THROUGH THE SERIES OF SCHOOL-BASED LESSON STUDY ACTIVITIES

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ABSTRACT

The aim of the study is to promote mathematical thinking at primary and secondary mathematics schools. The backbone of theoretical framework is Katagiri's notions of mathematical thinking which is consist of 3 (three) components: attitude toward mathematics, method to learn mathematics and content of mathematics. The study has uncovered that for the 4th Grade Students of Primary School, their thinking of the concept of Lowest Common Multiple (LCM) were much contributed by teacher's employing real-life contexts as a starting point for their learning and were influenced by the connection among the strands of mathematical concepts developed previously e.g. the concept of factor of numbers and by the connection with meaningful problems in the real world. For the 5th Grade Students of Primary School, their thinking of the volume of pyramid, prism, cone and cube were varied; it depends on the geometrical form of the object. For the 8th grade students of Junior High School, in learning the total area of a right circular cylinder and sphere as well as the volume of a right circular cone, they employed many mathematical method covers: inductive thinking, analogical thinking, deductive thinking, abstract thinking, thinking that simplifies, thinking that generalizes, thinking that specializes, thinking that symbolize, thinking that express with numbers, quantifies, and figures.

Key Word: primary mathematics, secondary mathematics, lesson study, mathematical thinking

A. INTRODUCTION

In the School-Based Curriculum, it was stated that mathematics in primary and secondary school should encourage the students to think logically, analytically, systematically, critically, creatively and be able to collaborate with others. The implementations of primary and secondary mathematics curriculum in class-rooms need to develop problem solving skills covering both closed and open problems. In solving the problems, students need to creatively develop many ways and alternatives, to develop mathematical models, and to estimate the results. Contextual and realistic approaches are recommended to be developed by the teachers to encourage mathematical thinking in primary schools. With these approaches, there is a hope that the students step-by-step learn and master mathematics enthusiastically. To make their teaching learning of primary mathematics more effective, teachers also need to develop resources such as information technology, teaching aids and other media.

The curriculum outlines the aims of teaching learning of mathematics are as follows: (1) to understand the concepts of mathematics, to explain the relationships among them and to apply them to solve the problems accurately and efficiently, (2) to develop thinking skills to learn patterns and characteristics of mathematics, to manipulate them in order to generalize, to proof

and to explain ideas and mathematics propositions, (3) to develop problems solving skills which covers understanding the problems, outlining mathmatical models, solving them and estimating the outcomes, (3) to communicate mathematics ideas using symbols, tables, diagrams and other media, and (4) to develop appreciations of the uses of mathematics in daily lifes, curiosity, consideration, and willingness to learn mathematics as well as tough and self-confidence.

A series of Lesson Study activities may be thought of as constituting a set of culturally organized activities carried out by teacher or a group of teacher to promote children's mathematical thinking. Many small group activities are flexible and do not have a clear end point, predetermined by the teacher. However, small group discussions offer an interesting context in which to explore the participation of children interacting among the others in naturally occurring open ended thinking. In general, when a task has a clear end point, it has been assumed that the children were thinking towards that point. These series of studies were particularly interested in the attitude and method to which the students develop mathematical thinking to learn mathematics. Experience indicates that teachers can employ Lesson Study to promote mathematical thinking. Teacher is perceived to be the subject of the research as well as to be the researcher. By proposing planning, doing and seeing, the study expected to uncover the aspects of students' mathematical thinking.

B. THEORETICAL FRAMEWORK

Students' interactions with adults and among themselves may promote complex cognitive processes in the form of mathematical thinking. The context in which tasks are performed has begun to form a significant aspect of analysis, it has been interpreted in a variety of ways, sometimes in very local terms, to refer to the conditions under which a particular task is performed, or a particular mathematical thinking is produced. A wider view suggested that the aspect of students' mathematical thinking offers a variety of potential meanings and interpretations, and that mathematical thinking involves a negotiation of shared context. Mathematical thinking happens in the context of teaching learning processes. Group discussion based on principles of promoting children's growth and development through play activities may encourage the students to explore, experiment, question, and talk. Teacher's efforts to facilitate their students to find various patterns of mathematical content can be seen as consistent with extending the students' experiences of mathematical thinking and encouraging social interaction among them.

1. Realistic Approach to Mathematical Thinking

Mathematics teaching can be perceived as a process of students' construction of mathematical meaning, where students can experience mathematical thinking in which a similar process compared to the process by which mathematics was invented. The meaning of invention is steps in learning outcome while the meaning of construction is the learning process. Moreover, the reinvention principle can also be inspired by informal solution procedures. *Realistic approach*, a real-world situation or a context problem is taken as the starting point of learning mathematics (Zulkardi, 2006). And then it is explored by horizontal mathematization activities. This means students organize the problem, try to identify the mathematical aspects of the

problem, and discover regularities and relations. Then, by using vertical mathematization students develop mathematical concepts. There are at least two interpretations of what is called mathematical concept? First, mathematical concept as a product, and second, mathematical concept as a process. Mathematical thinking is actually not the simple one to elaborate. There are many theories of mathematical thinking. It can be philosophical, psychological, or contextual. Second, the idea of mathematics as a human activity is stressed. Mathematics education organized as a process of *guided reinvention*, where students can experience a similar process compared to the process by which mathematics was invented. The meaning of invention is steps in learning processes while the meaning of guided is the instructional environment of the learning process. Moreover, the reinvention principle can also be inspired by informal solution procedures. Informal strategies of students can often be interpreted as anticipating more formal procedures (Zulkardi, 2006).

Two types of mathematization which were formulated explicitly in an educational context by Treffers, 1987, in Zulkardi, 2006, are horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. The following activities are examples of horizontal mathematization: identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known mathematical problem (Zulkardi, 2006). On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. The following activities are example of vertical mathematization: representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing. *Realistic approach*, a real-world situation or a context problem is taken as the starting point of learning mathematics. And then it is explored by horizontal mathematization activities. This means students organize the problem, try to identify the mathematical aspects of the problem, and discover regularities and relations. Then, by using vertical mathematization students develop mathematical concepts. Following figure illustrates the process of reinvention in which both the horizontal and vertical mathematization take place in order to develop basic concepts of mathematics or formal mathematical language.

2. The Aspects of Mathematical Thinking

Katagiri (2004) elaborates that mathematical thinking involves 3 (three) important aspects of thinking i.e. attitude, method and content. Specifically, he indicated that the aspects of mathematical thinking covers the following:

a. Mathematical Thingking related to Attitudes

1) Attempting to grasp one's own problems or objectives or substance clearly, by oneself: (a) Attempting to have questions, (b) Attempting to maintain a problem consciousness, (c) Attempting to discover mathematical problems in phenomena

2) Attempting to take logical actions: (a) Attempting to take actions that match the objectives, (b) Attempting to establish a perspective, (c) Attempting to think based on the data that can be used, previously learned items, and assumptions
3) Attempting to express matters clearly and succinctly: (a) Attempting to record and communicate problems and results clearly and succinctly, (b) Attempting to sort and organize objects when expressing them, (c) Attempting to seek better things, (d) Attempting to raise thinking from the concrete level to the abstract level, (e) Attempting to evaluate thinking both objectively and subjectively, and to refine thinking, (f) Attempting to economize thought and effort

b. Mathematical Thinking Related to Mathematical Methods

1) Inductive thinking, 2) Analogical thinking, 3) Deductive thinking, 4) Integrative thinking (including expansive thinking), 5) Developmental thinking, 6) Abstract thinking (thinking that abstracts, concretizes, idealizes, and thinking that clarifies conditions), 7) Thinking that simplifies, 8) Thinking that generalizes, 9) Thinking that specializes, 10) Thinking that symbolize, 11) Thinking that express with numbers, quantifies, and figures

c. Mathematical Thinking Related to Mathematical Contents

1) Clarifying sets of objects for consideration and objects excluded from sets, and clarifying conditions for inclusion (Idea of sets), 2) Focusing on constituent elements (units) and their sizes and relationships (Idea of units), 3) Attempting to think based on the fundamental principles of expressions (Idea of expression), 4) Clarifying and extending the meaning of things and operations, and attempting to think based on this (Idea of operation), 5) Attempting to formalize operation methods (Idea of algorithm), 6) Attempting to grasp the big picture of objects and operations, and using the result of this understanding (Idea of approximation), 7) Focusing on basic rules and properties (Idea of fundamental properties), 8) Attempting to focus on what is determined by one's decisions, finding rules of relationships between variables, and to use the same (Functional Thinking), 9) Attempting to express propositions and relationships as formulas, and to read their meaning (Idea of formulas)

This is mathematical thinking, which differs from simple knowledge or skills. It is evident that mathematical thinking serves an important purpose in providing the ability to solve problems on one's own as described above, and that this is not limited to this specific problem. Therefore, the cultivation of a number of these types of mathematical thinking must be the aim of this class. Katagiri, S. (2004) lays out the followings as mathematical thinking related to mathematical method: inductive thinking, analogical thinking, deductive thinking, integrative thinking (including expansive thinking), developmental thinking, abstract thinking (thinking that abstracts, concretizes, idealizes, and thinking that clarifies conditions), thinking that simplifies, thinking that generalizes, thinking that specializes, thinking that symbolize, thinking that express with numbers, quantifies, and figures. Teaching should focus on mathematical thinking includes mathematical method. Questions related to mathematical thinking and method must be posed based on a perspective of what kinds of questions must be asked.

C. METHOD

The aim of the study was to promote *mathematical thinking* of primary and secondary students. The study provided teachers the opportunity to see teaching and learning in primary

and secondary mathematics classroom in a real context. The study guided the teachers to focus on planning, implementation, observation, and reflection of their mathematical teaching in such away that their students were to perform *mathematical thinking*. By looking at real contexts of mathematical teaching learning processes, the researcher and the teachers were able to develop a common understanding of what should be planned, done and reflected to promote mathematical thinking. The study also provided opportunities for teachers to carefully examine students' *mathematical thinking* by observing and discussing real mathematics problems.

The study was in the form of *School-Based Lesson Study* in which some teachers of Primary and Secondary Schools were participated to establish lesson study goal and develop lesson study cycles by developing common vision of systematic and consistent pedagogical approach to facilitate students need in performing their mathematical thinking. The steps of developing School-Based Lesson Study covered recruiting teachers, developing the theme, planning the Lesson Study, preparing observation and reflecting the results.

1. Recruiting the Teachers

Lesson study teams composed of 4th-grade teachers and 5^{th}-grade teachers of Primary Schools and 8^{th}-grade teachers of Junior High Schools who work in three different schools: SD MIN I Yogyakarta, SD Percobaan Bulaksumur Yogyakrta and SMP N II Depok Yogyakarta. The researcher facilitated the teachers to provide perspective and a broader view of the issues as well as to serve as outside commentator, evaluator, or outside advisor. The researcher emphasized that the selected teachers should come in with the mindset of being a learner and ready to share and to communicate findings. The researcher and the selected teachers built open communication and set time-table for the related activities.

2. Focusing the Study on the Approach and the Aspects of Mathematical Thinking

The lesson study theme captures the school goals as well as the academic content goals for students to develop approaches and to perform *mathematical thinking*. The researcher exposed the important of mathematical thinking in the sense of students' thinking, educational theories, and international trends. The selected teachers choose a subject area in which to focus on *mathematical thinking*. They needed to identify a unit or lesson on which to focus on *mathematical thinking* and thoroughly discuss the unit and agree about what they are trying to achieve with the lesson. They also needed to expect what did they want students to know and be able to perform mathematical thinking. The researcher strived that the selected teachers must understand how their lesson would significantly supporting and facilitating mathematical thinking. To achieve this goal the researcher carried out firstly the socialization of Katagiri's notions of mathematical thinking.

3. Planning the study lesson.

Prior the study implementation, the researcher shared and discussed with the teachers to prepare lessons related to the topic. The researcher and the selected teachers developed the lessons and set the stage for the observation in which the lesson and the learning processes would be reflected. A piece of planning the lesson included the schema of student responses to various

aspects of the lesson and preparing appropriate teacher responses as well as the logical implication of mathematical thinking. There was conformity among the goal of the overall Lesson Study, the aim of teaching and the aim for student learning. The researcher and the selected teachers developed lesson design and lesson plan to bring these goals. The developed Lesson Plan referred to the School-Based Curriculum (KTSP)

The selected teachers implemented teaching learning processes while the researchers collect data on students' mathematical thinking and their aspects. Some other teachers and the researcher were set to observe the study lesson in a scheduled time and place. The researcher and the selected teachers shared the data collected covering of the evidence that goals for promoting students' mathematical thinking; and then found out the solutions of how to improve the lessons. Lesson debriefing was proposed to give the chance for the selected teacher to reflect his/her teaching; while getting inputs from other teachers or researcher.

4. Preparing for the observation.

In while teaching, the researcher collected the data that need for debriefing. The data covered the comments of students and the work students produce during the lesson. At some occasions the researcher needed to observer closely the work and comments of particular students. The researcher and the selected teachers prepared copies of the lesson plan, teaching aids, and any students' worksheets that students would be using. The study prepared the classroom so that the observers can circulate freely among students during whole-class teaching. The researcher developed instrument to investigate the structure of lesson, the schema of interaction, and the schema of mathematical thinking in the frame of the effort of achieving the mathematical competences.

5. Teaching and observing the lesson.

In order to observe intensively, the researcher develop Observation Instrument and Observation Check-List consisting of the elaboration of Katagiri's notions of mathematical thinking related to the series of events in which the students strived to perform their *mathematical thinking*. Having observed mathematical classroom, the researcher strived to enable the data to be learned so much about the lesson being taught.

7. Reflecting the results

The researcher motivated the teachers that the lesson study experience was well worth the time; however, the value of collaborative between the researcher and the teacher allowed for considerations certain aspect that may not think about when they are planned alone. The teachers may reflect their teaching activities within their schema of perception of *mathematical thinking*. However, the teacher might find wonderful inputs from the researcher of the different angle of sight.

D. RESULTS

1. Student Thinking on The Concept of Least Common Multiple (LCM)

The subjects of the study were the 4^{th} grade students of Primary School. The aim of the lesson was to encourage the students to understand and to apply factors and multiple of numbers to solve problems. The specific aim of the lesson was to encourage the students to understand the

Least Common Multiple (LCM), to determine the Least Common Multiple (LCM), and solve problems which is related to LCM. The search in this lesson indicated that the students strived to develop *horizontal mathematization* through identifying or describing the specific mathematics, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem.

In the effort of to think mathematically on the problems of understanding the Least Common Multiple (LCM) and determining the Least Common Multiple (LCM), the series of sentences produces by the group indicated first *horizontal mathematization* then followed by *vertical mathematizaion*. Most of the students employed subtraction, addition, multiplication and division to list multiple of 7 and 8. They indicated Common Multiple , as the mathematical concepts to answer the common Shintas activities in one year. Students' reflection of *mathematical thinking* employed transition from daily language of mathematical language i.e. from common activities to common multiple. There was a student who jumped their concept to LCM due to he got it from "*informal private lesson*". In performing the *Vertical Mathematization* the students need the assistances from the teacher. The teacher encouraged the students to list more the multiple of 7 and the multiple of 8 and encouraged them to indicate the common multiple of 7 and 8.

In the effort of discovering relations, the students discovered the relationship between "common activities" and "common multiple" i.e. 7 days and 8 days compare with "multiple of 7 and 8 = 56". In the aspect of discovering regularities, the students found that the concepts of regularities arouse from the concepts of "routine activities". The students recognized the isomorphic aspect in different mathematics problems i.e. the regularities emerged from isomorphic activities such as "swimming" and "gardening", "study club", "laboratory activities" or "going to library". There are the key concepts reflecting by the key word of how the students can transfer the real world problems to mathematical problem e.g. the concepts of "common", "regular", "routine", "number of", etc. Students' thinking of the concept of LCM were much contributed by teacher's employing real-life contexts as a starting point for their learning and simultaneously affected by the use of their own productions of formulas and strategies. In thinking the concept of LCM, interactions between teacher and students, students and students are the essential activities. Students' thinking of the concepts of LCM were influenced by the connection among the strands of mathematical concepts developed previously e.g. the concept of factor of numbers and by the connection with meaningful problems in the real world.

2. Students' Mathematical Attitude in Learning the Volume of Cube and Rectangular Parallelepiped

The subjects of the study were the 5^{th} grade students of Primary School. The aim of the research was to promote mathematical attitude through teaching learning the Volume of Cube and Rectangular Parallelepiped. The aim of the lesson was to encourage the students to find the volume of Cube and Rectangular Parallelepiped and applying them to solve related problems. The specific aim of the lesson were to find the volume of Cube and Rectangular Parallelepiped and to solve the problems related to the volume of Cube and Rectangular Parallelepiped. Most of

the students strived very hard to understand the concepts of Pyramid, Prism, Cone and Cube (Lesson Object). There are some ways in which the students strive to understand the concept: manipulation of the Model of Three Dimensional Geometrical Object, questioning to the teacher, questioning to other students, manipulation of mathematics net. Some students got the concept of geometrical shapes from informal learning i.e. from their parents or from additional lesson outside the school. The students were able to indicate the similar of geometrical shapes in daily lives; they also tried to identify the function of identical geometrical shape in daily live. The students perceived that their teacher has important role in helping them to perform *mathematical thinking.*

Different geometrical shapes have different level to be understood. The concept of a cone was the most difficult for the students to understand. The students have difficulties how to calculate the number of the side of the cone. To solve the problems some students delivered the questions to the teacher and the other asked to their classmates. The students tried to employ their pre-requisite knowledge in clarifying the difficult concepts. Some students developed the step in order to understand the difficult concepts i.e. by asking first about the nature of the concept of a cone and then to ask to the teacher about its characteristics. However, some students inevitably jumped without any pattern due to have no systematic knowledge of geometrical shapes. There were the students who tended to be silent and passive if they still do not understand the difficult concepts. Students' effort to understand the difficult concept of geometrical shapes depended on the context and the schema of teaching. If the teacher communicate with the students in less formal, the students felt have no constraint to ask to their teachers. Some students perceived that their teacher should provide the complete and good quality of teaching aids. However, they also perceived that they enjoy getting assignments from the teacher. Most of the students employ inductive thinking i.e. by trial and error to answer teacher's questions; some of them tried to sketch the geometrical shapes and compare with different size of the models. The students tended to re-state the explanations and get attention from their teacher and their classmates to confirm whether their ideas were true.

3. Students' Mathematical Method in Learning the Total Area of a Right Circular Cylinder and Sphere as well as the Volume of a Right Circular Cone

The subjects of the study were the 8^{th} grade students of Junior High School. The aim of the lesson was to understand the characteristics of cylinder, cone, sphere and to determine their measures. The specific aims of the lesson were to identify the formula of the total area of right circular cylinder and to identify the formula of the area of sphere. The students manipulated *Concrete Mode*l of the *Right Circular Cylinder, Sphere and Right Circular Cone* in order to identify its components. They performed *mathematical abstractions* when the teacher gave them some questions or when the teacher let them to work in group. Some students defined the *concept* of *Right Circular Cylinder* as its *functions* in daily life e.g. *"A Right Circular Cylinder is the storage to keep something like pen, pencil, etc."* There were students who defined *a Sphere*

by giving the example in daily life e.g. ball, tennis-ball, etc. Students' *abstractions* of Sphere resulted the investigation of its components i.e. the *radius* and *diameter*.

Students' *inductive thinking* involved *Concretization and method of abstraction* in the area of *problem formation* and *comprehension.* When the students, they who had known the certain concepts, were paced to perform *inductive thinking* they tend to reconfirm their concepts. *Inductive thinking* was spread from the beginning activities to the ultimate accomplishment when the students were paced to do so. The students developed *method of abstraction* to observe the given model of right circular cylinder and strived to identify the components of the right circular cylinder in order to define the concept of right circular cylinder. Students' *inductive thinking* were also related to *establishing perspective* in which the students employed concrete model to search the total area of right circular cylinder and brook-down the model of right circular cylinder into its components: two congruent circles and one oblong.

Logical organization of mathematical concept happened in all context of mathematical method: *idealization, abstraction, deduction, induction and simplification. Logical organizations* of mathematical concept can be indicated from the following example of students' questions: Why the lateral area of cylinder is equal to the area of its rectangle?, Why the volume of cylinder is equal to three times the volume of its cone?, What happened if we do not carefully cover the surface of the sphere in which we use the rope for twisting around?, and Is it true that that the area of the surface of sphere is equal to 4 times the area of its circle? Problem *formation* and *comprehension* emerged when the students: observe given model of right circular cylinder, observe given model of Sphere, and observe given model of right circular cone; identify the components of the right circular cylinder, sphere and right circular cone; define the concept of right circular cylinder, sphere and right circular cone; and get questions and notices from teacher to search the concepts.

E. CONCLUSION

The evidences indicated that, in term of the realistic approach, mathematical thinking can be performed through identifying or describing the specific mathematics, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems; transferring a real world problem to a mathematical problem. Mathematical thinking was always started when the teacher posed the prepared problems written in the *Work Sheet.* The students employed their pre-requisite knowledge to perform mathematical thinking. The students employed different ways to perform *schematizing, formulating* and *visualizing.* The series of sentences produces by the group indicated first *horizontal mathematization* then followed by *vertical mathematizaion*. In performing the *vertical mathematization* the students need the assistances from the teacher.

The students recognized the *isomorphic aspect* in different mathematics problems i.e. the key concepts reflecting by the key word of how the students can transfer the real world problems to mathematical problem. Students' thinking of the concepts of mathematics was influenced by the connection among the strands of mathematical concepts developed previously. The students

tried to employ their pre-requisite knowledge in clarifying the difficult concepts and developed the step in order to understand the difficult concepts. Most of the students employ *inductive thinking* i.e. by trial and error to answer teacher's questions; some of them tried to sketch the geometrical shapes and compare with different size of the models. The students tended to re-state the explanations and get attention from their teacher and their classmates to confirm whether their ideas were true.

There were many ways in which the students *idealized* the geometrical concept. Students' *inductive thinking* involved *concretization and method of abstraction* in the area of *problem formation and comprehension.* When the students, they who had known the certain concepts, were paced to perform *inductive thinking* they tend to reconfirm their concepts. *Inductive thinking* was spread from the beginning activities to the ultimate accomplishment when the students were paced to do so. Students' *inductive thinking* were also related to *establishing perspective* in which the students employed concrete model to search the total area of right circular cylinder and brook-down the model of right circular cylinder into its components: two congruent circles and one oblong. *Logical organization* of mathematical concept happened in all context of mathematical method: *idealization, abstraction, deduction, induction and simplification. Problem formation* and *comprehension* emerged when the students observe mathematical models

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