

**LESSON STUDY ON MATHEMATICAL THINKING:
Developing Mathematical Methods in Learning the Total Area of a
Right Circular Cylinder and Sphere as well as the Volume of a Right
Circular Cone of the Indonesian 8th Grade Students**

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In this study, the researchers strived to uncover the aspects of students attempts in developing mathematical methods in learning the total area of a right circular cylinder and sphere as well as the volume of a right circular cone of the 8th grade students of Junior High School. The results of the research describe students' attempts in inductive thinking, analogical thinking, deductive thinking, abstract thinking, thinking that simplifies, thinking that generalizes, thinking that specializes, thinking that symbolize, thinking that express with numbers, quantifies, and figures. Students' mathematical methods can be traced through the schema of teaching learning activities.

INTRODUCTION

The Decree of Sisdiknas No. 20 year 2003 insists that Indonesian Educational System should develop intelligence and skills of individuals, promote good conduct, patriotism, and social responsibility, should foster positive attitudes of self reliance and development. Improving the quality of teaching is one of the most important tasks in raising the standard of education in Indonesia. It was started in June 2006, based on the Ministerial Decree No 22, 23, 24 year 2006, Indonesian Government has implemented the new curriculum for primary and secondary education, called KTSP "School-Based Curriculum". This School-based curriculum combines two paradigms in which, one side stress on students competencies while on the other side concerns students' learning processes. The School-Based Secondary Junior mathematics curriculum outlines that the aims of teaching learning of mathematics are as follows:

1. to understand the concepts of mathematics, to explain the relationships among them and to apply them in solving the problems accurately and efficiently.

2. to develop thinking skills in learning patterns and characteristics of mathematics, to manipulate them in order to generalize, to prove and to explain ideas and mathematics propositions.
3. to develop problem solving skills which cover understanding the problems, outlining mathematical models, solving them and estimating the outcomes.
4. to communicate mathematics ideas using symbols, tables, diagrams and other media.
5. to develop appreciations of the use of mathematics in daily lives, curiosity, consideration, and to encourage willingness and self-confidence in learning mathematics.

This is why the aim of mathematics education from now on is still urgently to promote mathematical method and to take it into actions. Above all, these lead to suggest that it needs to conduct classroom-based research to investigate the necessary driving factors towards students' ability to develop mathematical method.

THEORETICAL FRAMEWORK

Katagiri, S. (2004) insists that the most important ability that children need to gain at present and in future, as society, science, and technology advance dramatically, are not the abilities to correctly and quickly execute predetermined tasks and commands, but rather the abilities to determine themselves to what they should do or what they should charge themselves with doing. Of course, the ability to correctly and quickly execute necessary mathematical problems is also necessary, but from now on, rather than adeptly to imitate the skilled methods or knowledge of others, the ability to come up with student's own ideas, no matter how small, and to execute student's own independence, preferable actions will be most important. Mathematical activities cannot just be pulled out of a hat; they need to be carefully chosen so that children form

concepts, develop skills, learn facts and acquire strategies for investigating and solving problems.

Mathematical method

Mathematical thinking has its diversity of simple knowledge or skills. It is evidence that mathematical thinking serves an important purpose in providing the ability to solve problems on one's own as described above, and this is not limited to this specific problem. Therefore, the cultivation of a number of these types of mathematical thinking should be the aim of mathematics teaching. Katagiri, S. (2004) lays out the followings as mathematical thinking related to mathematical method:

inductive thinking, analogical thinking, deductive thinking, integrative thinking (including expansive thinking), developmental thinking, abstract thinking (thinking that abstracts, concretizes, idealizes, and thinking that clarifies conditions), thinking that simplifies, thinking that generalizes, thinking that specializes, thinking that symbolize, thinking that express with numbers, quantifies, and figures.

Questions for Eliciting Mathematical Method

Teaching should focus on mathematical thinking including mathematical method. Questions related to mathematical thinking and method must be posed based on a perspective of what kinds of questions to ask. Katagiri, S. (2004) indicates that question must be created so that problem solving process elicits mathematical thinking and method. He lists question analysis designed to cultivate mathematical thinking as follows:

a. Problem Formation and Comprehension

- 1) What is the same? What is shared? (Abstraction)
- 2) Clarify the meaning of the words and use them by oneself. (Abstraction)
- 3) What (conditions) are important? (Abstraction)
- 4) What types of situations are being considered? What types of situations are being proposed? (Idealization)
- 5) Use figures (numbers) for expression. (Diagramming, quantification)
- 6) Replace numbers with simpler numbers. (Simplification)
- 7) Simplify the conditions. (Simplification)

8) Give an example. (Concretization)

b. Establishing a Perspective

- 1) Is it possible to do this in the same way as something already known? (Analogy)
- 2) Will this turn out the same thing as something already known? (Analogy)
- 3) Consider special cases. (Specialization)

c. Executing Solutions

- 1) What kinds of rules seem to be involved? Try collecting data. (Induction)
- 2) Think based on what is known (what will be known). (Deduction)
- 3) What must be known before this can be said? (Deduction)
- 4) Consider a simple situation (using simple numbers or figures). (Simplification)
- 5) Hold the conditions constant. Consider the case with special conditions. (Specialization)
- 6) Can this be expressed as a figure? (Diagramming)
- 7) Can this be expressed with numbers? (Quantification)

d. Logical Organization

- 1) Why is this (always) correct? (Logical)
- 2) Can this be said more accurately? (Accuracy)

RESEARCH METHOD

The study was aimed at promoting students to develop mathematical method in learning the total area of a right circular cylinder and sphere and also the volume of a right circular cone. The approach used in the study was descriptive-qualitative of Lesson Study in two classes: the 8th grade of Junior High School, class A and the 8th grade of Junior High School, class B. The design of the research included: preparation (PLAN), implementation (DO), and reflection (SEE). The instrument used for collecting data consists of questionnaire, interview, observation of the lesson, and VTR of the Lesson. The research was began with two series of discussions between teachers and lectures and followed by observing and reflecting two lesson activities in the class, as the following description:


LESSON PLAN I



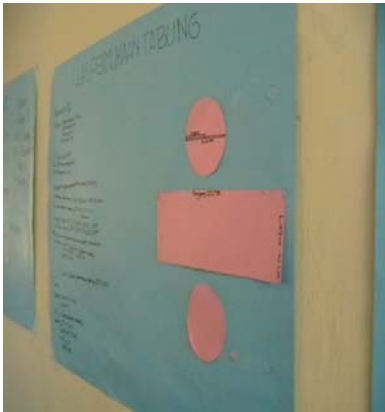
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|---------------------------|---|---|
| Day | : | Thursday, May, 24, 2007, |
| Date | : | 07.00-09.00 |
| Junior High School | : | SMP NEGERI DEPOK II, Yogyakarta, Indonesia |
| Grade/Sem/year | : | 8/Sem II/2007 |

| | | |
|----------------------------|---|--|
| Teacher | : | Siwi Pujiastuti SPd |
| Class | : | A |
| Number of Students | : | 40 |
| Standard Competency | : | To understand the characteristics of cylinder, cone, sphere and to determine their measures. |
| Basic Competencies | : | To identify the formula of the total area of right circular cylinder; to identify the formula of the area of sphere. |
| Teaching Scenario | : | <ol style="list-style-type: none"> 1. Apperception 2. Developing concepts 3. Reflection and presentation 4. Conclusion and closing |

VIDEOTAPED LESSON I Part A :




Aim : To identify the formula of the total area of right circular cylinder

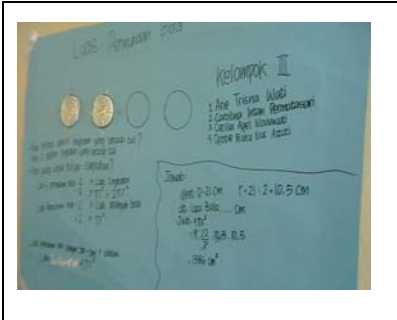
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|  | <p>Introduction:</p> <p>Problem Formation and Comprehension</p> <ul style="list-style-type: none"> - Teacher let the students observe given model of right circular cylinder (<i>concretization and induction</i>) - Teacher let the students identify the components of the right circular cylinder (<i>method of abstraction</i>) - Teacher let the students define the concept of right circular cylinder (<i>method of abstraction</i>) - During the whole class teaching process, teacher encouraged students' abstraction (<i>method of abstraction</i>) |
|--|---|

| | |
|---|---|
|  | <p>Group Work and Discussion: Establishing a Perspective</p> <ol style="list-style-type: none"> Students employed concrete model to search the total area of right circular cylinder (<i>employing concrete model to express the concept and induction</i>) Students broke-down the model of right circular cylinder into its components: two congruent circles and one oblong. (<i>employing concrete model to express the concept and induction</i>) Students learned that the height of right circular cylinder is equal to the width of its rectangle; and the circumference of the circle is equal to the length of rectangle. (<i>employing concrete model to express the concept and induction</i>) |
|  | <p>Group Work and Discussion: Executing Solutions</p> <ul style="list-style-type: none"> Students learned that the lateral area of right circular cylinder is equal to the area of its rectangle. (<i>analogy of concept and induction</i>) Students learned that the total area of right circular cylinder is equal to the area of its rectangle plus the area of its two circles. (<i>analogy of concept and induction</i>) |
|  | <p>Students' Reflection :</p> <ul style="list-style-type: none"> Students presented that the lateral area of right circular cylinder is equal to the area of its rectangle i.e. LATERAL AREA = HEIGHT X CIRCUMFERENCE OF CIRCLE = $t \times 2 \pi r$ Some students needed to have clarification that the lateral area of right circular cylinder is equal to the area of its rectangle. (<i>logical organization, analogy of concept and induction</i>) Students presented that the total area of right circular cylinder is equal to the area of its rectangle plus the area of its two circles i.e. TOTAL AREA = LATERAL AREA + 2 (AREA OF CIRCLE) = $t \times 2 \pi r + 2 (\pi r^2) = 2 \pi r (t + r)$ |

VIDEOTAPED LESSON I, Part B:

Aim : to identify the formula of the area of sphere.

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|---|--|
|  | <p>Introduction: Problem Formation and Comprehension</p> <ul style="list-style-type: none">- Teacher let the students observe given model of Sphere (Concretization and Induction)- Teacher let the students identify the components of Sphere (Abstraction)- Teacher let the students define the concept of Sphere (method of abstraction)- Teacher's explained the way to find the area of the surface of Sphere. |
|  | <p>Group Work and Discussion: Establishing a Perspective</p> <ul style="list-style-type: none">d. Students employed concrete model of a half Sphere to search the area of its surface (<i>employing concrete model to express the concept and induction</i>)e. Students prepared the supporting facilities e.g. diagram of circles that has similar radius with the Sphere. (<i>employing concrete model to express the concept and induction</i>) |
|  | <p>Group Work and Discussion: Executing Solutions</p> <ul style="list-style-type: none">- Students learned that the area of Sphere is equal to the area of its cover (<i>logical organization, analogy of concept and induction</i>)- Students learned that the area of the surface of a half Sphere is equal to the double of the area of its circles. (<i>logical organization, analogy of concept and induction</i>) |



Students' Reflection :

- Students presented that the area of the surface of a Sphere is equal to four times the area of its circles.
- AREA of SPHERE = 4 X AREA of CIRCLE**
- Some students needed to have clarification whether their formula was correct? (*logical organization, analogy of concept and induction*)

LESSON PLAN II

| | | |
|----------------------------|---|--|
| Day | : | Saturday, May, 26, 2007 |
| Date | : | 07.00-09.00 |
| Junior High School | : | SMP NEGERI DEPOK II, Yogyakarta, Indonesia |
| Grade/Sem/year | : | 8/Sem II/2007 |
| Teacher | : | Siwi Pujiastuti SPd |
| Class | : | B |
| Number of Students | : | 40 |
| Standard Competency | : | To understand the characteristics of cylinder, cone and sphere and to determine their measures. |
| Basic Competency | : | To identify the formula of the volume of right circular cone. |
| Teaching Scenario | : | <ol style="list-style-type: none"> 1. Apperception 2. Developing concepts 3. Reflection and presentation 4. Conclusion and closing |

VIDEOTAPED LESSON II




Aim : To identify the formula of the volume of right circular cone.



Introduction:

Problem Formation and Comprehension

- Teacher let the students observe given model of right circular cone. (*concretization and induction*)
- Teacher let the students identify the components of the right circular cone. (*method abstraction*)
- Teacher let the students define the concept of right circular cone. (*method abstraction*)
- During the whole class teaching process, teacher encouraged students' abstraction (*method of abstraction*)

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|  | <p>Group Work and Discussion:</p> <p>Establishing a Perspective</p> <p>f. Students learned the teacher's guide to understand the procedures how to search the volume of right circular cone. (<i>employing concrete model to express the concept and induction</i>)</p> <p>g. Students observed and manipulated the concrete model of right circular cone and right circular cylinder (<i>employing concrete model to express the concept and induction</i>)</p> |
|  | <p>Group Work and Discussion:</p> <p>Executing Solutions</p> <ul style="list-style-type: none"> - Students collected data of the measurement of the volume of cone in comparison with the volume of cylinder. (<i>logical organization, analogy of concept and induction</i>) - Students learned that the volume of right circular cylinder is equal to three times the volume of right circular cone. (<i>logical organization, analogy of concept and induction</i>) |
|  | <p>Students' Reflection :</p> <ul style="list-style-type: none"> - Students presented that the volume of right circular cone is equal to one third of the volume of right circular cylinder. (<i>logical organization, analogy of concept and induction</i>) - Some students needed to have clarification whether their formula was correct (<i>logical organization, analogy of concept and induction</i>) |

ANALYSIS OF DATA

a. Problem Formation and Comprehension

The students manipulated *Concrete Model* of the *Right Circular Cylinder*, *Sphere* and *Right Circular Cone* in order to identify its components. They performed *mathematical abstractions* when the teacher gave them some questions or when the teacher let them work in group. Some students defined the *concept* of *Right Circular Cylinder* as its *functions* in daily life e.g. “A *Right Circular Cylinder* is the storage to keep something like pen, pencil, etc.” The teacher encouraged the students to perform *mathematical abstractions* of the *Concrete Model* of the *Right Circular Cylinder* viz. to indicate its components such as circles, height, and the radius of its circle. There were students who defined a *Sphere* by giving the example of daily life e.g. ball, tennis-ball, etc. Students’ *abstractions* of *Sphere* resulted the investigation of its components i.e. the *radius* and *diameter*.

At the first step, most of the students defined the right circular cone through *characterization of its shape*, e.g. “A *right circular cone* is a thing like triangle”, “A *right circular cone* is composed by three dimensional triangle”, and “A *right circular cone* is composed by many circles – the higher the circle the smaller it does”. There were many ways in which the students *idealized* the geometrical concept. They mostly confirmed the concept to the teacher and asked their mates. Sometimes they performed their *idealization* by commenting other works. Some students asked the teacher why the *lateral area of cylinder is equal to the area of its rectangle* and why the *volume of cylinder is equal to three times the volume of its cone*.

b. Establishing a Perspective

Working in group triggered the students to develop analogical thinking of mathematical concepts. *Analogical thinking* happened when the students perceived

that finding the lateral area of *Right Circular Cylinder* is similar to finding the area of its rectangle; and, finding the area of *Sphere* is similar to finding the area of its surface i.e. covering its surface by twisting around the rope. In sum, the concepts of geometrical shapes are mostly perceived to be analogical with examples in daily life e.g. the *right circular cone was perceived as a traditional hat*. In performing their *analogical thinking* the students frequently used strategic terminologies such as “*similar to*”, “*comparison with*”, “*the example of*”, and “*the function of*”.

Most of the students perceived that the given task by the teacher as the cases that need special consideration. Therefore, most of them considered more seriously on the ways to find the formulas of the total area of right circular cylinder and the formula of the area of sphere as well as the formula of the volume of right circular cone. Some students still paid attention on the concepts of cylinder, sphere, and cone. There was a student who wanted a clarification on the form of circle bases of cone whether it is convex, concave or plane. After getting input from teacher or their mates, the students usually *considered special cases* including corrected the formulas and made some notes at their works. Some students *considered* the use of $\pi = \frac{22}{7}$ or $\pi = 3.14$. One student expressed that for the bigger radius, we use $\pi = \frac{22}{7}$ and for the smaller radius we use $\pi = 3.14$.

c. Executing Solutions

Students’ *inductive thinking* involved *Concretization and method of abstraction* in the area of *Problem Formation and Comprehension*. When the students who known the certain concept, were paced to perform *inductive thinking* they tend to reconfirm their concepts. *Inductive thinking* was spread from the beginning activities to the ultimate accomplishment when the students were paced to do so. The students observed the given model of right circular cylinder and strived to identify the

components of the right circular cylinder in order to define the concept of right circular cylinder (method of abstraction). Students' *inductive thinking* was also related to *establishing perspective* in which the students employed concrete model to search the total area of right circular cylinder and broke-down the model of right circular cylinder into its components: two congruent circles and one oblong.

There were some steps in which the students perform *inductive thinking*, as following:

Inductive thinking of finding the total area of Right Circular Cylinder:

- Step 1: Observing the *Concrete Model of Right Circular Cylinder*
- Step 2: Manipulating the model and learning the teacher's guide
- Step 3: Drawing the components of the *Right Circular Cylinder* i.e. the bottom circle, the top circle and its rectangle.
- Step 4: Determining the area of its components
- Step 5: Adding up the total area

Inductive thinking of finding the area of Sphere:

- Step 1: Observing *Concrete Model of a half Sphere*
- Step 2: Manipulating the model and learning the teacher's guide
- Step 3: Twisting around of the model with the rope
- Step 4: Thinking inductively that the length of rope needed to twist around a half model of sphere determines the area of circle.
- Step 5: Finding out that the area of a half sphere is equal to two times the area of its circle.

Inductive thinking of finding the volume of cone:

- Step 1: Observing *Concrete Model of Right Circular Cone and its Right Circular Cylinder*
- Step 2: Manipulating the model and learning the teacher's guide
- Step 3: Practicing to fill up the *Right Circular Cone* with *sand* which was acted of measuring out by the *Right Circular Cone*
- Step 4: Thinking inductively to find out the volume of *Right Circular Cone* compare with the volume of its *Right Circular Cone*
- Step 5: Finding out that the volume of the *Right Circular Cone* is equal to one third of the volume of its *Right Circular Cylinder*

d. Logical Organization

Logical organization of mathematical concept happened in all context of mathematical method: *idealization, abstraction, deduction, induction and simplification*. *Logical organizations* of mathematical concept can be indicated from the following example of students' questions:

1. Why is the lateral area of cylinder equal to the area of its rectangle?
2. Why is the volume of cylinder equal to three times the volume of its cone?
3. What happens if we do not carefully cover the surface of the sphere in which we use the rope for twisting around?
4. Is it true that the area of the surface of sphere is equal to 4 times the area of its circle?

DISCUSSION

Krygowska (1980) in Bonomo M.F.C indicates that mathematics would have to be applied to natural situations, any where real problems appear, and to solve them, it is necessary to use the mathematical method. The knowledge, skills, and mathematical methods are the foundation to achieve the knowledge on science, information, and other learning areas in which mathematical concepts are central; and to apply mathematics in the real-life situations. This study uncovered that teacher has important role to encourage their students to develop mathematical methods. The students performed *mathematical method* when they found difficulties or when they were asked by the teacher. Most of the students reflected that they paid attention on the perfect of the *Concrete Model* of geometrical shape. However, their consideration on the perfect form of the models did not indicate that they performed *mathematical idealization* as one of mathematical method.

Student's reflection:

- Researcher : What is the effect on your calculation of the volume if a *right circular cone* is made up from “*a very thick metal plat*”
- Student : It's okey. No, problem.
- Researcher : Compare to a *right circular cone* if it made up from “*a very thin metal plat*” How can you determine the volume?
- Student : In “*a very thin metal plat cone*” I can fill up *more sand*. But if the radius and the height of the cone are similar, they should have similar volume.
- Researcher : So, what do you think about the different between the cone that made up from “*a very thick metal plat*” and cone that made up from “*a very thin metal plat*” ?
- Students : Yes, it will be very different. I am sorry for my initial statement.

Meanwhile, David Tall (2006) states that success in mathematical thinking depends on the effect of met-befores, the compression to rich thinkable concepts, and the building of successive levels of sophistication both powerful and simple. In this research, one aspect of mathematical method i.e. *simplifications* happened when the students perceived that the concept of *right circular cone* is similar to the concept of triangle or circle. In this case, they *simplified* the concepts through manipulation of *Concrete Models*. They also performed simplification when they broke down the formula to solve the problems. They mostly simplified the concepts when they had got some questions from the teacher; or, when they worked in group. Ultimately, when the teacher asked for the students to write the results, the students got that: 1) the total area of *Right Circular Cylinder* is equal to the area of its rectangle plus two times the area of its circles, 2) the area of *Sphere* is equal to four times the area of its circle, and 3) the volume of *right circular cone* is equal to one third of the volume of its cylinder.

The students developed *inductive thinking* when they uncovered that the height of right circular cylinder is equal to the width of its rectangle; and the circumference of the circle is equal to the length of rectangle. They continued to perform *inductive thinking* until they found the formula of the lateral area of right circular cylinder; the

formula of sphere, and the formula of the volume of *Right Circular Cone*. Students' *schema of inductive thinking* seemed in line with Katagiri's claim that *inductive thinking* covers:

- 1) Attempting to gather a certain amount of data, 2) Working to discover rules or properties in common between these data, 3) Inferring that the set that includes that data (the entire domain of variables) is comprised of the discovered rules and properties, and 4) Confirming the correctness of the inferred generality with new data

In a different context, Stacey, K (2006) suggests that a key component of mathematical thinking is having a disposition to looking at the world in a mathematical way, and an attitude of seeking a logical explanation. While Katagiri, S. (2004) claims that students' logical actions include: attempting to take actions that match with the objectives; attempting to establish a perspective; and attempting to think based on the data that can be used, previously learned items, and assumptions. In this research, the aspects of *logical organization* of mathematical concept emerged after the students put into practice mathematical procedures in their group. However, there were evidences that it was difficult for the students to practice mathematical procedures. Students' inappropriate organization of mathematical procedures appeared when the students had difficulties in performing mathematical procedure into practice with *concrete model*. In searching the formula of the volume of right circular cone, there were some students who hesitated what to fill-up with sand. Should it be a cone or cylinder? In searching the formula of the total area of *a right circular cylinder*, there was a question from the student, why the total area is the result of addition and not the result of multiplication.

CONCLUSION

In this Lesson Study, the researchers had sought to uncover the picture in which the teacher strived to promote mathematical methods in learning the total area of a right circular cylinder and sphere as well as the volume of a right circular cone. The striking

results of the study can be stated that *students' mathematical methods* can be traced through the schema of teaching learning activities as follows:

1. Problem Formation and Comprehension were emerged when the students:
 - a. observed given model of right circular cylinder, observed given model of Sphere, and observed given model of right circular cone
 - b. identified the components of the right circular cylinder, sphere, and right circular cone
 - c. defined the concept of right circular cylinder, sphere, and right circular cone
 - d. got questions and notices from teacher to search the concepts
2. Establishing a Perspective were emerged when the students:
 - a. employed concrete model to search the total area of right circular cylinder, the area of sphere and the volume of right circular cone
 - b. learned that the height of right circular cylinder is equal to the width of its rectangle; and the circumference of the circle is equal to the length of rectangle
 - c. learned the teacher's guide to understand the procedures how to search the volume of right circular cone
 - d. broke-down the model of right circular cylinder into its components
3. Executing Solutions were emerged when the students:
 - a. tried to find out the lateral area of right circular cylinder
 - b. tried to find out the total area of right circular cylinder
 - c. tried to find out the area of sphere
 - d. collected the data of the measurement of the volume of cone in comparison with the volume of cylinder

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