

THE ROLE OF KANT'S THEORY OF KNOWLEDGE IN SETTING UP THE EPISTEMOLOGICAL FOUNDATION OF MATHEMATICS

SUMMARY

A. Background

*Mathematics*¹, by its nature, always has an inclination toward the right, and, for this reason, has long withstood the spirit of the *time* that has ruled since the Renaissance; i.e., the *empiricist* theory of mathematics, such as the one set forth by Mill, did not find much support. Indeed, *mathematics* has evolved into ever higher abstractions, away from matter and to ever greater clarity in its foundations e.g. by giving an exact foundation of the *infinitesimal calculus* and the complex numbers - thus, away from *scepticism*. However, around the turn of the century, it is the *antinomies of set theory*, contradictions that allegedly appeared within mathematics, whose significance is exaggerated by *sceptics* and *empiricists* and which are employed as a pretext for the leftward upheaval.

Epistemological *foundationalism*² seeks for a solid ground of mathematical cognition to shield it from arbitrariness, non-conclusiveness, and vulnerability to historical circumstances as well as to guarantee certainty and truth of mathematical knowledge. It is noted that in any version of epistemological *foundationalism* there is an element of absolutism viz. epistemological standards, such as truth, certainty, universality, objectivity, rationality, etc. According to an *empiricist* version of *foundationalism*³, the basic elements of knowledge are the truths which are certain because of their causes

rather than because of the arguments given for them.

¹ Gödel, K., 1961, "The modern development of the foundations of mathematics in the light of philosophy". Retrieved 2003 <<http://www.marxists.org/glossary/people/g/o.htm#godel-kurt>>

² Tuchanska, B., 1999, "Is a Non-Foundationalist Epistemology Possible?", Retrieve 2004 <<http://www.Google.com/Tuchanska>>

³ -----, 1990, *Kant, Rorty and transcendental argumentation*, Paper was written at Section de Philosophie, Université de Fribourg

The rising of Kant's theory of knowledge, as the epistemological foundations in seeking the solid ground on knowledge⁴, was influenced by at least two distinct epistemological foundations i.e. from its root of *Empiricist foundationalism* and *Rationalist foundationalism*. According to an *empiricist* version of *foundationalism*⁵ there are basic elements of knowledge that the truths are certain because of their causes rather than because of the arguments given for them; they believe in the existence of such truths because they presuppose that the object which the proposition is about imposes the proposition's truth. It⁶ is an *empiricist* version of *foundationalism* that can be characterized as consisting of two assumptions: (a) there are true claims which, if known, would allow us to derive all knowledge of the architecture of existence; and (b) such claims are given.

To discover which concepts and judgments are foundational for our knowledge, *rationalist foundationalism* finds a source of cognitive act, that is, an act in which we discover fundamental ideas and truths⁷. This is of course an intellectual act but not an act of reasoning since it itself requires premises. It is either an act of intellectual intuition or an act of *self-reflection*, *self-consciousness*, i.e., *Cartesian Cogito*; such an act not only reveals the ultimate foundations of knowledge but also gives us certainty referring to their epistemological value: we know for sure that they are indubitable, necessary, and - consequently - true. The foundational ideas and judgments are indubitable since they are involved in acts of the clear and distinct knowing-that; they are necessary as ultimate premises from which other judgments are deducible; and they are true since they are involved in acts of correct and accurate knowing-about.

If *rationalists*, such as Plato, Descartes, Leibniz, or Spinoza, believe that all knowledge is already present in human mind before any cognitive activity begins. They cannot construct their *foundationalist program* as the search for privileged, fundamental, and true propositions. On the other hand, the search for foundations of cognition and

⁴ Tuchanska, B.,1999, "Is a Non-Foundationalist Epistemology Possible?", Retrieve 2004
<http://www.Google.com/Tuchanska>

⁵ Rorty in Tuchanska, B.,1999, "Is a Non-Foundationalist Epistemology Possible?", Retrieve 2004
<http://www.Google.com/Tuchanska>

⁶ Krausz in Tuchanska, B.,1999, "Is a Non-Foundationalist Epistemology Possible?", Retrieve 2004
<http://www.Google.com/Tuchanska>

⁷ Tuchanska, B.,1999, "Is a Non-Foundationalist Epistemology Possible?", Retrieve 2004
<http://www.Google.com/Tuchanska>

knowledge does not mean for Kant to establish basic ideas and truths from which the rest of knowledge can be inferred. It means to solve the question of *how is cognition as a relation between a subject and an object possible*, or - in other words - how *synthetic representations* and their objects can establish connection, obtain necessary relation to one another, and, as it were, meet one another.

Relating to those problems, in his *theory of knowledge*, Kant proposes to epistemologically set mathematics upon the secure path of science. Kant⁸ claims that the true method rests in the realization of mathematics can only have certain knowledge that is *necessarily presupposed a priori* by reason itself. In particular, the *objective validity of mathematical knowledge*, according to Kant, rests on the fact that it is based on the a priori forms of our sensibility which condition the possibility of experience. However, mathematical developments in the past two hundred years have challenged *Kant's theory of mathematical knowledge* in fundamental respects.

This research, therefore, investigates the role of *Kant's theory of knowledge* in setting up the epistemological foundations of mathematics. The material object is the epistemological foundation of mathematics and the formal object of this research are Kant's notions in his theory of knowledge.

B. Problem Formulation

The research problem consists of: what and how is the epistemological foundation of mathematics?, what and how is the Kant's Theory of Knowledge?; and, how and to what extent Kant's theory of knowledge has its roles in setting up the epistemological foundation of mathematics.

C. Bibliographical Review

Having reviewed related references, the researcher found two other researches on similar issues. Those are of Thomas J. McFarlane, 1995, i.e. "*Kant and Mathematical Knowledge*", and of Lisa Shabel, 2003, i.e. "*Mathematics in Kant's Critical Philosophy*".

⁸ McFarlane, T.J1995,. "Kant and Mathematical Knowledge.". Retreived 2004 <<http://www.integralscience>>

The first, McFarlanne, in his research, has critically examined the challenges non-Euclidean geometry poses to Kant's theory and then, in light of this analysis, briefly speculates on how we might understand the foundations of mathematical knowledge; by previously examined Kant's view of mathematical knowledge in some of its details.

While in the second, Shabel has strived to clarify the conception of *synthetic* a priori cognition central to the critical philosophy, by shedding light on Kant's account of the construction of mathematical concepts; and carefully examined the role of the production of diagrams in the mathematics of the early moderns, both in their practice and in their own understanding of their practice. While, in this research, the researcher strives to investigate the role of Kant's philosophy of mathematics in setting up the epistemological foundation of mathematics.

D. Theoretical Review

The philosophy of mathematics has it aims to clarify and answer the questions about the status and the foundation of mathematical objects and methods that is ontologically clarify whether there exist mathematical objects, and epistemologically clarify whether all meaningful mathematical statements have objective and determine the truth. Perceiving that the *laws of nature* and the *laws of mathematics* have a similar status, the very real world of mathematical objects forms the mathematical foundation; however, it is still a big question how we access them. Although some modern theories in the philosophy of mathematics *deny the existence of foundations* in the original sense; some philosophers focused on human cognition as the origin of the reliability of mathematics and proposed to find mathematical foundations only in human thought, as Kant did, not in any objective outside mathematics.

While the *empiricist* and *rationalist* versions of *foundationalism* strive to establish the foundation of mathematics as justifying the *epistemologically valid*, i.e., *certain*, *necessary*, *true*, Kant established forms and categories to find the *ultimate conditions* of the possibility of the objectivity of cognition, i.e., of the fact that we cognize mathematics. The historical development of the reflection on knowledge and cognition of mathematics shows that the most notorious exemplars of *foundationalism* have overgrown themselves and gave birth to criticisms undermining their postulates and

conclusions. The ideal of *justificationism* has been abandoned in consequence of the demonstration that neither inductive confirmation nor deductive falsification can be conclusive. The *foundationalist program* of Descartes is also radically modified or even rejected.

*Kant's theory of knowledge*⁹ insists that instead of assuming that our ideas, to be true, must conform to an external reality independent of our knowing, objective reality should be known only in so far as it conforms to the essential structure of the knowing mind. Kant¹⁰ maintains that objects of experience i.e. *phenomena*, may be known, but that things lying beyond the realm of possible experience i.e. *noumena* or *things-in-themselves* are unknowable, although their existence is a necessary presupposition. *Phenomena*, in which it can be perceived in the pure forms of sensibility, *space*, and *time* must possess the characteristics that constitute our categories of understanding. Those categories, which include causality and substance, are the source of the structure of phenomenal experience.

Kant¹¹ sums up that all three disciplines—*logic*, *arithmetic* and *geometry*—are *synthetic* as disciplines independent from one another. In the *Critique of Pure Reason* and the *Prolegomena to Any Future Metaphysics*, Kant¹² argues that the *truths of geometry* are *synthetic a priori truths*, and *not analytic*, as most today would probably assume. *Truths of logic* and *truths* that are merely true by definition are "*analytic*" because they depend on an "*analysis*", or *breakdown*, of a concept into its components, without any need to bring in external information. *Analytic truths* are thus necessarily *a priori*. *Synthetic truths*, on the other hand, require that a concept to be "*synthesized*" or combined with some other information, perhaps another concept or some sensory data, to produce something truly new.

Modern philosophy after Kant presents some important criteria that distinguish the foundations of mathematics i.e. pursuing the foundation in the logical sense, in the philosophy of mathematics, in the philosophy of language, or in the sense of

⁹ -----, “Encyclopedia—Kant, ImmanuelKant”, Retrieved 2004 <<http://www.factmonster.com/encyclopedia.html>>

¹⁰ Ibid.

¹¹ Randall, A., 1998, A Critique of the Kantian View of Geometry, Retrieved 2004 <<http://home.ican.net/~arandall/Kant/Geometry/>>

¹² Ibid.

epistemology. The dread of the story of the role of Kant's theory of knowledge to the foundation of mathematics can be highlighted e.g. from the application of Kant's doctrine to algebra and his conclusion that geometry is the science of *space* and since *time* and *space* are pure sensuous forms of intuition, therefore the rest of mathematics must belong to *time* and that algebra is the science of pure *time*.

E. Method of Research

This research employs literal study on *Kant's critical philosophy* and on the philosophy of mathematics. This research employs various approaches to describe the works of Immanuel Kant, specifically epistemology, and to investigate its contribution to the philosophy of mathematics. Qualitative approach was employed to identify and describe coherently some related concepts, in a narrative form; in a certain occasion, the researcher needs to develop in depth of understanding and description provided by textual sources. The related references have several features to take into consideration in such away that understanding of *Kant's epistemology* and its contribution to the philosophy of mathematics will be gained through a holistic perspective. The researcher, in striving to understand the concepts, at any occasion, strived to compare and contrast the notions from different philosophical threads.

The researcher strives to reflect and interpret the related texts of critical philosophy that is most of them are the works of Immanuel Kant as the primary data. Argumentations and expositions were in analytical and logical form in such away that the researcher formed the body of the essay fit together. On the other hand, the researcher need to be flexible and open because the purpose was to learn how past intentions and events were related due to their meaning and value. Lack of understanding of Kant's theory of knowledge was over came by phenomenological approach in which the researcher began with the acknowledgement that there was a gap between author's understanding and the clarification or illumination of the features.

F. Research Result and Discussion

Kant's most significant contribution to modern philosophy, by his own direct assertion, is the recognition that mathematical knowledge , holds the key to defeating the Humean skeptical work, as the *synthetic a priori judgments* are possible. The *vicious dichotomy* in the foundation of mathematics maintains that there are two absolutely *incommensurable* types of knowledge, the *a priori analytic* and *a posteriori synthetic*, each deriving from two radically distinct sources, reason and experience. This division makes the product of reason, logic, empty and relegates knowledge of nature as merely particular and therefore blind. Kant's insight is to recognize that mathematical knowledge seems to bridge this dichotomy in a way that defeats the skeptical claim. Mathematical thinking is both *a priori in the universality and necessity of its results* and *synthetic* in the expansively ampliative promise of its inquiry.

Kant's proof consists of mathematical knowledge, in which under the action of cognition, leads to mathematical knowledge of the conclusion even though the truth-conditions of the former may not cover those of the latter. *Kant's epistemological mathematic* is the principle that *an inference is gapless* when one grasps a mathematical architecture in which a mathematical justification of the conclusion is seen as a development of a mathematical justification of the premises. Prior to modern empiricism, analysis was the "*taking apart*" of a mathematical problem while synthesis referred to the process of reconstructing that same problem in a logically deductive form. *Euclidean Geometry* is erroneously taken as synonymous with both ancient geometry and this same *synthetic* form of presentation.

Kant's use of mathematics is not the same as that made by Frege; his basic claim was that Kant used a geometrical model to subsume arithmetic within the real of the *synthetic a priori*. Kant's model of mathematical reasoning only actually puts the smaller part of geometry in the *synthetic a priori*. By recognizing that the *metaphysical gap* can be represented as within the mathematical, rather than as between the mathematical and the dynamical uses of reason, we can realize, as did Plato and Descartes, that the productive synthesis of the transcendental schemata is exactly the mathematical or "*figurative*" representation of the metaphysical. The *figurative schemata of mathematics*

can exemplify the pure concepts of reason as adeptly as they capture the possible objects of experience. Kant clearly had some part of this model in mind when he defined the mathematical as the *constructio of concepts*.

Kant believes the judgments of mathematics and Newtonian physics to be instances of *genuine knowledge*. Mathematics and science are objective and universally valid, because all human beings know in the same way. Mathematician after Kant, Frege, of course, agreed with Kant on geometry, and considered it to be *synthetic a priori*; but different *logicist* might claim that their reduction laid out the 'true nature' of geometrical objects as strictly logical. Taking a particularly *logicist* definition of *rigour* for granted; more than that of mathematician pre-Kant. Kant goes on to identify this illegitimate use of logic as *organon* as one of the primary sources of paradox in the contemporary philosophy of mathematics, and indeed one of the fundamental points of Kant's theory of knowledge is that pure reason, including logic, cannot go beyond a close connection with objects as given in experience. This is precisely why the three alternatives to *logicism* can be considered as Kantian; *logicists* insist on attention to the connection between knowledge of mathematics and the content of that knowledge.

Kant's theory of knowledge recognizes broad notions of mathematics and intuition, so that both algebra and geometry are accommodated, but it can easily do this while maintaining a distinction between the mathematical and the universal. So, it should now be obvious why the challenge that, e.g. the Poincaréan epistemologist presents to *logicism* is much more daunting than a mere question of adequacy; rejecting the use of logic as an *organon* of knowledge means *logicism* is incoherent from the start. Even questions like the consistency of some proposed neo-Fregean system that attempts to demonstrate that geometry is analytic are meaningless on this view. And, even though logic is accepted as an *organon* among most contemporary philosophers of mathematics, this is no reason to consider the Kantian views vanquished.

Ultimately, it can be said that *Kant's theory of knowledge* has inherently philosophical interest, contemporary relevance, and defensibility argument to remain essentially intact to the foundations of mathematics no matter what one may ultimately think about controversial of his metaphysics and epistemology of transcendental idealism. In term of the current tendency of the philosophy of mathematics, *Kant's theory*

of knowledge is in line with the perception that understanding of mathematics can be supported by the nature of perceptual faculties. Accordingly, mathematics should be intuitive. What is needed in the foundation of mathematics is then a more sophisticated theory of sensible intuition. Kronecker in Smith N.K., for example, advocated a return to an intuitive basis of mathematics due to the fact that abstract non-intuitive mathematics was internally inconsistent; and this is the other crisis in the *epistemological foundation of mathematics*.

The important results of the research can specifically be exposed as follows:

1. Kant's Theory of Knowledge Synthesizes the Foundation of Mathematics

In the sphere of Kant's '*dogmatic*' notions, his *theory of knowledge*, in turn, can be said to lead to *un-dogmatization* and *de-mythologization* of *mathematical foundations* as well as rages the institutionalization of the research of *mathematical foundations* in which it encourages the mutual interactions among them. Kant¹³ insists that a *dogmatist* is one who assumes that human reason can comprehend *ultimate reality*, and who proceeds upon this assumption; it expresses itself through three factors viz. *rationalism*, *realism*, and *transcendence*.

The role of *Kant's theory of knowledge*, in the sense of *de-mythologization* of *mathematical foundations*, refers to history of the mathematical *myth* from that of Euclid's to that of contemporary philosophy of mathematics. The *myth* of Euclid: "*Euclid's Elements contains truths about the universe which are clear and indubitable*", however, today advanced student of *geometry* to learn *Euclid's proofs* are *incomplete* and *unintelligible*. *Kant's theory of knowledge* implies to the critical examinations of those *myths*. In fact, being a *myth* doesn't entail its truth or falsity. *Myths* validate and support institutions in which their truth may not be determinable. Those latent mathematical *myths* are almost *universally accepted*, but they are not *self-evident or self-proving*. From a different perspective, it is possible to question, doubt, or reject them and some people do reject them.

Smith, N. K. concerns with Kant's conclusion that there is no *dwelling-place* for permanent settlement obtained only through perfect certainty in our *mathematical*

¹³ *Ibid.* p. 13

knowledge, alike of its objects themselves and of the limits which all our knowledge of object is enclosed. In other word¹⁴, *Kant's theory of knowledge* implies to *undogmatization* and *de-mythologization* of *mathematical foundations* as well as to rage the institutionalization of the *research of mathematical foundations*. In term of these perspectives, Kant considers himself as contributing to the further advance of the eighteenth century *Enlightenment* and in the future prospect of *mathematics philosophy*.

2. Kant's Theory of Knowledge Contributes to Epistemological Foundation of Mathematics

Kant (1783) in “*Prolegomena to Any Future Metaphysics*”, claims that the conclusions of mathematicians proceed according to the *law of contradiction*, as is demanded by all *apodictic certainty*. Kant¹⁵ says that it is a great mistake for men persuaded themselves that the *fundamental principles* were known from *the same law*. Further, Kant¹⁶ argues that *the reason* that for a *synthetical proposition* can indeed be comprehended according to *the law of contradiction* but only by presupposing another *synthetical proposition* from which it follows. Further, Kant¹⁷ argues that *all principles of geometry* are *no less analytical*; and that the proposition “*a straight line is the shortest path between two points*” is a *synthetical proposition* because *the concept of straight* contains *nothing of quantity*, but only *a quality*.

Meanwhile, Shabel L. believes that Kant explores an *epistemological explanation* whether *pure geometry* ultimately provides a structural description of certain features of empirical objects. According to Shabel L.¹⁸, Kant requires his first articulation that *space* is *a pure form of sensible intuition* and argues that, in order to explain the *pure geometry* without *paradox*, one must take the concept of *space* to be *subjective*, such that it has its source in our cognitive constitution. Kant¹⁹ perceives that *epistemological foundation of*

¹⁴ Smith, N. K., 2003, “*A Commentary to Kant's Critique of Pure Reason*”, New York: Palgrave Macmillan, p. 21

¹⁵ *Ibid*

¹⁶ *Ibid.*

¹⁷ *Ibid.*

¹⁸ Shabel, L., 1998, “*Kant's “Argument from Geometry”*”, Journal of the History of Philosophy, The Ohio State University, p.19

¹⁹ *Ibid. p.20*

geometry is only possible under the *presupposition* of a given way of explaining our pure intuition of *space* as the form of our *outer sense*.

For Kant²⁰ and his contemporaries, the *epistemological foundations of mathematics* consists amount of a view to which our *a priori* mental representation of *space-temporal intuition* provides us with the original cognitive object for our mathematical investigations, which ultimately produce *a mathematical theory of the empirical world*. However²¹, Kant's account of *mathematical cognition* serves still remains unresolved issues. Shabel L.²² concludes that the great attraction of *Kant's theory of knowledge* comes from the fact that other views seem unable to do any better.

3. Kant's Theory of Sensible Intuition Contributes to Constructive and Structural Mathematics

For Kant²³, to set up the *foundation of mathematics* we need to start from the very initially step analysis of *pure intuition*. Kant means by a “*pure intuition*” as an intuition purified from particulars of experience and conceptual interpretation. i.e., we start with experience and abstract away from concepts and from particular sensations. The impressions made by *outward thing* which is regarded as *pre-established forms* of sensibility i.e. *time* and *space*. *Time*²⁴ is no empirical conception which can be deduced from experience and *a necessary representation* which lies at the foundation of all intuitions. It is given *a priori* and, in it alone, is any reality of phenomena possible; it disappears, but it cannot be annihilated. *Space* is an intuition, met with in us *a priori*, *antecedent* to any perception of objects, a pure, not an empirical intuition. These two *forms of sensibility*, inherent and invariable to all experiences, are subject and prime facts of consciousness in the *foundation of mathematics*.

According to Kant²⁵, mathematics depends on those of *space* and *time* that means that the *abstract extension of the mathematical forms* embodied in our experience

²⁰ *Ibid.p.34*

²¹ *Ibid. p.34*

²² In Wilder, R. L. , 1952, “*Introduction to the Foundation of Mathematics*”, New York, p.205

²³ ----, 2003, “*Kant's Mathematical Epistemology*”, Retrieved 2004 <<http://www.wesleyan.edu/phil/courses/202/s00/pdfs/phil202s0011a.pdf>>

²⁴, “*Immanuel Kant, 1724–1804*”, Retrieved 2004 <<http://www.alcott.net/alcott/home/champions/Kant.html>>

²⁵ Wilder, R. L. , 1952, “*Introduction to the Foundation of Mathematics*”, New York, p.198

parallels an *extension of the objective world* beyond what we actually perceive. Wilder R.L. points out the arguments for the claim that intuition plays an essential role in *mathematics* are inevitably *subjectivist* to a degree, in that they pass from a direct consideration of the *mathematical statements* and of what is required for their truth verifying them. The dependence of *mathematics*²⁶ on *sensible intuition* gives some plausibility to the view that the possibility of *mathematical representation* rests on the *form of our sensible intuition*. This conception²⁷ could be extended to the *intuitive verification* of elementary propositions of the arithmetic of *small numbers*. If these propositions really are evident in their *full generality*, and hence are *necessary*, then this conception gives some insight into the nature of this evidence.

Other writer, Johnstone H.W. in Sellar W. ascribes that Kant's sensible intuition account the role in *foundation of mathematics* by the productive imagination in perceptual geometrical shapes. *Phenomenological reflection*²⁸ on the structure of perceptual geometrical shapes, therefore, should reveal the *categories*, to which these objects belong, as well as the manner in which objects perceived and perceiving subjects come together in the perceptual act. To dwell it we need to consider Kant's distinction between (a) *the concept of an object*, (b) *the schema of the concept*, and (c) *an image of the object*, as well as his explication of the distinction between *a geometrical shape* as object and *the successive manifold* in the apprehension of a geometrical shape.

Johnstone H.W.²⁹ highlights Kant's notion that the *synthesis* in connection with perception has two things in mind (1) *the construction of mathematical model as an image*, (2) *the intuitive formation of mathematical representations as a complex demonstratives*. Since *mathematical intuitions* have *categorical form*, we can find this *categorical form* in them and arrive at *categorical concepts of mathematics* by abstracting from experience.

²⁶ Ibid. p.198

²⁷ Ibid. p.198

²⁸ Johnstone, H.W., 1978, in Sellars, W., 1978 “*The Role Of The Imagination In Kant's Theory Of Experience*”, Retrieved 2003 <<http://www.ditext.com/index.html>>

²⁹ Ibid

4. The Relevance of Kant's Theory of Knowledge to Contemporary Foundation of Mathematics

The relevance of Kant's theory of knowledge to the contemporary foundation of mathematics can be traced from the notions of contemporary writers. Jørgensen, K.F.(2006) admits that *a philosophy of mathematics* must square with *contemporary mathematics* as it is carried out by actual mathematicians. This³⁰ leads him to define a very general notion of constructability of mathematics on the basis of a generalized understanding of *Kant's theory of schema*. Jørgensen, K.F. further states that *Kant's theory of schematism* should be taken seriously in order to understand his *Critique*. It was science which Kant wanted to provide a foundation for. He says that one should take *schematism* to be a very central feature of *Kant's theory of knowledge*.

Meanwhile, Hanna, R. insists that Kant offers an account of human rationality which is essentially oriented towards *judgment*. According to her, Kant also offers an account of the nature of *judgment*, the nature of *logic*, and the nature of the various irreducibly different kinds of *judgments*, that are essentially oriented towards the anthropocentric empirical referential meaningfulness and truth of the proposition. Further, Hanna, R.³¹ indicates that the rest of *Kant's theory of judgment* is then thoroughly cognitive and non-reductive.

In Kant³², *propositions* are systematically built up out of directly *referential terms* (*intuitions*) and *attributive* or *descriptive terms* (*concepts*), by means of unifying acts of our *innate* spontaneous cognitive faculties. This *unification* is based on *pure logical constraints* and under *a higher-order unity* imposed by our faculty for rational *self-consciousness*. Furthermore³³ all of this is consistently combined by Kant with *non-conceptualism* about intuition, which entails that *judgmental rationality* has a *pre-rational* or *proto-rational cognitive* grounding in more basic *non-conceptual cognitive capacities* that we share with various non-human animals. In these ways, Hanna, R.³⁴ concludes that *Kant's theory of knowledge* is the *inherent philosophical interest*,

³⁰ Jørgensen, K.F., 2006, "Philosophy of Mathematics" Retrieved 2006 <<http://akira.ruk.dk/~frovin/work.htm>>

³¹ Hanna, R., 2004, "Kant's Theory of Judgment", Stanford Encyclopedia of Philosophy, Retrieved 2004, <<http://plato.stanford.edu/cgi-bin/encyclopedia/archinfo.cgi?entry=kant-judgment>>

³² Ibid.

³³ Ibid.

³⁴ Ibid.

contemporary relevance, and defensibility remain essentially intact no matter what one may ultimately think about his controversial metaphysics of transcendental idealism.

In the sense of *contemporary foundation of mathematics*, Hers R.³⁵ notifies that in providing truth and certainty in mathematics Hilbert implicitly referred Kant. He.³⁶ pointed out that, like Hilbert, Brouwer was sure that *mathematics* had to be established on a sound and *firm foundation* in which *mathematics* must start from the intuitively given. The name *intuitionism*³⁷ displays its descent from Kant's intuitionist theory of mathematical knowledge. Brouwer follows Kant in saying that *mathematics* is founded on *intuitive truths*. As it was learned that Kant though *geometry* is based on *space intuition*, and *arithmetic* on *time intuition*, that made both geometry and arithmetic "synthetic a priori". About geometry, Frege³⁸ agrees with Kant that it is *synthetic intuition*.

F. Conclusion

The conclusion of this dissertation can be taken as follows:

1. The *philosophy of mathematics* has aims to clarify and answer the questions about the *status* and the *foundation* of mathematical *objects* and *methods*, that is, ontologically clarify whether there mathematical objects *exist*, and epistemologically clarify whether all meaningful mathematical statements have objective and determine the truth. A simplistic view of the *philosophy of mathematics* indicates that there are four main schools i.e. *Platonism*, *Logicism*, *Formalism*, and *Intuitionism*.
2. *Pre-Kant's philosophy of mathematics* is organized as a debate between *Rationalists* and *Empiricists*. Kant begins the *philosophy of mathematics* with a focus on *mathematics knowers* and their epistemic relationship to *theorems* and *proofs* viz. epistemology of mathematics. The *role of Kant's theory of knowledge* in setting up the epistemological foundation of mathematics emerges from Kant's efforts to set forth *epistemological foundation of mathematics* based on the *synthetical a priori*

³⁵ Hersh, R., 1997, "What is Mathematics, Really?", London: Jonathan Cape, p.162

³⁶ Ibid.p.162

³⁷ Ibid. p.162

³⁸ In Hersh, R., 1997, "What is Mathematics, Really?", London: Jonathan Cape, p.162

principles in which he believes that the *judgments of mathematics* is an *instances of genuine knowledge*.

3. Kant's most significant contribution to *modern philosophy* is the recognition that *mathematical knowledge* is possible. The *privileging* of mathematical thought after Kant seems to derive from Kant's earlier distinguishing of the *models of intuition* and *thought*. *Kant's epistemological mathematic* is the principle that an inference is gapless when one grasps a *mathematical architecture* in which a mathematical justification of the conclusion is seen as a development of a mathematical justification of the *premises*.
4. Kant believes that the *judgments of mathematics* and *Newtonian physics* to be instances of *genuine knowledge*. *Mathematics* and *science* are *objective* and *universally valid*, because all human beings know in the same way. *Kant's theory of knowledge* recognizes broad notions of *mathematics* and *intuition*, so that both *algebra* and *geometry* are accommodated. *Kant's theory of knowledge* has inherently *philosophical interest*, *contemporary relevance*, and *defensibility argument* to remain essentially intact to the *foundations of mathematics* no matter what one may ultimately think about controversial of his *metaphysics* and *epistemology of transcendental idealism*.
5. In term of the current tendency of the *philosophy of mathematics*, *Kant's theory of knowledge* is in line with the perception that *understanding of mathematics* can be supported by the nature of perceptual faculties. There are at least two philosophical lines in which they have different position of epistemological problems in the *epistemological foundation of mathematics*. The *first line* perceives that *mathematics* should be limited by the nature of perceptual faculties. The *second line* perceives that problems in *mathematics* are not consistent with *perceptual abilities*, but do *not* limit *mathematics* to what is able to intuit.

REFERENCE

- , “Encyclopedia—Kant, ImmanuelKant”, Retrieved 2004 <<http://www.factmonster.com/encyclopedia.html>>
- , 1990, *Kant, Rorty and transcendental argumentation*, Paper was written at Section de Philosophie, Université de Fribourg

- Gödel, K., 1961, “*The modern development of the foundations of mathematics in the light of philosophy*”. Retrieved 2003 <<http://www.marxists.org/glossary/people/g/o.htm#godel-kurt>>
- Hanna, R., 2004, “*Kant's Theory of Judgment*”, Stanford Encyclopedia of Philosophy, Retrieved 2004, <<http://plato.stanford.edu/cgi-bin/encyclopedia/archinfo.cgi?entry=kant-judgment>>
- Hers, R., 1997, “*What is Mathematics*”, Really?, London: Jonathan Cape, pp.9
- Jørgensen, K.F., 2006, “Philosophy of Mathematics” Retrieved 2006 <<http://akira.ruk.dk/~frovin/work.htm>>
- Kant, I., 1781, “*Critic of Pure Reason*”, Translated by J.M.D. Meiklejohn
- Kant, I., 1783. “*Prolegomena to Any Future Metaphysic.*” Trans. Paul Carus.. Retrieved 2003 <www.phil-books.com/>
- Kant, I. “*Prolegomena To Any Future Metaphysics, First Part Of The Transcendental Problem: How Is Pure Mathematics Possible?*” Remark 1, 28, *The Kant Project*. Retrieved 2003 <kant@math.tu-berlin.de>
- Kant, I. “*The Critic of Pure Reason.*” Trans. Meiklejohn, J.M.D. New York: The MacMillan Company. 1914. Retrieved 2003 <<http://www.encarta.msn.com/>>
- Kant, I. “*Theoretical Philosophy 1755-1770.*” Trans. David Walford. Cambridge: Cambridge University Press. 1992
- Kant, I., 1781, “*Prolegomena: Preamble On The Peculiarities Of All Metaphysical Cognition, Sect. 2. Concerning The Kind Of Cognition Which Can Alone Be Called Metaphysical*”, Trans. Paul Carus.. Retrieved 2003 <www.phil-books.com/>
- Kant, I., 1787, *The Critic of Pure Reason: First Part, Transcendental Aesthetic*, translated by F. Max Muller
- Kant, I., 1790, *The Critic of Judgment*, translated by James Creed Meredith
- Kant, I., 1931, “*The Critique of Judgment (tr. J.Bernard)*”, New York: The MaCmillan Company.
- Kant, I., 1992, “*Theoretical Philosophy 1755-1770 (tr. By David Walford)*”, Cambridge: Cambridge University Press
- Koetsier, T., 1991, “*Lakatos' Philosophy of Mathematics, A Historical Approach*”, Retrieved 2004 <<http://www.peccate.com/>>
- Krausz in Tuchanska, B., 1999, “*Is a Non-Foundationalist Epistemology Possible?*”, Retrieved 2004 <<http://www.Google.com/Tuchanska>>
- McFarlane, T.J. 1995, “*Kant and Mathematical Knowledge.*”. Retrieved 2004 <<http://www.integralscience>>
- Randall, A., 1998, “*A Critique of the Kantian View of Geometry*”, Retrieved 2004 <<http://home.ican.net/~arandall/Kant/Geometry/>>
- Randall, A., 1998, “*In Defence of Transcendental idealism*”, Retrieved 2003, <<http://home.ican.net/>>
- Rorty in Tuchanska, B., 1999, “*Is a Non-Foundationalist Epistemology Possible?*”, Retrieve 2004 <<http://www.Google.com/Tuchanska>>
- Sellars, W., 1976, “*Kant's Transcendental Idealism*”, Collections of Philosophy:165-181.
- Sellars, W., 1978, “*The Role Of The Imagination In Kant's Theory Of Experience*”, Retrieved 2003 <<http://www.ditext.com/index.html>>

- Shabel, L., 1998, “*Kant on the ‘Symbolic Construction’ of Mathematical Concepts*”, Pergamon Studies in History and Philosophy of Science , Vol. 29, No. 4. pp. 591
- Shabel, L., 2003, “*Reflections on Kant’s concept (and intuition) of space*”, Studies In History and Philosophy of Science Part A Volume 34, Issue 1 , Elsevier Science Ltd, Retrieved 2003 <<http://www.sciencedirect.com/science?>>
- Shabel, L.,1999, “*Kant’s Argument from Geometry*”, The Ohio State University forthcoming in the Journal of the History of Philosophy
- Smith, N. K., 2003, “*A Commentary to Kant’s Critique of Pure Reason: Kant on Arithmetic*, pp. 128-133”, New York: Palgrave Macmillan
- Tuchanska, B.,1999, “*Is a Non-Foundationalist Epistemology Possible?*”, Retrieve 2004 <<http://www.Google.com/Tuchanska>>
- Wilder,R.L., 1952, “*Introduction to the Foundation of Mathematics*”, New York